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Adaptive multi-dimensional Taylor network tracking control for a class of stochastic nonlinear systems with unknown input dead-zone

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ABSTRACT In this paper, multi-dimensional Taylor network (MTN) tracking control scheme is proposed for a class of stochastic nonlinear systems with unknown input dead-zone. The multi-dimensional Taylor networks (MTNs) are used to approximate the nonlinearities, and then, an adaptive MTN controller is constructed via backstepping technique. It is proved that the design MTN controller ensures that all signals of the closed-loop system remain bounded in probability, and the tracking error eventually converges to an arbitrarily small neighborhood around the origin in the sense of mean quartic value. Finally, two numerical examples and one practical example are given to demonstrate the effectiveness of the proposed design method.

INDEX TERMS Tracking control, stochastic nonlinear systems, dead-zone, backstepping technique, multi-dimensional Taylor network.

I. INTRODUCTION

In the past several decades, the stability analysis and controller design of nonlinear systems have received considerable attention, and many research results obtained for nonlinear systems have been successfully applied in the many practice engineering, such as underwater vehicle-manipulator system [1], cable-driven manipulators [2,3] and so on. As an important brand of control theory, the adaptive control of stochastic nonlinear systems has been extensively studied because of stochastic disturbance often exists in practical systems. Many control approaches obtained for deterministic systems, such as Lyapunov stability theory [4], LaSalle invariance principle [5, 6], backstepping technique [7-10] and neural network [11-13], have been successfully extended to the stochastic nonlinear systems, for example, Pan and Basar [14] first generalised backstepping to the stochastic systems and achieved good results, since then, this technique has become a popular design tool for stochastic nonlinear systems and a battery of extensions have been made for different stochastic nonlinear systems under different assumptions [15-17]. At the same time, adaptive control approach is often used to handle the control design of stochastic nonlinear systems, for example, by using equivalent-input disturbance (EID) approach, Sakthivel et al. [18] studied the tracking control problem for networked control systems with actuator faults and external disturbances. In addition, by combining the backstepping design technique with the adaptive control approach, many interesting achievements have been achieved; see, for example, Wang et al. [17] proposed an adaptive neural decentralized control scheme for a class of multiple-input multiple-output (MIMO) stochastic nonlinear systems with strong interconnected. Xie and Tian [16] studied the state-feedback stabilization problem for high-order stochastic nonlinear systems with nonlinear parameterization. Zhao et al. [19] considered the tracking control problem for a class of switched stochastic nonlinear systems. Bu using EID approach, Sakthivel et al. [20] developed a repetitive control scheme for stochastic nonlinear systems with time-varying delay.

On the other hand, the method of approximation-based adaptive neural network (NN) has been successfully applied to control of stochastic nonlinear systems, and many interesting results have been reported, see, for example, Wang et al. [13] proposed an adaptive neural control scheme for a class of single-input single-output

(SISO) stochastic nonlinear systems. For a class of SISO stochastic nonlinear systems with unknown time-delay, Zhou et al. [21] proposed a direct adaptive NN control scheme. Li et al. [22] investigated the adaptive NN output-feedback stabilization problem for a class of MIMO stochastic nonlinear systems. A novel adaptive NN tracking control scheme was proposed in [23] for a class of MIMO nonlinear time-delay systems with block-triangular structure. Li et al. [24] proposed a novel decentralized adaptive neural control scheme for a class of uncertain MIMO nonlinear systems with time-delay. However, most of the results were too complex to be feasible for real applications, the reasons are chiefly as follows: (1) The long training time leads to most NNs can't meet the quick performance of the control systems. (2) The real-time performance of most NNs is unsatisfactory due to the computation complexity of each neuron. Therefore, it is still a meaningful and challenging task to construct a simple but effective adaptive control algorithm for the stochastic nonlinear systems.

Despite many developments have been achieved, the mentioned above control approaches only focus on the stochastic nonlinear systems without nonsmooth nonlinear characteristics. It is well known that nonsmooth nonlinear characteristics such as dead-zone, backlash, hysteresis and saturation are frequently exist in industrial control systems. Among them, dead-zone often exist in hydraulic valves, motors, amplifiers and even in biomedical actuation systems [25], which is a source of instability. Therefore, severely stochastic nonlinear systems with unknown input dead-zone may face great challenges in achieving desired stability. Therefore, the investigation on stochastic systems with unknown input dead-zone have received considerable attention in recently years [26-28]. The most formidable task in the controller design is to handle the nonlinearity inherit from dead-zone. By introducing characteristic function, Zhang and Ge [29, 30] denoted the dead-zone output as a simple linear system with bounded disturbance and a static time-varying gain. And then, this idea has become a popular design approach for the nonlinear systems with dead-zone [31-33]. However, as far as we know, the tracking control problem of stochastic nonlinear systems with unknown input dead-zone, which is a significant and challenging task in both theoretical analysis and practical applications.

In recent years, approximation-based on multi-dimensional Taylor network (MTN) control schemes have been found to be a credible and accurate method for the control of nonlinear systems, and many interesting results have been obtained, for example, Kang and Yan [34, 35] studied the problem of asymptotic tracking and dynamic regulation of SISO nonlinear systems with the help of MTN. A MTN control approach was proposed in [36] for a class of nonlinear time-varying delay system with an inaccurate model. However, most of the results focused on deterministic systems without stochastic disturbances. Han and Yan [37] first extend the MTN approach to a class of stochastic nonlinear systems. By combining dynamic surface control (DSC) technique and backstepping technique, Han [38] proposed an adaptive MTN control scheme for a class of stochastic nonlinear systems with immeasurable states. To the best of our knowledge, until now still no adaptive MTN control schemes have been proposed for stochastic nonlinear systems with unknown input dead-zone, which motivates our research.

Inspired by previous work, this paper tries to investigate the adaptive MTN tracking control problem for a class of stochastic nonlinear systems with unknown input dead-zone. Firstly, based on [29, 30], the dead-zone output is represented as a simple linear system. Secondly, in the controller design, MTNs are used to approximate the unknown nonlinearities, and a novel adaptive MTN scheme is developed via backstepping technique. It is proved that the proposed adaptive MTN control approach can guarantee that all the signals in the closed-loop system are remain bounded in probability and the tracking error eventually converges to a small area around the origin in the sense of mean quartic value. The main contributions of this paper are listed as follows:

1. For the first time, the MTN approach is utilized to solve the tracking problem of a class of stochastic nonlinear systems with unknown input dead-zone. Up to now, the MTN-based control approach for the nonlinear systems [34-36] and stochastic nonlinear systems [37, 38] cannot be directly used to deal with the problem of nonlinear systems with input dead-zone. In this paper, we successfully extend the MTN-based method to stochastic nonlinear systems with unknown input dead-zone.
2. In the controller design, the unknown nonlinearities of the stochastic nonlinear system are successfully estimated using the universal approximation capabilities of MTN. Compared with the existing adaptive NN control approach [32, 39, 40] for stochastic nonlinear systems with unknown input dead-zone, the computation complexity of the proposed control method is significantly reduced thanks to the simple structure of MTN. Further, the middle layer of MTN consists of an array of polynomials, which contains only addition and multiplication, and no lengthy calculation is involved. Therefore, we may conclude that the control method designed in this paper has a good real-time performance.

The rest of this paper is organized as follows. The system description and preliminaries are given in Section II. The adaptive MTN controller design procedure and the stability analysis of the closed-loop system are given in Section III. Section IV gives three simulation examples to illustrate the effectiveness of the proposed control scheme. Finally, this paper is concluded in Section V.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

The following notations are used throughout this paper. $\mathbb{R}_+$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times r}$ denote the set of all positive real numbers, the real n-dimensional space, and the set of all $n\times r$ real matrices, respectively. For a given vector or matrix $\boldsymbol{x}$, $\boldsymbol{x}^{\mathrm{T}}$

stands for its transpose and $\mathrm{Tr}(\boldsymbol{x})$ denotes its trace when $\boldsymbol{x}$ is square, and $\|\boldsymbol{x}\|$ denotes the Euclidean norm of a vector $\boldsymbol{x}$. $\mathcal{C}^i$ denotes the set of all functions with continuous i-th partial derivative. I_i is $i\times i$ identity matrix. And $P\{\mathcal{A}\}$ represents the probability of event $\mathcal{A}$.

A. STOCHASTIC STABILITY

Consider the following stochastic nonlinear system

$$\mathrm{d}\boldsymbol{x} = f(\boldsymbol{x})\mathrm{d}t + h(\boldsymbol{x})\mathrm{d}\boldsymbol{\omega} \tag{1}$$

where $\boldsymbol{x}\in\mathbb{R}^n$ is the system state, and $\boldsymbol{\omega}$ is a r-dimensional independent standard Wiener process defined on the complete probability space $(\mathscr{W},\mathscr{F},\mathscr{P})$ with $\mathscr{W}$ being a sample space, $\mathscr{F}$ being a σ-field, and $\mathscr{P}$ being a probability measure. $f:\mathbb{R}^n\to\mathbb{R}^n$ and $h:\mathbb{R}^n\to\mathbb{R}^{n\times r}$ stand for unknown smooth nonlinear functions with $f(\mathbf{0})=0$, $h(\mathbf{0})=0$.

Definition 1 [8,13]: For any given $V(\boldsymbol{x})\in\mathcal{C}^2$, associated with the stochastic nonlinear system (1), define the differential operator $\mathcal{L}$ as follows:

$$\mathcal{L}V(\boldsymbol{x}) = \frac{\partial V}{\partial \boldsymbol{x}} f + \frac{1}{2}\mathrm{Tr}\left(g^{\mathrm{T}}\frac{\partial^2 V}{\partial \boldsymbol{x}^2} g\right) \tag{2}$$

Definition 2 [13]: The solution process $\{x(t), t\ge 0\}$ of the stochastic nonlinear system (1) is said to be bounded in probability, if

$$\lim_{\varsigma\to\infty}\sup_{0\le t<\infty} P\{\|x(t)\|>\varsigma\}=0 \tag{3}$$

Lemma 1 [41]: Consider the stochastic nonlinear system (1), if there exists a positive definite, radially unbounded, twice continuously differentiable Lyapunov function $V(\boldsymbol{x}):\mathbb{R}^n\to\mathbb{R}$, and constants $\mathrm{a}>0$, $\mathrm{b}>0$ such that

$$\mathcal{L}V(\boldsymbol{x}) \le -\mathrm{a}V(\boldsymbol{x}) + \mathrm{b} \tag{4}$$

then (i) the system has a unique solution almost surely; (ii) the system is bounded in probability.

Proof See Theorem 1 in [41].

Lemma 2 (Young's inequality): For $\forall(x,y)\in\mathbb{R}^2$ and $\forall\varepsilon>0$, the following inequality holds

$$xy \le \frac{\varepsilon^{\mathrm{p}}}{\mathrm{p}}|x|^{\mathrm{p}} + \frac{1}{q\varepsilon^{\mathrm{q}}}|y|^{\mathrm{q}} \tag{5}$$

where $\mathrm{p}>1$, $\mathrm{q}>1$ and $(\mathrm{p}-1)(\mathrm{q}-1)=1$.

B. SYSTEM DESCRIPTIONS

Consider the following stochastic nonlinear system

$$\begin{cases} \mathrm{d}x_i = \left(x_{i+1} + f_i(\bar{\boldsymbol{x}}_i)\right)\mathrm{d}t + h_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)\mathrm{d}\omega \\ \quad 1\le i\le n-1 \\ \mathrm{d}x_n = \left(u + f_n(\bar{\boldsymbol{x}}_n)\right)\mathrm{d}t + h_n^{\mathrm{T}}(\bar{\boldsymbol{x}}_n)\mathrm{d}\omega \\ y = x_1 \end{cases} \tag{6}$$

where $\boldsymbol{x}=[x_1,x_2,\cdots,x_n]^{\mathrm{T}}\in\mathbb{R}^n$ and $y\in\mathbb{R}$ are the state vector and the system output, respectively, $\bar{\boldsymbol{x}}_i=[x_1,\cdots,x_i]^{\mathrm{T}}\in\mathbb{R}^i$. $f_i(\cdot):\mathbb{R}^i\to\mathbb{R}$ and $h_i(\cdot):\mathbb{R}^i\to\mathbb{R}^r$ are unknown smooth nonlinear functions satisfy $f_i(\mathbf{0})=0$ and $h_i(\mathbf{0})=0$. $u\in\mathbb{R}$ is the output of an unknown dead-zone defined as follows:

$$u = D(v) = \begin{cases} g_r(v), & v\ge b_r \\ 0, & b_l<v<b_r \\ g_l(v), & v\le b_l \end{cases} \tag{7}$$

where b_l and b_r are unknown parameters satisfy $b_l<0$ and $b_r>0$, and v is the input of the dead-zone, and the dead-zone defined in (7) is shown in Fig. 1.

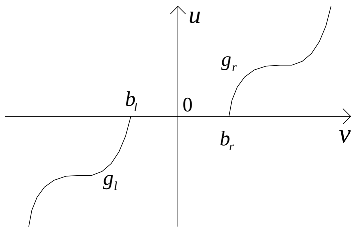

FIGURE 1. Nonsymmetric nonlinear dead-zone model.

The following assumptions are required to facilitate control system design.

Assumption 1: The output of the dead-zone (7) is not available, i.e. u is not available.

Assumption 2 [30]: For the unknown dead-zone defined in (7), $g_l(v)$ and $g_r(v)$ are smooth functions, and there exist four unknown positive constants k_{l0}, k_{l1}, k_{r0} and k_{r1} such that

$$0<k_{l0}\le g_l'(v)\le k_{l1},\quad \forall v\in(-\infty,b_l] \tag{8}$$

$$0<k_{r0}\le g_r'(v)\le k_{r1},\quad \forall v\in[b_r,+\infty) \tag{9}$$

where $g_l'(v)=\left.\frac{dg_l(z)}{dz}\right|_{z=v}$ *and* $g_r'(v)=\left.\frac{dg_r(z)}{dz}\right|_{z=v}$, *and there exists a known positive constant β_0 satisfies $\beta_0\le\min\{k_{l0},k_{r0}\}$.*

Based on Assumption 2 and [29, 30] the dead-zone (7) can be rewritten as

$$u = D(v) = K^{\mathrm{T}}(t)\Phi(t)v + d(v) \tag{10}$$

where

$$\Phi(t) = \left[\varphi_r(t),\varphi_l(t)\right]^{\mathrm{T}} \tag{11}$$

$$K(t) = \left[K_r(v(t)),K_l(v(t))\right]^{\mathrm{T}} \tag{12}$$

$$d(v) = \begin{cases} -g_r'(\xi_r(v))b_r, & v\ge b_r \\ -\left[g_l'(\xi_l(v))+g_r'(\xi_r(v))\right]v, & b_l<v<b_r \\ -g_l'(\xi_l(v))b_l, & v\le b_l \end{cases} \tag{13}$$

with

$$\varphi_r(t)=\begin{cases}1, & v(t)>b_l\\ 0, & v(t)\le b_l\end{cases}$$

$$\varphi_l(t)=\begin{cases}1, & v(t)<b_r\\ 0, & v(t)\ge b_r\end{cases}$$

$$K_r(v(t))=\begin{cases}0, & v(t)\le b_l\\ g_r'(\xi_r(v(t))), & b_l<v(t)<+\infty\end{cases}$$

$$K_l(v(t))=\begin{cases}g_l'(\xi_l(v(t))), & -\infty<v(t)<b_r\\ 0, & v(t)\ge b_r\end{cases}$$

$$\xi_l(v)\in\begin{cases}(v,b_l), & v<b_l\\ (b_l,v), & b_l\le v<b_r\end{cases}$$

$$\xi_r(v)\in\begin{cases}(b_r,v), & b_r<v\\ (v,b_r), & b_l<v\le b_r\end{cases}$$

and $|d(v)|\le\tilde{k}$, $\tilde{k}$ is an unknown positive constant with $\tilde{k}=(k_{l1}+k_{r1})\max\{b_r,-b_l\}$.

Remark 1 According to (8), (9), (11) and (12), we can easily get

$$\beta_0\le K^{\mathrm{T}}(t)\Phi(t)\le k_{l1}+k_{r1} \tag{14}$$

Assumption 3: The desired trajectory y_d and its time derivatives up to the n-th order are continuous and bounded.

C. MULTI-DIMENSIONAL TAYLOR NETWORK [38]

A typical MTN with an input layer, a middle layer and an output layer, and its structure is shown in Figure 2. Its mathematical expression can be defined as

$$f_{\mathrm{MTN}}(\boldsymbol{s})=\boldsymbol{\theta}^{\mathrm{T}}P_m(\boldsymbol{s}) \tag{15}$$

where $\boldsymbol{s}=[s_1,\cdots,s_n]^{\mathrm{T}}\in\mathbb{R}^n$ and $\boldsymbol{\theta}=[\theta_1,\cdots,\theta_l]^{\mathrm{T}}\in\mathbb{R}^l$ are the input vector and the weight vector, respectively. The elements of the middle layer $P_m(\boldsymbol{s})$ are $\prod_{i,j=1}^{n}s_i^{\sigma_i}s_j^{\sigma_j}$, with $\sigma_i\in\mathbb{R}_+$ and $\sigma_j\in\mathbb{R}_+$ satisfy $1\le\sigma_i+\sigma_j\le m$, m is the highest power of the middle layer. i.e. $P_m(\boldsymbol{s})=[\underbrace{s_1,\cdots,s_n}_{1\text{ item}},\underbrace{s_1^2,s_1s_2,\cdots,s_n^2}_{2\text{ item}},\cdots,\underbrace{s_n^m}_{m\text{ item}}]^{\mathrm{T}}$, where n and l are the number of input and middle layer of MTN, respectively.

Remark 2: As shown in [34, 42-44], the value of l can be determined by m and n.

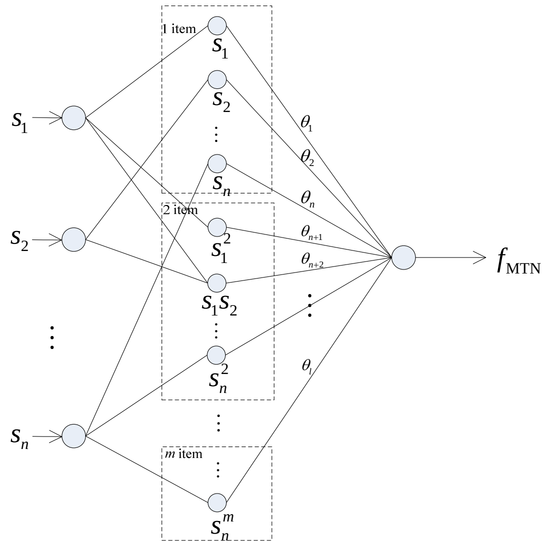

FIGURE 2. The structure of MTN.

In this paper, on a compact set $D\in\mathbb{R}^n$, an unknown smooth nonlinear function $f(\boldsymbol{s})$ will be approximated by the following MTN

$$f(\boldsymbol{s})=\boldsymbol{\theta}^{*\mathrm{T}}P_m(\boldsymbol{s})+\delta(\boldsymbol{s}) \tag{16}$$

where $\delta(\boldsymbol{s})$ is the MTN inherent approximation error and $\boldsymbol{\theta}^*$ is the ideal constant weight vector defined as

$$\boldsymbol{\theta}^*=:\arg\min_{\boldsymbol{\theta}\in\mathbb{R}^l}\left\{\sup_{s\in D}f(\boldsymbol{s})-\boldsymbol{\theta}^{\mathrm{T}}P_m(\boldsymbol{s})\right\}.$$

III. MAIN RESULTS

A. ADAPTIVE MTN CONTROL DESIGN

First of all, introduce the following coordinate transformation:

$$z_i=x_i-\alpha_{i-1},\ i=1,2,\cdots,n \tag{17}$$

where α_{i-1} is the intermediate virtual control signal to be designed and $\alpha_0=y_d$.

Then, according (6) and (17), we have

$$\begin{cases}\mathrm{d}z_1=(x_2+f_1-\dot{y}_d)\mathrm{d}t+\bar{h}_1^{\mathrm{T}}\mathrm{d}\omega\\ \mathrm{d}z_i=(x_{i+1}+f_i-L_{\alpha_{i-1}})\mathrm{d}t+\bar{h}_i^{\mathrm{T}}\mathrm{d}\omega\\ \quad 2\le i\le n-1\\ \mathrm{d}z_n=(u+f_n-L_{\alpha_{n-1}})\mathrm{d}t+\bar{h}_n^{\mathrm{T}}\mathrm{d}\omega\end{cases} \tag{18}$$

where

$$L_{\alpha_{i-1}}=\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1}+f_j)+\sum_{j=0}^{i-1}\frac{\partial\alpha_{j-1}}{\partial y_d^{(j)}}y_d^{(j+1)}+\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}_{i-1}}\dot{\hat{\boldsymbol{\theta}}}_{i-1}+\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}\bar{h}_p^{\mathrm{T}}\bar{h}_q\ (i=2,\cdots,n)$$

and $\bar{h}_1=h_1$, $\bar{h}_i=h_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}h_j\ (i=2,\cdots,n)$.

For notational simplicity, the functions $f_i(\bar{\boldsymbol{x}}_i)$, $h_i(\bar{\boldsymbol{x}}_i)$ and $\bar{h}_i(\bar{\boldsymbol{x}}_i)$ are abbreviated as f_i, h_i and $\bar{h}_i$, respectively.

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Next, a MTN-based control design procedure will be developed via backstepping.

Step 1: According to (18), one has

$$\mathrm{d}z_1 = \left(x_2 + f_1 - \dot{y}_d\right)\mathrm{d}t + \bar{h}_1^{\mathrm{T}}\mathrm{d}\omega \tag{19}$$

Consider the stochastic Lyapunov function candidate as

$$V_1 = \frac{1}{4}z_1^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\tilde{\boldsymbol{\theta}}_1 \tag{20}$$

where $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$ is the parameter error, $\Gamma_1 = \Gamma_1^T > 0$ is any constant matrix.

By Definition 1 and (19) we have

$$\begin{aligned}\mathcal{L}V_1 &= z_1^3\left(x_2 + f_1 - \dot{y}_d\right) + \frac{3}{2}z_1^2\left\|\bar{h}_1\right\|^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \\ &\le z_1^3\left(x_2 + \bar{f}_1\right) - \frac{3}{4}z_1^4 + \frac{3}{4}l_1^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1\end{aligned} \tag{21}$$

where $\bar{f}_1 = f_1 - \dot{y}_d + \frac{3}{4}z_1 + \frac{3}{4l_1^2}z_1\left\|\bar{h}_1\right\|^4$ with $l_1 \in \mathbb{R}_+$ being a design constant.

It is obvious that $\bar{f}_1$ is an unknown function, according to (16), we can employ a MTN to estimate it, in other words, for any given $\varepsilon_1 > 0$, there exists a MTN $\boldsymbol{\theta}_1^{\mathrm{T}}P_{m_1}\left(\boldsymbol{z}_1\right)$ such that

$$\bar{f}_1 = \boldsymbol{\theta}_1^{\mathrm{T}}P_{m_1}\left(\boldsymbol{z}_1\right) + \delta_1\left(\boldsymbol{z}_1\right), \left|\delta_1\left(\boldsymbol{z}_1\right)\right| \le \varepsilon_1 \tag{22}$$

where $\boldsymbol{z}_1 = \left[z_1\right]^{\mathrm{T}}$ and $\delta_1\left(\boldsymbol{z}_1\right)$ are the input of MTN and the approximation error, respectively.

Note $z_2 = x_2 - \alpha_1$, and substituting (22) into (21), we have

$$\begin{aligned}\mathcal{L}V_1 &\le z_1^3 z_2 + z_1^3\alpha_1 + z_1^3\boldsymbol{\theta}_1^{\mathrm{T}}P_{m_1} + z_1^3\delta_1 \\ &\quad - \frac{3}{4}z_1^4 + \frac{3}{4}l_1^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1\end{aligned} \tag{23}$$

Taking the intermediate control signal α_1 as

$$\alpha_1 = -k_1 z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}}P_{m_1}\left(\boldsymbol{z}_1\right) \tag{24}$$

where $k_1 \in \mathbb{R}_+$ is a design constant.

Then by Lemma 2, we have

$$z_1^3 z_2 \le \frac{3}{4}z_1^4 + \frac{1}{4}z_2^4 \tag{25}$$

$$z_1^3\alpha_1 = -k_1 z_1^4 - z_1^3\hat{\boldsymbol{\theta}}_1^{\mathrm{T}}P_{m_1}\left(\boldsymbol{z}_1\right) \tag{26}$$

$$z_1^3\delta_1 \le \left|z_1^3\right|\varepsilon_1 \le \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4 \tag{27}$$

substituting (25) (26) and (27) into (23), we have

$$\begin{aligned}\mathcal{L}V_1 &\le -\left(k_1 - \frac{3}{4}\right)z_1^4 + \frac{1}{4}z_2^4 + \frac{1}{4}\varepsilon_1^4 \\ &\quad + \frac{3}{4}l_1^2 + \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\left(z_1^3 P_{m_1}\left(\boldsymbol{z}_1\right) - \Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1\right)\end{aligned} \tag{28}$$

Step $i\left(2 \le i \le n-1\right)$*:* According to (18), one has

$$\mathrm{d}z_i = \left(x_{i+1} + f_i - L_{\alpha_{i-1}}\right)\mathrm{d}t + \bar{h}_i^{\mathrm{T}}\mathrm{d}\omega \tag{29}$$

Consider the stochastic Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{4}z_i^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i \tag{30}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ is the parameter error, $\Gamma_i = \Gamma_i^T > 0$ is any constant matrix.

By Definition 1 and (29), we have

$$\begin{aligned}\mathcal{L}V_i &= \mathcal{L}V_{i-1} + z_i^3\left(x_{i+1} + f_i - L_{\alpha_{i-1}}\right) \\ &\quad + \frac{3}{2}z_i^2\bar{h}_i^{\mathrm{T}}\bar{h}_i - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i\end{aligned} \tag{31}$$

From Lemma 2, the following inequality holds:

$$\frac{3}{2}z_i^2\bar{h}_i^{\mathrm{T}}\bar{h}_i \le \frac{3}{4}l_i^{-2}z_i^4\left\|\bar{h}_i\right\|^4 + \frac{3}{4}l_i^2 \tag{32}$$

where $l_i \in \mathbb{R}_+$ being a design parameter.

Then, by (31) and (32), and repeating the procedure taken in Step 1, we have

$$\begin{aligned}&\mathcal{L}V_i \\ &\le z_i^3\left(z_{i+1} + \alpha_i + \bar{f}_i\right) - \left(k_1 - \frac{3}{4}\right)z_1^4 - \sum_{j=2}^{i-1}\left(k_j - 1\right)z_j^4 \\ &\quad + \frac{1}{4}\sum_{j=1}^{i-1}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{i-1}l_j^4 - \frac{1}{2}z_i^4 + \frac{3}{4}l_i^2 \\ &\quad + \sum_{j=1}^{i-1}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j^3 P_{m_j} - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right) - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i\end{aligned} \tag{33}$$

where

$$\begin{aligned}\bar{f}_i &= f_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(x_{j+1} + f_j\right) - \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} \\ &\quad - \frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}_{i-1}}\dot{\hat{\boldsymbol{\theta}}}_{i-1} - \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}h_p^{\mathrm{T}}h_q \\ &\quad + \frac{3}{4}l_i^{-2}z_i\left\|\bar{h}_i\right\|^4 + \frac{3}{4}z_i\end{aligned} \tag{34}$$

It is obvious that $\bar{f}_i$ is an unknown function, according to (16), we can employ a MTN to estimate it, in other words, for any given $\varepsilon_i > 0$, there exists a MTN $\boldsymbol{\theta}_i^{\mathrm{T}}P_{m_i}\left(\boldsymbol{z}_i\right)$ such that

$$\bar{f}_i = \boldsymbol{\theta}_i^{\mathrm{T}}P_{m_i}\left(\boldsymbol{z}_i\right) + \delta_i\left(\boldsymbol{z}_i\right), \left|\delta_i\left(\boldsymbol{z}_i\right)\right| \le \varepsilon_i \tag{35}$$

where $\boldsymbol{z}_i = \left[z_1, \cdots, z_i\right]^{\mathrm{T}}$ and $\delta_i\left(\boldsymbol{z}_i\right)$ are the input of MTN and the approximation error, respectively.

By Lemma 2 and (35), we have

$$z_i^3\bar{f}_i \le z_i^3\boldsymbol{\theta}_i^{\mathrm{T}}P_{m_i}\left(\boldsymbol{z}_i\right) + \frac{3}{4}z_i^4 + \frac{1}{4}\varepsilon_i^4 \tag{36}$$

Taking the intermediate control signal α_i as

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}P_{m_i}\left(\boldsymbol{z}_i\right) \tag{37}$$

where $k_i \in \mathbb{R}_+$, then by Lemma 2, we have

$$z_i^3 z_{i+1} \le \frac{3}{4}z_i^4 + \frac{1}{4}z_{i+1}^4 \tag{38}$$

$$z_i^3\alpha_i = -k_i z_i^4 - z_i^3\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}P_{m_i}\left(\boldsymbol{z}_i\right) \tag{39}$$

substituting (36) (38) and (39) into (33), we have

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$$\begin{aligned}\mathcal{L}V_i \le& -\left(k_1-\frac{3}{4}\right)z_1^4-\sum_{j=2}^{i}\left(k_j-1\right)z_j^4+\frac{1}{4}z_{i+1}^4\\&+\frac{1}{4}\sum_{j=1}^{i}\varepsilon_j^4+\frac{3}{4}\sum_{j=1}^{i}l_j^4+\sum_{j=1}^{i}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j^3P_{m_j}-\Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right)\end{aligned} \tag{40}$$

Step n: According to (18), one has

$$\mathrm{d}z_n=\left(u+f_n-L_{\alpha_{n-1}}\right)\mathrm{d}t+\bar{h}_n^{\mathrm{T}}\mathrm{d}\omega \tag{41}$$

Consider a stochastic Lyapunov function candidate as

$$V_n=V_{n-1}+\frac{1}{4}z_n^4+\frac{1}{2}\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\Gamma_n^{-1}\tilde{\boldsymbol{\theta}}_n \tag{42}$$

where $\tilde{\boldsymbol{\theta}}_n=\boldsymbol{\theta}_n-\hat{\boldsymbol{\theta}}_n$ is the parameter error, $\Gamma_n=\Gamma_n^T>0$ is any constant matrix.

Similar to the design in Step $i\left(2\le i\le n-1\right)$, and take (10) into account, it is easy to obtain

$$\begin{aligned}\mathcal{L}V_n \le& -\left(k_1-\frac{3}{4}\right)z_1^4-\sum_{j=2}^{n-1}\left(k_j-1\right)z_j^4\\&+\sum_{j=1}^{n-1}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j^3P_{m_j}\left(\boldsymbol{z}_j\right)-\Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right)+\frac{1}{4}\sum_{j=1}^{n-1}\varepsilon_j^4\\&+\frac{3}{4}\sum_{j=1}^{n-1}l_j^4+\frac{1}{4}z_n^4+z_n^3\left(K^{\mathrm{T}}\left(t\right)\Phi\left(t\right)v+d\left(v\right)+\bar{f}_n\right)\\&-\frac{3}{4}z_n^4+\frac{3}{4}l_n^2-\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\Gamma_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n\end{aligned} \tag{43}$$

where $l_n\in\mathbb{R}_+$ is a design parameter, and

$$\begin{aligned}\bar{f}_n=& f_n-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(x_{j+1}+f_j\right)-\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)}\\&-\frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol{\theta}}_{n-1}}\dot{\hat{\boldsymbol{\theta}}}_{n-1}-\frac{1}{2}\sum_{p,q=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_p\partial x_q}h_p^{\mathrm{T}}h_q\\&+\frac{3}{4}l_n^{-2}z_n\parallel\bar{h}_n\parallel^4+\frac{3}{4}z_n\end{aligned} \tag{44}$$

It is obvious that $\bar{f}_n$ is an unknown function, according to (16), we can employ a MTN to estimate it, in other words, for any given $\varepsilon_n>0$, there exists a MTN $\boldsymbol{\theta}_n^{\mathrm{T}}P_{m_n}\left(\boldsymbol{z}_n\right)$ such that

$$\bar{f}_n=\boldsymbol{\theta}_n^{\mathrm{T}}P_{m_n}\left(\boldsymbol{z}_n\right)+\delta_n\left(\boldsymbol{z}_n\right),\left|\delta_n\left(\boldsymbol{z}_n\right)\right|\le\varepsilon_n \tag{45}$$

where $\boldsymbol{z}_n=\left[z_1,\cdots,z_n\right]^{\mathrm{T}}$ and $\delta_n\left(\boldsymbol{z}_n\right)$ are the input of MTN and the approximation error, respectively.

By Lemma 2 and (45), we have

$$z_n^3\bar{f}_n\le z_n^3\boldsymbol{\theta}_n^{\mathrm{T}}P_{m_n}\left(\boldsymbol{z}_n\right)+\frac{3}{4}z_n^4+\frac{1}{4}\varepsilon_n^4 \tag{46}$$

Since $K^{\mathrm{T}}\left(t\right)\Phi\left(t\right)\ge\beta_0$ and β_0 is a known positive constant, and choose the actual input v as

$$v=-\frac{1}{\beta_0}\left[k_n\left|z_n\right|+\left|\hat{\theta}_n^{\mathrm{T}}P_{m_n}\right|\right]\mathrm{sgn}\left(z_n\right) \tag{47}$$

where $k_n\in\mathbb{R}_+$, then, according to Lemma 2, we have

$$z_n^3K^{\mathrm{T}}\left(t\right)\Phi\left(t\right)v\le -k_nz_n^4-\left|z_n^3\hat{\theta}_n^{\mathrm{T}}P_{m_n}\right| \tag{48}$$

Since $\left|d\left(v\right)\right|\le\tilde{k}$, according to Lemma 2, we have

$$z_n^3d\left(v\right)\le\left|z_n^3\right|\left|d\left(v\right)\right|\le\frac{3}{4}z_n^4+\frac{1}{4}\tilde{k}^4 \tag{49}$$

Substituting (46) (48) and (49) into (43), we have

$$\begin{aligned}\mathcal{L}V_n \le& -\sum_{j=1}^{n}c_jz_j^4+\sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j^3P_{m_j}\left(\boldsymbol{z}_j\right)-\Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right)\\&+\frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4+\frac{3}{4}\sum_{j=1}^{n}l_j^4+\frac{1}{4}\tilde{k}^4\end{aligned} \tag{50}$$

where $c_j=\left(k_j-1\right)>0,\ j=1,2,\cdots,n$.

Then, the main results of this paper can be summarized by the following theorem.

B. STABILITY ANALYSIS

Theorem 1: Consider the stochastic nonlinear system in (6) with the input dead-zone defined in (7). If the actual control law is chosen as

$$v=-\frac{1}{\beta_0}\left[k_n\left|z_n\right|+\left|\hat{\boldsymbol{\theta}}_n^{\mathrm{T}}P_{m_n}\left(\boldsymbol{z}_n\right)\right|\right]\mathrm{sgn}\left(z_n\right) \tag{51}$$

with the intermediate virtual control signals $\alpha_i\left(i=1,\cdots,n-1\right)$ *described as*

$$\alpha_i=-k_iz_i-\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}P_{m_i}\left(\boldsymbol{z}_i\right) \tag{52}$$

and the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_i\left(i=1,\cdots,n\right)$ *defined as*

$$\dot{\hat{\boldsymbol{\theta}}}_i=\Gamma_iP_{m_i}\left(\boldsymbol{z}_i\right)z_i^3-\Gamma_i\eta_i\hat{\boldsymbol{\theta}}_i \tag{53}$$

where $\Gamma_i=\Gamma_i^{\mathrm{T}}>0$ *are the adaptation gain matrices,* $\eta_i>0$ *and* $k_i>1$ *are design parameters. Then, for any bounded initial condition, all the signals in the closed-loop system are bounded in probability and the tracking error converges to an arbitrarily small neighborhood around the origin in the sense of mean quartic value.*

Proof: According to the adaptive MTN control design process, choose the following stochastic Lyapunov function as

$$V=V_n=\frac{1}{4}\sum_{i=1}^{n}z_i^4+\frac{1}{2}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i \tag{54}$$

It follows from (50) that

$$\begin{aligned}\mathcal{L}V_n \le& -\sum_{j=1}^{n}c_jz_j^4+\sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j^3P_{m_j}\left(\boldsymbol{z}_j\right)-\Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right)\\&+\frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4+\frac{3}{4}\sum_{j=1}^{n}l_j^4+\frac{1}{4}\tilde{k}^4\end{aligned} \tag{55}$$

Substituting (53) into (55), we have

$$\mathcal{L}V_n\le-\sum_{j=1}^{n}c_jz_j^4+\sum_{j=1}^{n}\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j+\frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4+\frac{3}{4}\sum_{j=1}^{n}l_j^4+\frac{1}{4}\tilde{k}^4 \tag{56}$$

For the term $\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j$, the following inequality is obvious:

$$\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j=\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(\boldsymbol{\theta}-\tilde{\boldsymbol{\theta}}_j\right)\le-\frac{\eta_j}{2}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j+\frac{\eta_j}{2}\left\|\boldsymbol{\theta}_j\right\|^2 \tag{57}$$

with

$$-\frac{\eta_j}{2}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j \le -\frac{\eta_j}{2\lambda_{\max}\left(\Gamma_j^{-1}\right)}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\theta}}_j \tag{58}$$

Substituting (57) and (58) into (56), we have

$$\begin{aligned}\mathcal{L}V_n \le & -\sum_{j=1}^{n} c_j z_j^4 - \frac{1}{2}\bar{\eta}_j\sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\Gamma_j^{-1}\tilde{\boldsymbol{\theta}}_j \\ & +\frac{1}{2}\sum_{j=1}^{n}\eta_j\left\|\boldsymbol{\theta}_j\right\|^2+\frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4+\frac{3}{4}\sum_{j=1}^{n}l_j^4+\frac{1}{4}\tilde{k}^4\end{aligned} \tag{59}$$

where $\bar{\eta}_j = \min\left\{\frac{\eta_j}{\lambda_{\max}\left(\Gamma_j^{-1}\right)} | j=1,2,\cdots,n\right\}$.

Let

$$\mathrm{a}_0 = \min\left\{4c_j, \bar{\eta}_j \mid j=1,2,\cdots,n\right\} \tag{60}$$

$$\mathrm{b}_0 = \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4+\frac{3}{4}\sum_{j=1}^{n}l_j^4+\frac{1}{2}\sum_{j=1}^{n}\eta_j\left\|\boldsymbol{\theta}_j\right\|^2+\frac{1}{4}\tilde{k}^4 \tag{61}$$

then, one has

$$\mathcal{L}V \le -\mathrm{a}_0 V + \mathrm{b}_0 \tag{62}$$

By Lemma 1 and inequality (62), and using the same arguments as [11,46,47], one can obtain that all the signals in the closed-loop system are bounded in probability and the tracking error converges to an arbitrarily small neighborhood around the origin in the sense of mean quartic value.

Remark 3: In this paper, a simple and effective adaptive MTN control scheme is proposed for a class of stochastic nonlinear systems with unknown input dead-zone. It should be noted that the simple here is only refer to the MTN controller and the ideas of controller design. From the design procedure, one can see that instead of using MTN to approximate each unknown smooth function of system, we lump all unknown functions into a suitable unknown function in each step of backsteeping, and then only a MTN is used to approximate it. In other words, n MTNs are required for the n-order nonlinear systems. Of course, it will increase the complexity of the design of the controller, however, it also increase the approximation precision.

IV. SIMULATION RESULTS

In this section, two numerical examples and one practical example are used to demonstrate the effectiveness of the proposed adaptive MTN control method.

Example 1 (numerical example): Consider the following three-order stochastic nonlinear system:

$$\begin{cases}\mathrm{d}x_1 = \left(x_2+0.5x_1^2e^{-x_1}\right)\mathrm{d}t + x_1\cos x_1\mathrm{d}\omega \\ \mathrm{d}x_2 = \left(x_3+x_1\cos\left(x_1x_2^2\right)\right)\mathrm{d}t + x_2\sin x_1\mathrm{d}\omega \\ \mathrm{d}x_3 = \left(u+x_1x_2x_3^3\right)\mathrm{d}t + x_1x_2^2\sin x_3\mathrm{d}\omega \\ y = x_1\end{cases} \tag{63}$$

and u is the output of the dead-zone $D(v)$ defined as:

$$u = D(v) = \begin{cases}0.1(v-2.5)^2+(v-2.5), & v\ge 2.5 \\ 0, & -1.5<v<2.5 \\ (v+1.5), & v\le -1.5\end{cases} \tag{64}$$

where x_1, x_2 and x_3 are the state variables, and y is the system output. u and v are the output and input of the dead-zone nonlinearity, respectively. The desired tracking trajectory y_d is taken as $y_d = 0.6\sin t$.

It is easy to verify that the dead-zone (64) satisfies Assumption 2, and the positive constant β_0 can be taken as 1. By Theorem 1, the virtual control laws, the actual control law and the adaptive laws are chosen, respectively, as

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} P_{m_i}\left(\boldsymbol{z}_i\right), \quad i=1,2 \tag{65}$$

$$v = -\frac{1}{\beta_0}\left[k_3\left|z_3\right|+\left|\hat{\boldsymbol{\theta}}_3^{\mathrm{T}} P_{m_3}\left(\boldsymbol{z}_3\right)\right|\right]\mathrm{sgn}\left(z_3\right) \tag{66}$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = \Gamma_i P_{m_i}\left(\boldsymbol{z}_i\right)z_i^3 - \Gamma_i\eta_i\hat{\boldsymbol{\theta}}_i, \quad i=1,2,3 \tag{67}$$

where $z_1 = x_1 - y_d$, $z_2 = x_2-\alpha_1$, $z_3 = x_3-\alpha_2$ and $\boldsymbol{z}_1 = z_1$, $\boldsymbol{z}_2 = \left[z_1, z_2\right]^{\mathrm{T}}$ and $\boldsymbol{z}_3 = \left[z_1,z_2,z_3\right]^{\mathrm{T}}$.

In the simulation, the design parameters are taken as: $k_1 = 7$, $k_2 = 2.1$, $k_3 = 1.25$, $\Gamma_1 = 0.5I_5$, $\Gamma_2 = 30I_9$, $\Gamma_3 = 5I_9$, $\eta_1 = 4$, $\eta_2 = 2$, $\eta_3 = 0.01$, $m_1 = 5$, $m_2 = 3$, $m_3 = 2$. And the initial conditions $\left[x_1(0), x_2(0), x_3(0)\right]^{\mathrm{T}} = [0,0,0]^{\mathrm{T}}$.

The simulation results are illustrated in Figures 3-6.

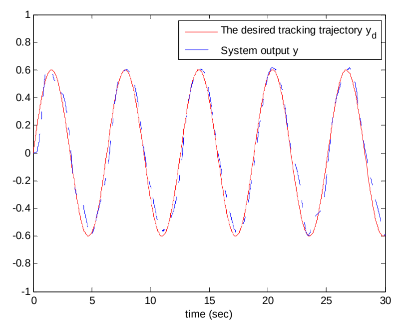

FIGURE 3. The desired tracking trajectory y_d and system output y of Example 1.

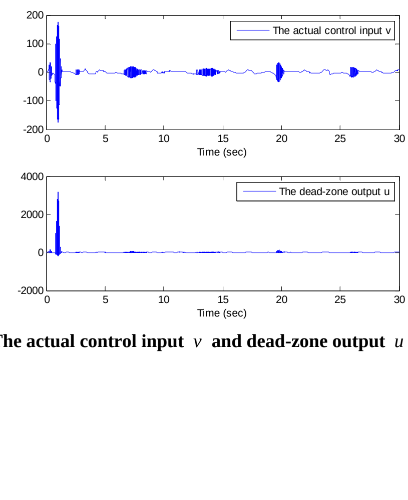

FIGURE 4. The actual control input v and dead-zone output u of Example 1.

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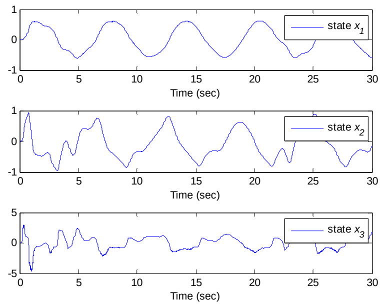

FIGURE 5. State variables x_1, x_2 and x_3 of Example 1.

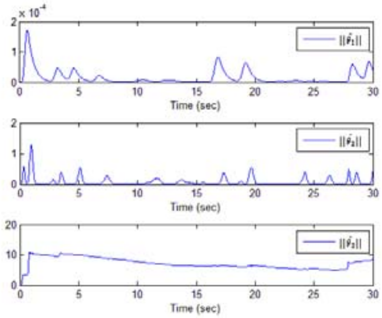

FIGURE 6. The norm of adaptive laws, i.e. $\|\hat{\theta}_1\|$, $\|\hat{\theta}_2\|$ and $\|\hat{\theta}_3\|$ of Example 1.

Figure 3 shows the system output y and the desired tracking trajectory y_d, from which we can see that the good tracking performance is obtained. Figure 4 displays the actual control input signal v and the dead-zone output u, from which we can see that a little large control effort is needed at the beginning. Figure 5 shows that the state variables x_1, x_2 and x_3 are bounded. Figure 6 depicts that the trajectories of the adaptation laws $\|\hat{\theta}_1\|$, $\|\hat{\theta}_2\|$ and $\|\hat{\theta}_3\|$ are boundedness. The presented simulation results demonstrate the good tracking and control performance for the stochastic nonlinear systems with unknown input dead-zone.

Remark 4 From Figures 3-6, we can see that the system output y effectively tracks the desired tracking trajectory y_d very fast. In addition, it can be seen that the states x_1, x_2 and x_3, the actual control input v, the norm of adaptive parameter laws $\|\hat{\theta}_1\|, \|\hat{\theta}_2\|$ and $\|\hat{\theta}_3\|$ are all bounded, according to (17), the intermediate virtual control signals α_1 and α_2 are also bounded. In other words, all the signals of the closed-loop system are bounded.

Example 2 (numerical example): To further validate the effectiveness of the proposed adaptive MTN controller, we consider the following third-order stochastic nonlinear system:

$$\begin{cases} dx_1 = \left(x_2 - 2x_1 \sin x_1\right) dt + 0.5x_1^2 d\omega \\ dx_2 = \left(x_2 + x_1 x_2^2\right) dt + x_2 \sin x_1 d\omega \\ dx_2 = \left(u + x_1 x_2 x_3^2\right) dt + x_2 \sin x_3 d\omega \\ y = x_1 \end{cases} \tag{68}$$

and u is the output of the dead-zone $D(v)$ defined as

$$u = D(v) = \begin{cases} 1.5(v-2), & v \geq 2 \\ 0, & -1.5 < v < 2.5 \\ v + 0.5, & v \leq -0.5 \end{cases} \tag{69}$$

with the initial states $x_1(0) = 0$, $x_2(0) = 0$ and $x_3(0) = 0$. The desired tracking trajectory y_d is taken as $y_d = 0.5(\sin(t) + 0.5\sin(0.4t))$.

It is easy to verify that the dead-zone (69) also satisfies Assumption 2, and the positive constant β_0 can be taken as 1. Similarly, Theorem 1 is used to design adaptive MTN controller for this system. We further apply the control approach (65), (66) and (67) to system (68). The design parameters are kept as that in Example 1 except $\Gamma_3 = 20I_9$ and $\eta_3 = 5$.

The simulation results are illustrated in Figures 7-10.

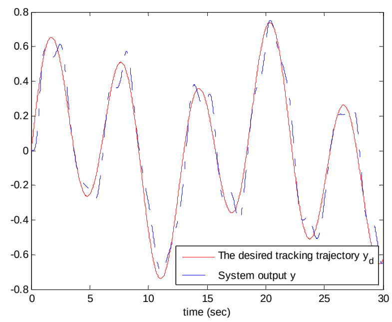

FIGURE 7. The desired tracking trajectory y_d and system output y of Example 2.

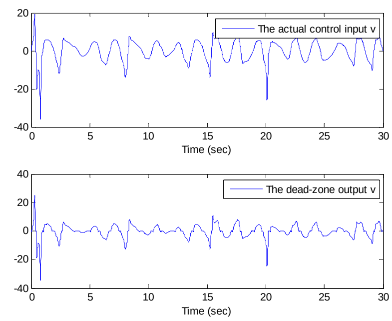

FIGURE 8. The actual control input v and dead-zone output u of Example 2.

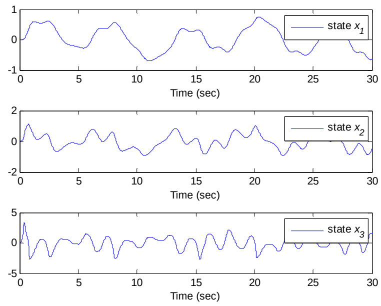

FIGURE 9. State variables x_1, x_2 and x_3 of Example 2.

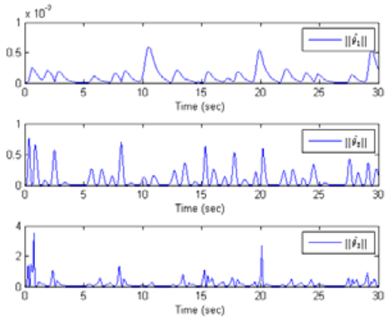

FIGURE 10. The norm of adaptive laws, i.e. $\|\hat{\theta}_1\|$, $\|\hat{\theta}_2\|$ and $\|\hat{\theta}_3\|$ of Example 2.

Figure 7 shows that the system output y tracks the desired trajectory y_d perfectly. Figures 8-10 display the actual control input v, the dead-zone output u, state variables x_1, x_2, x_3 and the norm of adaptive laws $\|\hat{\boldsymbol{\theta}}_1\|$, $\|\hat{\boldsymbol{\theta}}_2\|$ and $\|\hat{\boldsymbol{\theta}}_3\|$, respectively. It can be seen that for the different stochastic nonlinear system, the control performance is still satisfactory, which further verify the effectiveness of the adaptive MTN control approach proposed in this paper.

On the other hand, to further demonstrate the advantages of our method, let us compare the proposed control method with RBF neural network (RBFNN) (The form of RBFNN is the same as [45]). The structure of RBF neural networks are chose as $\hat{\boldsymbol{W}}_1^{\mathrm{T}} S_1(\boldsymbol{z}_1)$ contains 7 nodes with centres spaced evenly in the interval $[-3, 3]$ and widths being equal to 10; $\hat{\boldsymbol{W}}_2^{\mathrm{T}} S_2(\boldsymbol{z}_2)$ contains 9 nodes with centres spaced evenly in the interval $[-8, 8]\times[-8, 8]$ and widths being equal to 10; $\hat{\boldsymbol{W}}_3^{\mathrm{T}} S_3(\boldsymbol{z}_3)$ contains 9 nodes with centres spaced evenly in the interval $[-8, 8]\times[-8, 8]\times[-8, 8]$ and widths being equal to 10. The simulation result is illustrated in Figure 11.

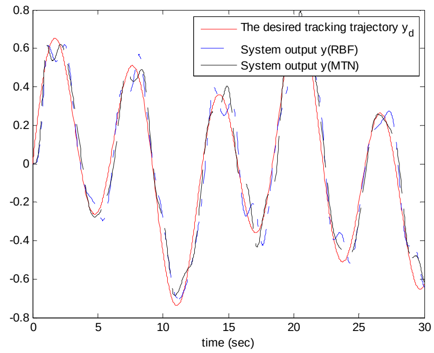

FIGURE 11. The tracking performances of MTNC and RBFNN.

Figure 11 shows the tracking performances of MTN and RBFNN, respectively. From Figure 11, we can see that, both MTN and RBFNN well realize the tracking control, but, the computation of the former is obviously less than that of the latter, that is to say, the former can get precise tracking result with low computational cost. As revealed by comparison, RBFNN is of high computational-complexity due to its exponential function, so it is difficult to achieve the real-time control with microcontrollers or embedded systems. However, only addition and multiplication are required by MTN easily achieve real-time control.

Remark 5 It should be noted that several results for stochastic nonlinear systems with unknown input dead-zone have been proposed using NN [40,41], the main difference between our result and the ones in [40,41] lies in the fact that a novel simple and effective adaptive MTN control algorithm is proposed. In addition, in [40,41], in order to alleviate the computational burden, only one adaptive parameter was involved in the proposed controller. However, the use of only one adaptive parameter may lose some approximation precision.

Remark 6 Theoretically speaking, for the stochastic nonlinear systems (6) with dead-zone (7), according to Theorem 1, the tracking performance could be reached when the design parameters $\eta_i > 0$ and $k_i > 1$, and matrices $\Gamma_i = \Gamma_i^{-1} > 0$. However, in practice, the design parameters should be selected appropriately to obtain a specific control objective.

Remark 7 From examples 1-2, it can be seen that the good tracking performances can be achieved by suitably choosing the design parameters. In fact, the dynamic performance and the tracking performance of the stochastic nonlinear systems (6) could be elevated by properly adjusting the design parameters in (61) and (62). Therefore, in practical applications, for a given control system and specific desired trajectory, we should properly adjust the parameters.

Example 3 (practical example): In order to test the application of the control method presented in this paper, we consider a nonlinear circuit or a pendulum moving in a

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viscous medium, whose dynamic can be described as a class of uncertain Duffng-Holmes chaotic system [48]:

$$\begin{cases} dx_1 = \left(x_2 + f_1\left(\bar{\mathbf{x}}_1\right)\right)dt + h_1^{\mathrm{T}}\left(\bar{\mathbf{x}}_1\right)d\omega \\ dx_2 = \left(u + f_2\left(\bar{\mathbf{x}}_2\right)\right)dt + h_2^{\mathrm{T}}\left(\bar{\mathbf{x}}_2\right)d\omega \\ y = x_1 \end{cases} \quad (70)$$

where $f_1\left(\bar{\mathbf{x}}_1\right) = 0$, $f_2\left(\bar{\mathbf{x}}_2\right) = -p_1x_1 - p_2x_2 - x_1^3 + q\cos\left(w_1t\right)$, $h_1^{\mathrm{T}}\left(\bar{\mathbf{x}}_1\right) = 0$, $h_2^{\mathrm{T}}\left(\bar{\mathbf{x}}_2\right) = 0.2x_2^2\cos x_1$, and u is the output of the dead-zone $D(v)$ defined as (7). The desired tracking trajectory y_d is taken as $y_d = 0.5\left(\sin(t) + 0.5\sin(0.4t)\right)$.

Similarly, Theorem 1 is used to design adaptive MTN controller for system (70). Therefore, the virtual control laws, the actual control law and the adaptive laws are chosen, respectively, as

$$\alpha_1 = -k_1z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}}P_{m_1}\left(\mathbf{z}_1\right) \quad (71)$$

$$v = -\frac{1}{\beta_0}\left[k_2\left|z_2\right| + \left|\hat{\boldsymbol{\theta}}_2^{\mathrm{T}}P_{m_2}\left(\mathbf{z}_2\right)\right|\right]\mathrm{sgn}\left(z_2\right) \quad (72)$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = \varGamma_iP_{m_i}\left(\mathbf{z}_i\right)z_i^3 - \varGamma_i\eta_i\hat{\boldsymbol{\theta}}_i, \quad i = 1,2 \quad (73)$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$ and $\mathbf{z}_1 = z_1$, $\mathbf{z}_2 = \left[z_1, z_2\right]^{\mathrm{T}}$. The simulation is carried out with the system parameters $p_1 = 1$, $p_2 = 0.25$, $q = 0.3$, $w_1 = 1$; the initial condition $\left[x_1(0), x_2(0)\right]^{\mathrm{T}} = [0,0]^{\mathrm{T}}$ and the design parameters $k_1 = 6$, $k_2 = 6$, $\varGamma_1 = 0.5I_5$, $\varGamma_2 = 20I_9$, $\eta_1 = 4$, $\eta_2 = 5$, $m_1 = 5$, $m_2 = 3$. To illustrate the effectiveness of the proposed control scheme, the simulation is done under two different dead-zones (64) and (69).

Case 1: u is the output of the dead-zone (64), the simulation results are illustrated in Figures 12-15:

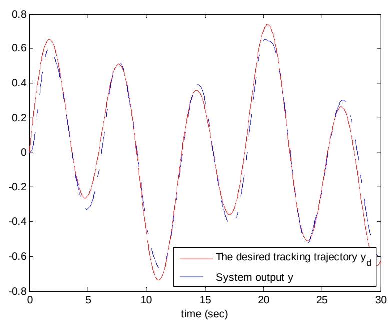

FIGURE 12. The desired tracking trajectory y_d and system output y of Example 3 of case 1.

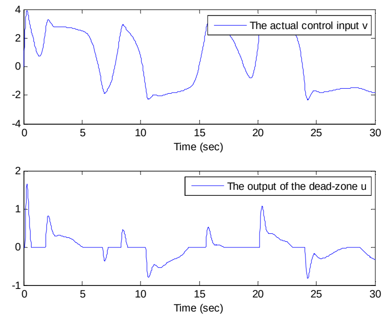

FIGURE 13. The actual control input v and the output of the dead-zone u of Example 3 of case 1.

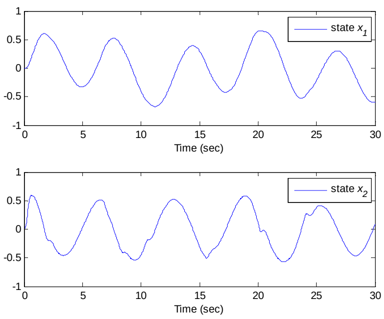

FIGURE 14. State variables x_1 and x_2 of Example 3 of case 1.

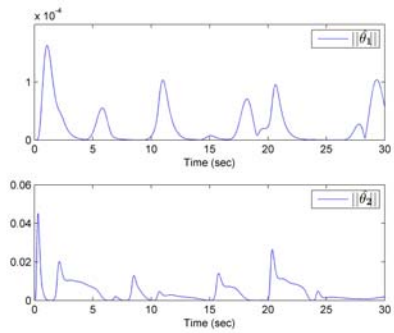

FIGURE 15. The norm of adaptive laws, i.e. $\|\hat{\theta}_1\|$ and $\|\hat{\theta}_2\|$ of Example 3 of case 1.

Case 2: u is the output of the dead-zone (69), the simulation results are illustrated in Figures16-19:

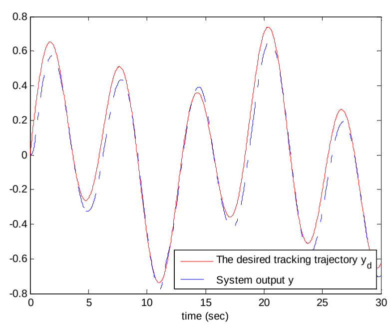

FIGURE 16. The desired tracking trajectory y_d and system output y of Example 3 of case 2.

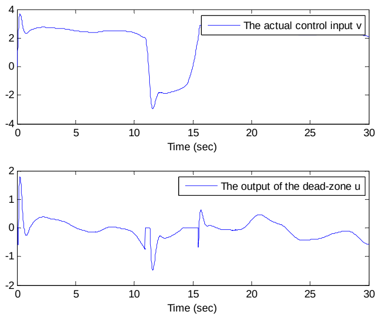

FIGURE 17. The actual control input v and the output of the dead-zone u of Example 3 of case 2.

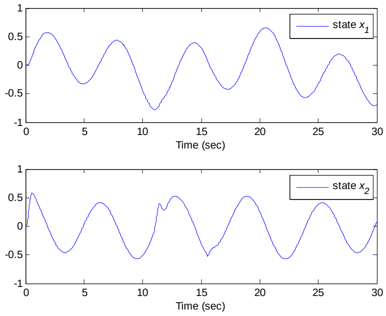

FIGURE 18. State variables x_1 and x_2 of Example 3 of case 2.

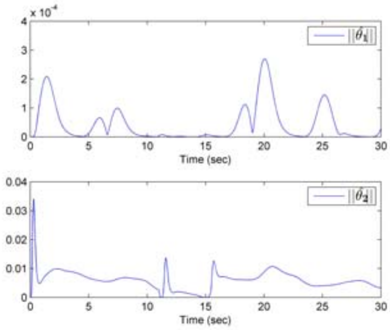

FIGURE 19. The norm of adaptive laws, i.e. $\|\hat{\theta}_1\|$ and $\|\hat{\theta}_2\|$ of Example 3 of case 2.

From Figures 12-19, we can get the following conclusions: i) The control approach designed in this paper can achieve the good control performance for a class of practice engineering systems; ii) For given stochastic nonlinear systems, the control approach designed in this paper can achieve the good control performance for different input dead-zones; iii) The simulation results further verify the effectiveness of the control approach designed in this paper.

V. CONCLUSIONS

In this paper, we successfully extend the adaptive MTN approach to a class of stochastic nonlinear systems with unknown input dead-zone. MTNs are used to deal with the unknown nonlinearities, meanwhile, the computation burden can be significantly reduced thanks to the simple structure of MTN. It is proved that the proposed control method can guarantee that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded. Finally, the effectiveness of the proposed control approach is illustrated by two numerical examples and one practical example.

There exist several topics left to be studied, e.g., how to develop an adaptive MTN output-feedback control strategy for the stochastic nonlinear systems (6) with unknown input dead-zone (7), and how to extend the proposed result to stochastic nonlinear systems with both state time-delay and unknown input dead-zone. These topics will be considered in a follow-up study.

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