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Impacts of High Frequency Spectral Tail of the Surface Gravity Waves on the Stokes Drift

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Abstract. The impacts of different wave spectrum formulations with high frequency tail on the Stokes drift are studied in this paper. Two wave spectrum formulations are described. One is from the Donelan and Pierson spectrum extended to the high frequency range (denoted by EDP spectrum) by Paskyabi and Jenkins, and the other is a combined spectrum (termed the Combi spectrum) provided by Tsagareli. Using the EDP spectrum and Combi spectrum, the Stokes drift velocities and Stokes transports are calculated for the different wind speeds. The results show that the high frequency spectral tail of the surface gravity waves obviously changes the Stokes drift velocity on the sea surface, but has little impact on the Stokes transport.

1. Introduction

Surface gravity waves are responsible for generating the Stokes drift in the oceanic upper layers. Stokes drift is able to transport the mass, heat and momentum in the upper ocean to the deeper water, providing nutrients for the marine life in the deep sea. Therefore, it is very significant to study the effects of surface waves and Stokes drift on the mixing in the oceanic mixed layer.

In recent years, many researchers have discussed the surface gravity waves and Stokes drift using the related ocean model. Webb and Fox-Kemper [13] evaluated the relationships between the wave spectrum moments and Stokes drift velocity according to the empirical spectral shapes and a third-generation wave model. Paskyabi and Jenkins [9] studied the effects of surface gravity wave on the upper ocean boundary layer by a one-dimensional vertical mixing model. Siadatmousavi et al. [10] used the SWAN and WAVEWATCH-III models to discuss the interaction of low and high frequency components in the wave spectral evolution.

However, the effects of high frequency portion of the surface gravity waves on the Stokes drift are still not studied. In this paper, we will use the EDP spectrum and Combi spectrum to calculate the impacts of high frequency spectral tail on the Stokes drift velocity and Stokes drift transport.



2. Material and method

2.1. EDP spectrum

The EDP spectrum is from Donelan and Pierson spectrum [3] when the frequency is less than f_{\max} (cut-off frequency), and is the f^{-5} tail at high frequencies used by the preceding spectrum. It has the following general form:

$$E(f) = \begin{cases} 0.0054 \frac{g^2}{(2\pi)^4} f_p^{-1} f^{-4} \exp\left[-\left(\frac{g}{2.4\pi f U_{10}}\right)^4\right] 1.7^\Gamma, & f \leq f_{\max} \\ E(f_{\max}) \cdot (f / f_{\max})^{-5}, & f > f_{\max} \end{cases} \quad (1)$$

Where $\Gamma = \exp\left[-1.22\left(\frac{2.4\pi U_{10} f}{g} - 1\right)^2\right]$, g is the acceleration due to gravity, U_{10} is the surface wind speed at the reference height 10 m, f is the frequency, and f_p is the frequency at the spectral peak, $f_p = g / (2.4\pi U_{10})$.

The directional spreading function $D(f, \theta)$ of the EDP spectrum is from Paskyabi and Jenkins [9]

$$D(f, \theta) = \frac{1}{2} \mu\left(\frac{f}{f_p}\right) \cdot \operatorname{sech}^2\left[\mu\left(\frac{f}{f_p}\right)\theta\right] \quad (2)$$

Where θ is the wave direction relative to the wind and μ is given by

$$\mu\left(\frac{f}{f_p}\right) = \begin{cases} 1.24, & 0 < f / f_p < \sqrt{0.31} \\ 2.61(f / f_p)^{1.3}, & \sqrt{0.31} \leq f / f_p < \sqrt{0.9} \\ 2.28(f_p / f)^{1.3}, & f / f_p \geq \sqrt{0.9} \end{cases} \quad (3)$$

The lower and upper limits of the frequency are separately taken as 0.01 Hz and 10 Hz, and the cut-off frequency f_{\max} is chosen as $\sqrt{10}f_p$. For a fully developed wind generated sea, the EDP spectrum with the frequency for $U_{10}=10$ m/s is shown as Figure 1. The energy flux value of EDP spectrum is $0.5398 \text{ m}^2\text{s}$ in the full frequency, and is $0.5304 \text{ m}^2\text{s}$ in the cut-off frequency.

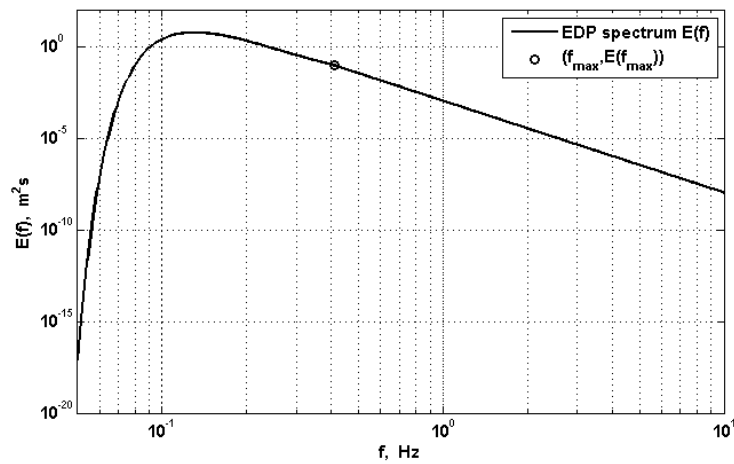


Figure 1. EDP spectrum changes with frequency at $U_{10}=10$ m/s for the fully developed sea.

$(f_{\max}, E(f_{\max}))$ is the cut-off point of EDP spectrum at the frequency $\sqrt{10}f_p$.

2.2. Combi spectrum

The Combi spectrum [1, 12] is modeled on the wave spectrum from Donelan et al. [2]. It exhibits an f^{-4} dependence close to the peak for the equilibrium range and an f^{-5} dependence for the high frequency range. The Combi spectrum is defined a

$$\begin{cases} F(f) = \alpha \frac{g^2}{(2\pi)^4} f_p^{-1} f^{-4} \exp\left[-\left(\frac{f}{f_p}\right)^{-4}\right] \gamma_D \exp\left[-\frac{(f-f_p)^2}{2\sigma^2 f_p^2}\right] & f \leq f_t \\ F(f) = \alpha \frac{g^2}{(2\pi)^4} f_t f_p^{-1} f^{-5} \exp\left[-\left(\frac{f}{f_p}\right)^{-4}\right] \gamma_D \exp\left[-\frac{(f-f_p)^2}{2\sigma^2 f_p^2}\right] & f > f_t \end{cases} \quad (4)$$

Where $f_t = 2.5g/\pi U_{10}$ is a transition frequency, $\alpha = 0.006\Omega^{0.55}$ ($0.8333 \leq \Omega < 5$) is the tail level (where Ω is the inverse wave age), $\sigma = 0.08[1 + 4\Omega^{-3}]$ is the peak width, $f_p = g\Omega/2\pi U_{10}$ is the spectral peak frequency, and $\gamma_D = \begin{cases} 1.7, & 0.8333 \leq \Omega < 1 \\ 1.7 + 6 \cdot \log(\Omega), & 1 \leq \Omega < 5 \end{cases}$. These parameters are from the standard JONSWAP spectrum [4]. For the fully-developed waves, the inverse wave age Ω is set as 0.8333.

The directional spreading function of the Combi spectrum is chosen as [7, 12]

$$D_{\text{dir}}(f, \theta) = \frac{2^{2s-1}}{\pi} \cdot \frac{\Gamma^2(s+1)}{\Gamma(2s+1)} \cdot \cos^{2s}\left(\frac{\theta}{2}\right) \quad (5)$$

Where the parameter s as defined by Mitsuyasu et al. [8] is:

$$s = \begin{cases} s_p \left(\frac{f}{f_p} \right)^5, & f < f_p \\ s_p \left(\frac{f}{f_p} \right)^{-2.5}, & f \geq f_p \end{cases} \quad (6)$$

$$s_p = 11.5 \cdot \left(\frac{U_{10}}{c_p} \right)^{-2.5} \quad (7)$$

Where s_p is the value of s at the peak frequency f_p , and c_p is the corresponding phase speed.

The lower and upper limits of the frequency are the same as those of the EDP spectrum, and the cut-off frequency f_{\max} is also chosen as $\sqrt{10}f_p$. The Combi spectrum with the frequency for $U_{10}=10$ m/s is shown as Figure 2 in the fully developed sea. The energy flux values of Combi spectrum are $0.5447 \text{ m}^2\text{s}$ and $0.5326 \text{ m}^2\text{s}$ in the full and cut-off frequency, respectively.

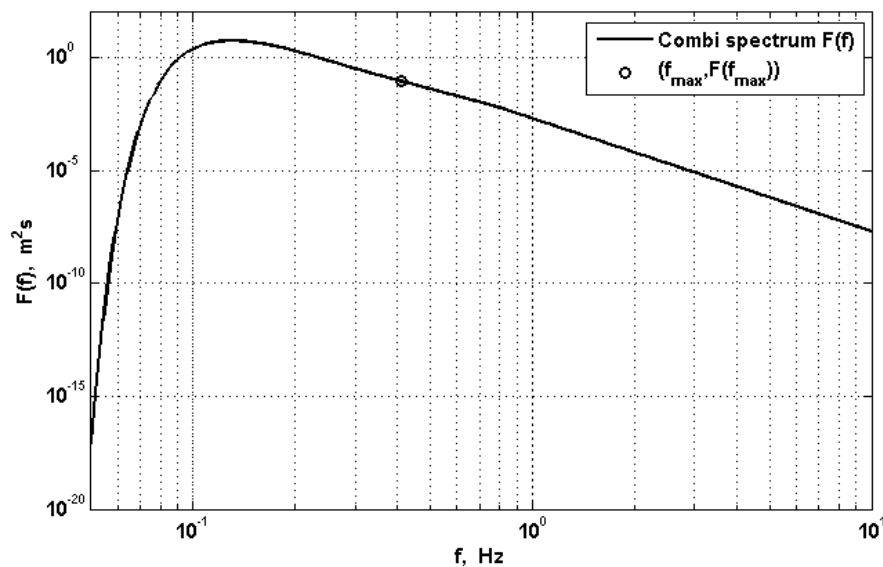


Figure 2. Combi spectrum changes with frequency at $U_{10}=10$ m/s for the fully developed sea.

$(f_{\max}, E(f_{\max}))$ is the cut-off point of Combi spectrum at the frequency $\sqrt{10}f_p$.

3. Result

The Stokes drift u_s is represented as [5, 6, 11]

$$u_s = 4\pi \iint f k e^{2kz} E(f, \theta) df d\theta, \quad (8)$$

Where $\mathbf{k} = (k \cos \theta, k \sin \theta)$ the wavenumber is vector and $E(f, \theta)$ is the directional frequency spectrum. Then, the Stokes drift velocities with the depth are separately calculated using the EDP spectrum and Combi spectrum, as shown in Figure 3 and Figure 4.

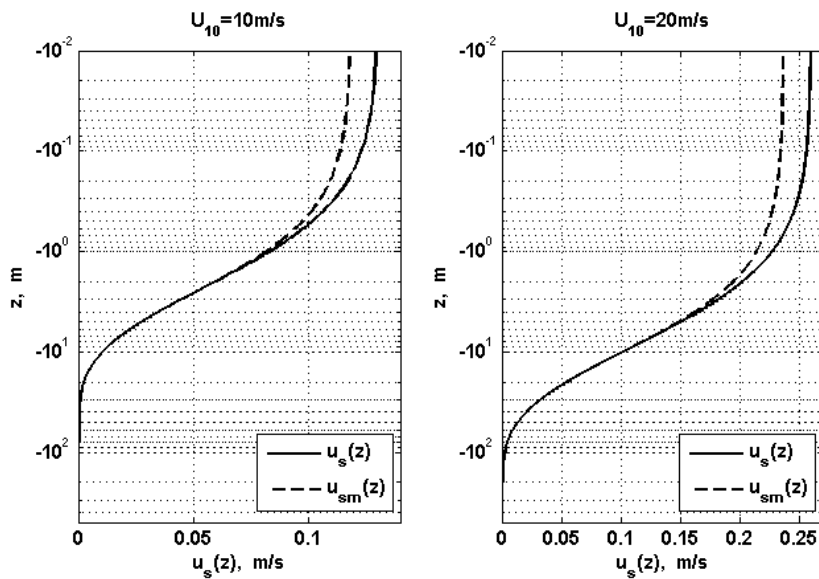


Figure 3. Stokes drift velocities with the depth for the wind speed 10 m/s (left) and 20 m/s (right) using the EDP spectrum. $u_{sm}(z)$ and $u_s(z)$ are the Stokes drift velocities in the cut-off and full frequency, respectively.

For the fully developed sea, according to the EDP spectrum, the Stokes drift velocities on the sea surface are 0.1184 m/s in the cut-off frequency and 0.1301 m/s in the full frequency for the wind speed 10 m/s, respectively. When the wind speed is 20 m/s, the Stokes drift velocities are separately 0.2358 m/s and 0.2588 m/s. The effect of high frequency tail on the Stokes drift velocity of sea surface is 9.88% and 9.75% under the wind speed 10 m/s and 20 m/s, respectively. The result shows that the high frequency spectral tail of the surface gravity waves obviously changes the Stokes drift velocity on the sea surface. But the increment of Stokes drift transport induced to the high frequency tail from -10 m to the ocean surface is about 1.55%, and is almost negligible.

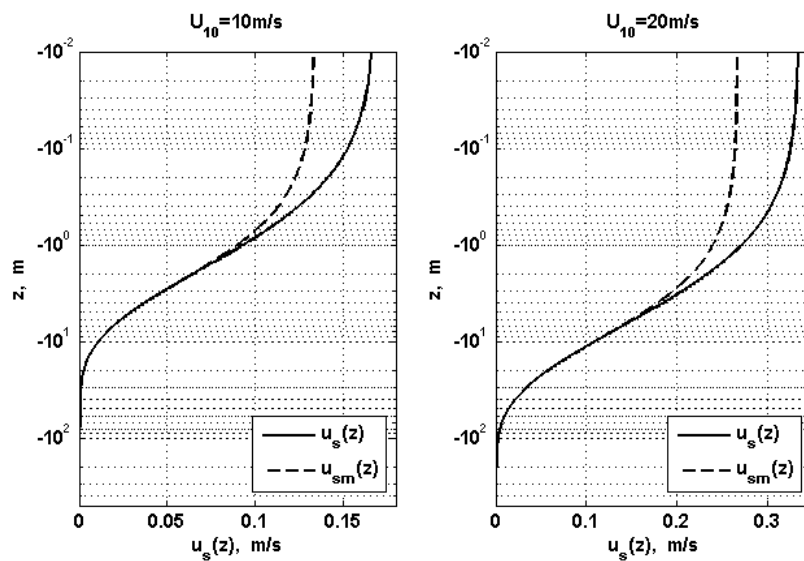


Figure 4. Stokes drift velocities with the depth for the wind speed 10 m/s (left) and 20 m/s (right) using the Combi spectrum. $u_{sm}(z)$ and $u_s(z)$ are the Stokes drift velocities in the cut-off and full frequency, respectively.

When the wind speed is 10 m/s, according to the Combi spectrum, the Stokes drift velocities on the sea surface are separately 0.1338 m/s in the cut-off frequency and 0.1679 m/s in the full frequency in the fully developed sea. The Stokes drift velocities are 0.2671 m/s and 0.3345 m/s for the wind speed 20 m/s, respectively. The increment of Stokes drift velocity generated by high frequency tail is 25.49% and 25.23% for the wind speed 10 m/s and 20 m/s, respectively. The result demonstrates that the high frequency portion of the surface gravity waves significantly influences the Stokes drift velocity on the sea surface. However, the increment of Stokes drift transport induced to the high frequency is only 3.34%, and is also ignored.

4. Conclusion

This paper used the EDP spectrum and Combi spectrum to calculate effects of high frequency tail of surface gravity waves on the Stokes drift. Two spectrums are the f^{-5} tail at high frequencies used by the preceding spectra, but the formulations are different. Under the conditions of different wind speeds, the Stokes drift velocities and transports were calculated according to the EDP spectrum and Combi spectrum.

For the fully developed sea, the impacts of high frequency tail on the Stokes drift velocities of the sea surface is 9.88% and 25.49% for the wind speed 10 m/s using the EDP spectrum and Combi spectrum, respectively. And for the wind speed 20 m/s, the effects are separately 9.75% and 25.23% by means of these two spectrums. Results indicate that the impacts of high frequency portion of surface gravity waves on the Stokes drift velocity are very significant. But the increments of Stokes drift transport induced to the high frequency tail from -10 m to the ocean surface are about 1.55% and 3.34% by the EDP spectrum and Combi spectrum, and are almost negligible.

Acknowledgments

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