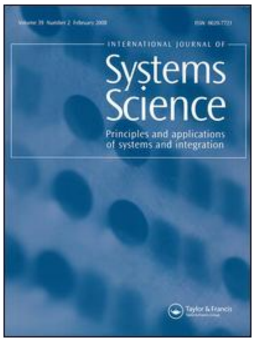

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Adaptive neural output feedback tracking control for a class of nonlinear systems

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ABSTRACT

In this paper, an adaptive neural output feedback control scheme based on backstepping technique and dynamic surface control (DSC) approach is developed to solve the tracking control problem for a class of nonlinear systems with unmeasurable states. Firstly, a nonlinear state observer is designed to estimate the unmeasurable states. Secondly, in the controller design process, radial basis function neural networks (RBFNNs) are utilised to approximate the unknown nonlinear functions, and then a novel adaptive neural output feedback tracking control scheme is developed via backstepping technique and DSC approach. It is shown that the proposed controller ensures that all signals of the closed-loop system remain bounded and the tracking error converges to a small neighbourhood around the origin. Finally, two numerical examples and one realistic example are given to illustrate the effectiveness of the proposed design approach.

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Neural network; nonlinear system; unmeasurable states; adaptive tracking control; dynamic surface control

1. Introduction

The control of nonlinear systems is an important branch of control theory. Therefore, the investigation on stability analysis and controller design of nonlinear systems has received considerable attention in the past decades, and many significant control methods have been reported for a series of different systems (Huang, Lin, & Yang, 2005; Nguang, 2000; Thakar, Trivedi, Bandyopadhyay, & Gandhi, 2017; Xiao & Yin, 2016). Among them, adaptive backstepping technique has become one of the most popular approaches to control of different nonlinear systems, such as high-order nonlinear systems (Sun & Liu, 2007), time-delay nonlinear systems (Jiao & Shen, 2005), large-scale nonlinear systems (Wu, 2002), stochastic nonlinear systems (Chen, Niu, & Zou, 2013), etc. However, most of the results focus on state-feedback control problem. Recently, a growing attention has been paid on output feedback control (Choi & Lim, 2005; Loría, 2016; Qian & Lin, 2002; Zhu & Pagilla, 2007). Compared with the state-feedback control, the output feedback control is easy to implement since it needs system output information only and has simple structure. However, the difficulty of the latter lies in the issue of estimating unmeasurable states and analysing the stability of the closed-loop systems. Therefore, it is still a meaningful and challenging task to design a simple an effective observer to estimate the unmeasured states for nonlinear systems. In summary, the research on output feedback control for nonlinear systems is important not only to the development of control theory itself but also to the practical engineering application.

On the other hand, in view of the excellent performance of neural network (NN), especially the capacity of study, self adapting, ability of parallel disposing, it shows great foreground in the control problem of nonlinear systems (Wang, Chen, & Lin, 2014), and many interesting results have been reported. For example, Wang and Huang (2002) developed an adaptive controller for a class of nonlinear systems with pure-feedback form. Liu, Li, Tong, and Chen (2016) proposed an adaptive neural control method for a class of nonlinear systems with full-state constraints. Wang, Chen, and Lin (2013) proposed an adaptive neural tracking control approach for a class of perturbed pure-feedback nonlinear systems. Zhang, Mei, Mao, and Chen (2005) studied the problem of direct adaptive neural control for a class of uncertain nonlinear systems with unknown dead-zone model and unknown constant control gain. Wang, Liu, and Liu (2016) proposed an adaptive neural decentralised control approach for a class of stochastic nonlinear interconnected systems. Han (2018) studied the problem of adaptive tracking control for a class of nonlinear systems with dynamic uncertainties, and proposed a novel adaptive neural control approach. Despite many

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developments have been achieved for the control of several different kinds of nonlinear systems based on NN, the above approaches are only focus on state-feedback control. It is well known that the most important advantage of output feedback control is that there are fewer feedback control loops compared to the full state feedback, and it is easier to implement in technology. Thus, the investigations on adaptive neural output feedback control for nonlinear systems have received more and more attention in recent years. Seshagiri and Khalil (2000) proposed an adaptive output feedback scheme for a class of continuous-time nonlinear systems represented by input-output models. Calise, Hovakimyan, and Idan (2001) proposed an adaptive output feedback controller for single-input single-output (SISO) nonlinear systems. Ren, Ge, Tee, and Tong (2010) proposed a NN-backstepping-based adaptive output controller for uncertain output feedback nonlinear systems. Chen and Ge (2015) proposed an adaptive neural output feedback control scheme for uncertain nonlinear systems with unknown hysteresis, external disturbances, and unmeasured states. Wang, Chen, Lin, Zhang, and Meng (2018) addressed the finite-time tracking problem for a class of nonlinear quantised systems and proposed an observer-based adaptive output-feedback control strategy. However, these results still inherit the open problem of the 'explosion of complexity' in the backstepping design procedure. The dynamic surface control (DSC) technique was first introduced in (Swaroop, Hedrick, Yip, & Gerdes, 2000) in order to avoid this problem for a class of nonlinear systems with the strict feedback form. Since then, this technique has widely been used in different systems (Chen, Tao, & Jiang, 2015; Wang & Huang, 2005; Zhang, Xia, & Yi, 2017). However, as far as we know, there are few results available on neural-network-based adaptive output-feedback tracking control for nonlinear systems. In fact, it is still a meaningful and challenging task to construct a simple but effective adaptive neural output-feedback control algorithm for nonlinear systems, which motivates our research.

Motivated by the aforementioned discussion, this paper focuses on the problem of adaptive neural control for nonlinear systems with unmeasurable system states. Radial basis function neural networks (RBFNNs) are used to handle with the unknown nonlinearities and a nonlinear state observer is designed to estimate the unmeasured states. And then, an observer-based adaptive neural output-feedback controller is designed via the backstepping approach. It is shown that the proposed controller ensures that all signals of the closed-loop system remain bounded and the tracking error converges to a small neighbourhood around the origin. The main contributions of this paper can be summarised as follows:

(1) Theoretically and technically, a novel adaptive backstepping-based neural output-feedback controller design scheme for a class of nonlinear systems with unmeasurable states is established. By designing a nonlinear state observer, the proposed adaptive control approach does not require that all the states of the system are measured directly.
(2) On the one hand, compared with (Zhang, Liu, & Ding, 2006), the controller designed in this paper is characterised by simple structure, less calculation and good real time property. On the other, different from the previous results (Jiang, Mareels, Hill, & Huang, 2004; Zhou, Shi, Lu, & Xu, 2011), nonlinear observer rather than linear observer is used to estimate the nonlinear state of the system, which improve the system state estimation accuracy.
(3) In the controller design process, on the one hand, we lump all unknown functions into a suitable unknown function that is approximated by only a RBFNN in each step of the backstepping. On the other hand, the DSC technique is introduced to overcome the problem of 'explosion of complexity', which inherit from the backstepping technique. Therefore, the computational burden of the designed controller is greatly reduced in terms of the above two aspects.

2. System description and preliminary

The following notations are used throughout this paper. $\mathbb{R}_+$ denotes the set of all non-negative real numbers, $\mathbb{R}^n$ indicates the real n-dimensional space, $\mathbb{R}^{n\times r}$ denotes the set of all $n \times r$ real matrices; For a given vector or matrix X, X^{T} stands for its transpose, $\mathrm{Tr}(X)$ denotes its trace when X is square, and $||X||$ denotes its Euclidean norm when X is a vector; I_i is $i \times i$ identity matrix.

2.1. Problem description

Consider a class of nonlinear systems in the following form:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(\bar{\boldsymbol{x}}_i) \quad 1 \le i \le n-1 \\ \dot{x}_n &= u + f_n(\bar{\boldsymbol{x}}_n) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\boldsymbol{x} = [x_1, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the state vector, the control input, and the system output, respectively, $\bar{\boldsymbol{x}}_i = [x_1, \ldots, x_i]^{\mathrm{T}} \in \mathbb{R}^i$. For each $i = 1, \ldots, n$, $f_i(\cdot) : \mathbb{R}^i \to \mathbb{R}$ is known smooth function with $f_i(\mathbf{0}) = 0$. In addition, it is assumed that only the output of the system is measurable.

The control objective of this paper is to design an adaptive neural controller such that the system output y follows the given reference signal y_d.

Write the system (1) in the following compact form:

$$\begin{aligned} \mathrm{d}\boldsymbol{x} &= A\boldsymbol{x} + F(\boldsymbol{x}) + Bu \\ y &= C\boldsymbol{x} \end{aligned} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad F(\boldsymbol{x}) = \begin{bmatrix} f_1(\bar{\boldsymbol{x}}_1) \\ f_2(\bar{\boldsymbol{x}}_2) \\ \vdots \\ f_n(\bar{\boldsymbol{x}}_n) \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^{\mathrm{T}}.$$

Throughout this paper, the following assumptions are necessary.

Assumption 2.1: (Arcak & Kokotovi, 2001; Fan & Arcak, 2003; Zhang et al., 2006): $F(\boldsymbol{x})$ is known, smooth and vanish at the origin. Moreover, there exist a matrix H and a known vector-valued function $h(\boldsymbol{x})$, such that $F(\boldsymbol{x}) = Hh(\boldsymbol{x})$, and $h(\boldsymbol{x})$ satisfies

$$\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}} + \left(\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)^{\mathrm{T}} \geq 0, \forall \boldsymbol{x} \in \mathbb{R}^n \tag{3}$$

Remark 2.1: It must be pointed out that the condition of the Assumption 2.1 is very harsh.

Assumption 2.2: (Arcak & Kokotovi, 2001; Fan & Arcak, 2003; Zhang et al., 2006)**:** Matrices A, C and H defined in system (2) and Assumption 2.1 satisfy the following LMI's

$$\begin{bmatrix} (A+LC)^{\mathrm{T}}Q_1 + Q_1(A+LC) + Q_2 & Q_1H + (I+KC)^{\mathrm{T}} \\ H^{\mathrm{T}}Q_1 + (I+KC) & 0 \end{bmatrix} \leq 0 \tag{4}$$

where $Q_1 = Q_1^{\mathrm{T}} > 0, Q_2 = Q_2^{\mathrm{T}} > 0, K = [k_1, \dots, k_n]^{\mathrm{T}}$ and $L = [l_1, \dots, l_n]^{\mathrm{T}}$.

Assumption 2.3: The reference signal y_d and its time derivatives up to the nth order are continuous and bounded.

2.2. Radial basis function neural network

In this paper, RBFNN will be used to approximate the unknown smooth nonlinear functions. For any continuous function $f(\boldsymbol{Z})$ defined on a compact $\mathbb{D}$, there exists a RBFNN $\boldsymbol{\theta}^{\mathrm{T}}S(\boldsymbol{Z})$, such that

$$f(\boldsymbol{Z}) = \boldsymbol{\theta}^{\mathrm{T}}S(\boldsymbol{Z}) + \delta(\boldsymbol{Z}), |\delta(\boldsymbol{Z})| \leq \varepsilon \tag{5}$$

where $\boldsymbol{\theta} = [\theta_1, \dots, \theta_l]^{\mathrm{T}} \in \mathbb{R}^l$ is the weight vector, $\boldsymbol{Z} = [z_1, \dots, z_l]^{\mathrm{T}} \in \mathbb{D} \subset \mathbb{R}^l$, l is the neural networks node number, $S(\boldsymbol{Z}) = [s_1(\boldsymbol{Z}), \dots, s_l(\boldsymbol{Z})]^{\mathrm{T}}$ is the basis function vector, $s_i(\boldsymbol{Z})$ is chosen as the commonly used Gaussian function with the form $s_i(\boldsymbol{Z}) = \exp(-(\boldsymbol{Z} - \boldsymbol{\mu}_i)^{\mathrm{T}}(\boldsymbol{Z} - \boldsymbol{\mu}_i)/\varpi_i^2), i = 1, 2, \dots, l,$ where $\boldsymbol{\mu}_i = (\mu_{i1}, \dots, \mu_{il})^{\mathrm{T}}$ is the centre of the receptive field, and ϖ_i is the width of the Gaussian function.

The optimal weight vector $\boldsymbol{\theta}^* \in \mathbb{R}^m$ is defined as

$$\boldsymbol{\theta}^* := \arg\min_{\boldsymbol{\theta} \in \mathbb{R}^l} \left\{ \sup_{Z \in D} |f(\boldsymbol{Z}) - \boldsymbol{\theta}^{\mathrm{T}}S(\boldsymbol{Z})| \right\} \tag{6}$$

3. Main results

In this section, first, we design an observer to estimate the unmeasured states, and then use the backstepping method to design an adaptive output-feedback controller.

3.1. Observer design

Based on Assumptions 2.1–2.2, we introduce the observer has the same form as in (Arcak & Kokotovi, 2001; Fan & Arcak, 2003; Zhang et al., 2006) to estimate the unmeasured states

$$\dot{\hat{\boldsymbol{x}}} = A\hat{\boldsymbol{x}} + L(C\hat{\boldsymbol{x}} - y) + F[\hat{\boldsymbol{x}} + K(C\hat{\boldsymbol{x}} - y)] + Bu \tag{7}$$

where matrices K and L are designed to satisfy the LMI's (4), and $\hat{\boldsymbol{x}} = [\hat{x}_1, \dots, \hat{x}_n]^{\mathrm{T}}$ is the observer state vector.

According to (2) and (7), the state observer error $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ satisfies

$$\dot{\tilde{\boldsymbol{x}}} = (A + LC)\tilde{\boldsymbol{x}} + F(\boldsymbol{x}) - F(\boldsymbol{v}) \tag{8}$$

where $\boldsymbol{v} = \hat{\boldsymbol{x}} + K(C\hat{\boldsymbol{x}} - y)$.

On the basis of Assumption 2.1, we have

$$\begin{aligned} F(\boldsymbol{x}) - F(\boldsymbol{v}) &= H(h(\boldsymbol{x}) - h(\boldsymbol{v})) \\ &= H(h(\boldsymbol{x}) - h(\boldsymbol{x} - \boldsymbol{\mu})) = H\phi(\boldsymbol{x}, \boldsymbol{\mu}) \end{aligned} \tag{9}$$

where $\boldsymbol{\mu} = \boldsymbol{x} - \boldsymbol{v}$ and $\phi(\boldsymbol{x}, \boldsymbol{\mu}) = h(\boldsymbol{x}) - h(\boldsymbol{x} - \boldsymbol{\mu})$.

Substituting (9) into (8), we can get the following error system

$$\dot{\tilde{\boldsymbol{x}}} = (A + LC)\tilde{\boldsymbol{x}} + H\phi(\boldsymbol{x}, \boldsymbol{\mu}) \tag{10}$$

Consider the following Lyapunov function

$$V_0 = \tilde{\boldsymbol{x}}^{\mathrm{T}} Q_1 \tilde{\boldsymbol{x}} \tag{11}$$

Then, the time derivative of V_0 along the solution of (10) is given by

$$\dot{V}_0 = \tilde{\boldsymbol{x}}^{\mathrm{T}}((A + LC)Q_1 + Q_1(A + LC)^{\mathrm{T}})\tilde{\boldsymbol{x}} + 2\tilde{\boldsymbol{x}}^{\mathrm{T}} Q_1 H\phi \tag{12}$$

According to Assumption 2.2, it is obvious that

$$\begin{aligned} &Q_1(A + LC) + (A + LC)^{\mathrm{T}} Q_1 \leq -Q_2 \\ &Q_1 H = -(I + KC)^{\mathrm{T}} \end{aligned} \tag{13}$$

Then, we have

$$\dot{V}_0 \leq -\tilde{\boldsymbol{x}}^{\mathrm{T}} Q_2 \tilde{\boldsymbol{x}} - 2\tilde{\boldsymbol{x}}^{\mathrm{T}}(I + KC)^{\mathrm{T}}\phi \tag{14}$$

By the definition of $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$, we have

$$\boldsymbol{\mu}^{\mathrm{T}} = \boldsymbol{x}^{\mathrm{T}} - \boldsymbol{\nu}^{\mathrm{T}} = \boldsymbol{x}^{\mathrm{T}} - (\hat{\boldsymbol{x}} - KC\tilde{\boldsymbol{x}})^{\mathrm{T}} = \tilde{\boldsymbol{x}}^{\mathrm{T}}(I + KC)^{\mathrm{T}} \tag{15}$$

According to Lemma 1 in (Zhang et al., 2006) and mean value theorem, we have

$$\phi(\boldsymbol{x}, \boldsymbol{\mu}) = h(\boldsymbol{x}) - h(\boldsymbol{\nu}) = \int_0^1 \left(\frac{\partial h}{\partial t}\right)_{t=\boldsymbol{x}+\vartheta\boldsymbol{\mu}} \boldsymbol{\mu}\,\mathrm{d}\vartheta \tag{16}$$

Based on (16) and Assumption 2.1, we have

$$\begin{aligned} &\boldsymbol{\mu}^{\mathrm{T}}\phi(\boldsymbol{x}, \boldsymbol{\mu}) \\ &= \frac{1}{2}\boldsymbol{\mu}^{\mathrm{T}}\left(\int_0^1 \left[\frac{\partial h}{\partial t} + \left(\frac{\partial h}{\partial t}\right)^{\mathrm{T}}\right]_{t=\boldsymbol{x}-\vartheta\boldsymbol{\mu}} \mathrm{d}\vartheta\right)\boldsymbol{\mu} \geq 0 \end{aligned} \tag{17}$$

Combining (14), (15) and (17), we have

$$\dot{V}_0 \leq -\tilde{\boldsymbol{x}}^{\mathrm{T}} Q_2 \tilde{\boldsymbol{x}} - 2\boldsymbol{\mu}^{\mathrm{T}}\phi \leq -\tilde{\boldsymbol{x}}^{\mathrm{T}} Q_2 \tilde{\boldsymbol{x}} \leq -\lambda_{\min}(Q_2)||\tilde{\boldsymbol{x}}|| \tag{18}$$

Remark 3.1: It should be noted that, for any design vectors K and L satisfy Assumption 2.2 are acceptable, in the design of observer (7).

Remark 3.2: In system (2), only the output y measurement is available, but now, the measurements of $[y, \hat{x}_2, \ldots, \hat{x}_n]^{\mathrm{T}}$ are available. Next, we will give the adaptive controller design process in the following section.

3.2. Adaptive neural controller design

By replacing the system states $x_2, x_3, \ldots, x_n$ with the observer states $\hat{x}_2, \hat{x}_3, \ldots, \hat{x}_n$, the entire system can be expressed as

$$\begin{cases} \dot{\tilde{\boldsymbol{x}}} = (A + LC)\tilde{\boldsymbol{x}} + H\phi(\boldsymbol{x}, \boldsymbol{\mu}) \\ \dot{y} = \hat{x}_2 + \tilde{x}_2 + \varphi_1(x_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 - l_2\tilde{x}_1 + \varphi_2(\bar{\hat{x}}_2, y) \\ \quad\vdots \\ \dot{\hat{x}}_n = u - l_n\tilde{x}_1 + \varphi_n(\bar{\hat{x}}_n, y) \end{cases} \tag{19}$$

where $\varphi_1(x_1) = f_1(x_1), \varphi_i(\bar{\hat{x}}_i, y) = F(\hat{x}_1 + k_1(\hat{x}_1 - y), \ldots, \hat{x}_i + k_i(\hat{x}_1 - y))$ with $i = 2, \ldots, n$.

Now, we introduce a change of coordinates as follows:

$$\begin{aligned} z_1 &= y - y_d \\ z_i &= \hat{x}_i - \alpha_{i,f}(y, \bar{\hat{x}}_{i-1}, \bar{\hat{\boldsymbol{\theta}}}_{i-1}), \quad i = 2, \ldots, n \end{aligned} \tag{20}$$

where $\bar{\hat{x}}_{i-1} = (\hat{x}_1, \cdots, \hat{x}_{i-1})$ and $\bar{\hat{\boldsymbol{\theta}}}_{i-1} = (\hat{\theta}_1, \cdots, \hat{\theta}_{i-1})$, $\alpha_{i,f}$ is the output of the first-order filter with α_{i-1} as the input.

Step 1: According to the coordinate transformation (20) with $i = 1$, we have

$$\dot{z}_1 = \hat{x}_2 + \tilde{x}_2 + \varphi_1 - \dot{y}_d \tag{21}$$

Consider the Lyapunov function as follows

$$V_1 = V_0 + \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\tilde{\boldsymbol{\theta}}_1 \tag{22}$$

where $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$ is the parameter error and $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$ is any constant matrix.

Then, the time derivative of V_1 is given by

$$\dot{V}_1 = \dot{V}_0 + z_1(\hat{x}_2 + \tilde{x}_2 + \varphi_1 - \dot{y}_d) - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \tag{23}$$

By completing the squares, we have

$$z_1\tilde{x}_2 \leq \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{x}_2^2 \leq \frac{1}{2}z_1^2 + \frac{1}{2}||\tilde{\boldsymbol{x}}||^2 \tag{24}$$

Now, substituting (18) and (24) into (23) gives

$$\begin{aligned} \dot{V}_1 &\leq -\lambda_{\min}(Q_2)||\tilde{\boldsymbol{x}}||^2 + z_1(\hat{x}_2 + \varphi_1 - \dot{y}_d) + \frac{1}{2}z_1^2 \\ &\quad + \frac{1}{2}||\tilde{\boldsymbol{x}}||^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \\ &= -\Xi||\tilde{\boldsymbol{x}}||^2 + z_1(\hat{x}_2 + \tilde{f}_1) - \frac{1}{2}z_1^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \end{aligned} \tag{25}$$

where $\Xi = \lambda_{\min}(Q_2) - \frac{1}{2}, \tilde{f}_1 = \varphi_1 - \dot{y}_d + z_1$.

Next, by virtue of the approximation property of RBFNN, a RBFNN can be employed to estimate $\tilde{f}_1$, that is to say, for any given $\varepsilon_1 > 0$, there exists a RBFNN $\boldsymbol{\theta}_1^{\mathrm{T}} S_1(\boldsymbol{z}_1)$ such that

$$\tilde{f}_1 = \boldsymbol{\theta}_1^{\mathrm{T}} S_1(\boldsymbol{z}_1) + \sigma_1(\boldsymbol{z}_1), |\sigma_1(\boldsymbol{z}_1)| \le \varepsilon_1 \tag{26}$$

where $\sigma_1(\boldsymbol{z}_1)$ denotes the approximation error and $\boldsymbol{z}_1 = [z_1]^{\mathrm{T}}$ is the input vector.

Taking the intermediate control signal α_1 as

$$\alpha_1 = -r_1 z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}} S_1(\boldsymbol{z}_1) \tag{27}$$

where $r_1 > 0$ is the design constant.

For the term $z_1(\hat{x}_2 + \tilde{f}_1)$, according to (26) and (27), we have

$$\begin{aligned} z_1(\hat{x}_2 + \tilde{f}_1) &\le z_1(\hat{x}_2 + (\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} + \hat{\boldsymbol{\theta}}_1^{\mathrm{T}}) S_1(\boldsymbol{z}_1)) + \frac{1}{2} z_1^2 + \frac{1}{2}\varepsilon_1^2 \\ &= z_1(\hat{x}_2 - \alpha_1) - r_1 z_1^2 + z_1 \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} S_1(\boldsymbol{z}_1) \\ &\quad + \frac{1}{2} z_1^2 + \frac{1}{2}\varepsilon_1^2 \end{aligned} \tag{28}$$

To avoid repeatedly differentiating α_1, a new variable $\alpha_{2,f}$ is introduced, and let α_1 pass through a first-order filter with time constant τ_2 to obtain $\alpha_{2,f}$ as

$$\tau_2 \dot{\alpha}_{2,f} + \alpha_{2,f} = \alpha_1, \alpha_{2,f}(0) = \alpha_1(0) \tag{29}$$

Substituting (28) and (29) into (25) gives

$$\begin{aligned} \dot{V}_1 &\le -\Xi ||\tilde{\boldsymbol{x}}||^2 + z_1(\hat{x}_2 - \alpha_1) - r_1 z_1^2 + \frac{1}{2}\varepsilon_1^2 \\ &\quad + \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} (z_1 S_1(\boldsymbol{z}_1) - \Gamma_1^{-1} \dot{\hat{\boldsymbol{\theta}}}_1) \end{aligned} \tag{30}$$

Due to $z_2 = \hat{x}_2 - \alpha_{2,f}$ and define the output error of this filter as $\chi_2 = \alpha_{2,f} - \alpha_1$, then we have

$$\begin{aligned} \dot{V}_1 &\le -\Xi ||\tilde{\boldsymbol{x}}||^2 + z_1 z_2 + z_1 \chi_2 - r_1 z_1^2 + \frac{1}{2}\varepsilon_1^2 \\ &\quad + \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} (z_1 S_1(\boldsymbol{z}_1) - \Gamma_1^{-1} \dot{\hat{\boldsymbol{\theta}}}_1) \end{aligned} \tag{31}$$

Step i $(2 \le i \le n-1)$**:** Similar to Step 1, a new variable $\alpha_{i+1,f}$ is introduced to develop the design process. Let the intermediate control signal α_i pass through a first-order filter with time constant τ_{i+1} to obtain $\alpha_{i+1,f}$ as

$$\tau_{i+1} \dot{\alpha}_{i+1,f} + \alpha_{i+1,f} = \alpha_i, \alpha_{i+1,f}(0) = \alpha_i(0) \tag{32}$$

Then, the output error of this filter $\chi_{i+1} = \alpha_{i+1,f} - \alpha_i$ satisfies

$$\dot{\chi}_{i+1} = -\frac{\chi_{i+1}}{\tau_{i+1}} + B_{i+1}(\bar{z}_i, \tilde{x}, \bar{\hat{\boldsymbol{\theta}}}_i, \bar{\chi}_{i+1}) \tag{33}$$

where $\bar{z}_i = [z_1, \cdots, z_i]^{\mathrm{T}}, \bar{\chi}_i = [\chi_1, \cdots, \chi_i]^{\mathrm{T}}, \bar{\hat{\boldsymbol{\theta}}}_i = [\hat{\boldsymbol{\theta}}_1, \cdots, \hat{\boldsymbol{\theta}}_i]^{\mathrm{T}}$ and

$$\begin{aligned} B_{i+1}(\bar{z}_i, \tilde{x}, \bar{\hat{\boldsymbol{\theta}}}_i, \bar{\chi}_{i+1}) &= -\frac{\partial \alpha_i}{\partial y}(\hat{x}_2 + \tilde{x}_2 + \varphi_1(x_1)) \\ &\quad - \sum_{j=1}^{i} \frac{\partial \alpha_i}{\partial \hat{\boldsymbol{\theta}}_j} \dot{\hat{\boldsymbol{\theta}}}_j - \sum_{j=2}^{i} \frac{\partial \alpha_i}{\partial \hat{x}_j} \dot{\hat{x}}_j - \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial \alpha_{j+1,f}} \dot{\alpha}_{j+1,f}. \end{aligned}$$

Remark 3.3: As state in (Li, Tong, & Li, 2015), for each $j = 2, \ldots, n$, B_{i+1} is a continuous function and there exists a positive constant σ_{i+1}, such that $|B_{i+1}| \le \sigma_{i+1}$.

According to the coordinate transformation (20), choose the Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2}\chi_i^2 + \frac{1}{2} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{\boldsymbol{\theta}}_i \tag{34}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ is the parameter error and $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ is any constant matrix.

By repeating the same procedure as that in Step 1, we have

$$\begin{aligned} \dot{V}_i &\le -\Xi ||\tilde{\boldsymbol{x}}||^2 + \sum_{j=1}^{i-1} z_j z_{j+1} + \sum_{j=1}^{i-1} z_j \chi_{j+1} - \sum_{j=1}^{i-1} r_j z_j^2 + \frac{1}{2} \sum_{j=1}^{i-1} \varepsilon_j^2 \\ &\quad + \sum_{j=1}^{i-1} \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} (z_j S_j(\boldsymbol{z}_j) - \Gamma_j^{-1} \dot{\hat{\boldsymbol{\theta}}}_j) + \sum_{j=1}^{i-1} \chi_{j+1} \left(-\frac{\chi_{i+1}}{\tau_{i+1}} + B_{i+1} \right) \\ &\quad + z_i(\hat{x}_{i+1} + \tilde{f}_i) - \frac{1}{2} z_i^2 + \chi_i \dot{\chi}_i - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \Gamma_i^{-1} \dot{\hat{\boldsymbol{\theta}}}_i \end{aligned} \tag{35}$$

where $\tilde{f}_i = \varphi_i - l_i \tilde{x}_1 - \dot{\alpha}_{i,f} + \frac{1}{2} z_i$.

Similarly, we employ the RBFNN $\boldsymbol{\theta}_i^{\mathrm{T}} S_i(\boldsymbol{z}_i)$ to approximate unknown function $\tilde{f}_i$, that is to say, for any given $\varepsilon_i > 0$, there exists a RBFNN $\boldsymbol{\theta}_i^{\mathrm{T}} S_i(\boldsymbol{z}_i)$ such that

$$\tilde{f}_i = \boldsymbol{\theta}_i^{\mathrm{T}} S_i(\boldsymbol{z}_i) + \sigma_i(\boldsymbol{z}_i), |\sigma_i(\boldsymbol{z}_i)| \le \varepsilon_i \tag{36}$$

where $\sigma_i(\boldsymbol{z}_i)$ denotes the approximation error and $\boldsymbol{z}_i = [z_1, \ldots, z_i]^{\mathrm{T}}$ is the input vector.

Taking the intermediate control signal α_i as

$$\alpha_i = -r_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} S_i(\boldsymbol{z}_i) \tag{37}$$

where $r_i > 0$ is the design constant.

For the term $z_i(\hat{x}_{i+1}+\tilde{f}_i)$, according to (36) and (37), we have

$$z_i(\hat{x}_{i+1}+\tilde{f}_i) \le z_i(\hat{x}_{i+1}-\alpha_i) - r_i z_i^2 + z_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} S_i(\boldsymbol{z}_i) + \frac{1}{2}z_i^2 + \frac{1}{2}\varepsilon_i^2 \tag{38}$$

Note that $z_{i+1} = \hat{x}_{i+1} - \alpha_{i+1,f}$ and substituting (38) into (35), we have

$$\begin{aligned}\dot{V}_1 \le & -\Xi||\tilde{\boldsymbol{x}}||^2 + \sum_{j=1}^{i} z_j z_{j+1} + \sum_{j=1}^{i} z_j\chi_{j+1} \\ & - \sum_{j=1}^{i} r_j z_j^2 + \frac{1}{2}\sum_{j=1}^{i}\varepsilon_j^2 \\ & + \sum_{j=1}^{i}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}(z_j S_j(\boldsymbol{z}_j) - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j) \\ & + \sum_{j=1}^{i}\chi_{j+1}\left(-\frac{\chi_{i+1}}{\tau_{i+1}} + B_{i+1}\right)\end{aligned} \tag{39}$$

Step n: According to the coordinate transformation (32) with $i = n$, choose the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{4}z_n^4 + \frac{1}{4}\chi_n^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\Gamma_n^{-1}\tilde{\boldsymbol{\theta}}_n \tag{40}$$

where $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n - \hat{\boldsymbol{\theta}}_n$ is the parameter error and $\Gamma_n = \Gamma_n^{\mathrm{T}} > 0$ is any constant matrix.

By employing (19) and taking (39) with $i = n-1$ into account, the time derivative of V_n is given by

$$\begin{aligned}\dot{V}_n = & -\Xi||\tilde{\boldsymbol{x}}||^2 + \sum_{j=1}^{n-1} z_j z_{j+1} + \sum_{j=1}^{n-1} z_j\chi_{j+1} - \sum_{j=1}^{n-1} r_j z_j^2 + \frac{1}{2}\sum_{j=1}^{n-1}\varepsilon_j^2 \\ & + \sum_{j=1}^{n-1}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}(z_j S_j(\boldsymbol{z}_j) - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j) + \sum_{j=1}^{n-1}\chi_{j+1}\left(-\frac{\chi_{i+1}}{\tau_{i+1}} + B_{i+1}\right) \\ & + z_n(u+\tilde{f}_n) - \frac{1}{2}z_n^2 + \chi_n\dot{\chi}_n - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\Gamma_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n\end{aligned} \tag{41}$$

where $\tilde{f}_n = \varphi_n - l_n\tilde{x}_1 - \dot{\alpha}_{n,f} + \frac{1}{2}z_n$.

Similarly, we employ the RBFNN $\boldsymbol{\theta}_n^{\mathrm{T}} S_n(\boldsymbol{z}_n)$ to approximate unknown function $\tilde{f}_n$, that is to say, for any given $\varepsilon_n > 0$, there exists a RBFNN $\boldsymbol{\theta}_n^{\mathrm{T}} S_n(\boldsymbol{z}_n)$ such that

$$\tilde{f}_n = \boldsymbol{\theta}_n^{\mathrm{T}} P_{m_n}(\boldsymbol{z}_n) + \sigma_n(\boldsymbol{z}_n), |\sigma_n(\boldsymbol{z}_n)| \le \varepsilon_n \tag{42}$$

where $\boldsymbol{z}_n = [z_1, \ldots, z_n]^{\mathrm{T}}$ and $\sigma_n(\boldsymbol{z}_n)$ being the approximation error.

Choosing the actual control input

$$u = -r_n z_n - \hat{\boldsymbol{\theta}}_n^{\mathrm{T}} S_n(\boldsymbol{z}_n). \tag{43}$$

where $r_n > 0$ is the design constant.

Then, the following inequality can be obtained

$$\begin{aligned}z_n(u+\tilde{f}_n) = & z_n(-r_n z_n - \hat{\boldsymbol{\theta}}_n^{\mathrm{T}} S_n(\boldsymbol{z}_n) \\ & + \boldsymbol{\theta}_n^{\mathrm{T}} S_n(\boldsymbol{z}_n) + \sigma_n(\boldsymbol{z}_n)) \\ & \le -r_n z_n^2 + z_n\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} S_n(\boldsymbol{z}_n) + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_n^2.\end{aligned} \tag{44}$$

And then, substituting (44) to (41), we have

$$\begin{aligned}\dot{V}_n = & -\Xi||\tilde{\boldsymbol{x}}||^2 + \sum_{j=1}^{n-1} z_j z_{j+1} + \sum_{j=1}^{n-1} z_j\chi_{j+1} - \sum_{j=1}^{n} r_j z_j^2 + \frac{1}{2}\sum_{j=1}^{n}\varepsilon_j^2 \\ & + \sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}(z_j S_j(\boldsymbol{z}_j) - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j) + \sum_{j=1}^{n}\chi_{j+1}\left(-\frac{\chi_{j+1}}{\tau_{j+1}} + B_{j+1}\right)\end{aligned} \tag{45}$$

By completing the squares, we have

$$\sum_{j=1}^{n-1} z_j z_{j+1} \le \sum_{j=1}^{n-1}\left(\frac{1}{2}z_j^2 + \frac{1}{2}z_{j+1}^2\right) \le \sum_{j=1}^{n} z_j^2 \tag{46}$$

$$\begin{aligned}\sum_{j=1}^{n-1} z_j\chi_{j+1} & \le \sum_{j=1}^{n-1}\left(\frac{1}{2}z_j^2 + \frac{1}{2}\chi_{j+1}^2\right) \\ & \le \frac{1}{2}\sum_{j=1}^{n} z_j^2 + \frac{1}{2}\sum_{j=2}^{n}\chi_j^2\end{aligned} \tag{47}$$

$$\begin{aligned}& \sum_{j=1}^{n-1}\chi_{j+1}\left(-\frac{\chi_{j+1}}{\tau_{j+1}} + B_{j+1}\right) \\ & \le -\sum_{j=1}^{n-1}\frac{\chi_{j+1}^2}{\tau_{j+1}} + \frac{1}{2}\sum_{j=1}^{n-1}\xi_{j+1}^2\sigma_{j+1}^2\chi_{j+1}^2 + \frac{1}{2}\sum_{j=1}^{n-1}\frac{1}{\xi_{j+1}^2}\end{aligned} \tag{48}$$

where $\xi_i (j = 2, \ldots, n-1)$ are any positive constant.

Combining (41) with (46), (47) and (48) gives

$$\begin{aligned}\dot{V}_n \le & -\Xi||\tilde{\boldsymbol{x}}||^2 + \sum_{j=1}^{n} z_i^2 + \frac{1}{2}\sum_{j=1}^{n} z_j^2 + \frac{1}{2}\sum_{j=1}^{n-1}\chi_{j+1}^2 \\ & - \sum_{j=1}^{n} r_j z_j^2 + \frac{1}{2}\sum_{j=1}^{n}\varepsilon_j^2 + \sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}(z_j S_j(\boldsymbol{z}_j) - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j) \\ & - \sum_{j=1}^{n-1}\frac{\chi_{j+1}^2}{\tau_{j+1}} + \frac{1}{2}\sum_{j=1}^{n-1}\xi_{j+1}^2\sigma_{j+1}^2\chi_{j+1}^2 + \frac{1}{2}\sum_{j=1}^{n-1}\frac{1}{\xi_{j+1}^2} \\ \le & -\Xi||\tilde{\boldsymbol{x}}||^2 - \sum_{j=1}^{n}\gamma_j z_j^2 \\ & + \frac{1}{2}\sum_{j=1}^{n-1}\left(1 - \frac{2}{\tau_{j+1}} + \xi_{j+1}^2\sigma_{j+1}^2\right)\chi_i^2\end{aligned}$$

$$+\sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}(z_jS_j(\boldsymbol{z}_j)-\Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j)+\frac{1}{2}\sum_{j=1}^{n}\varepsilon_j^2+\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2} \tag{49}$$

where $\gamma_j = r_j - \frac{3}{2} > 0$.

3.3. Stability analysis

The main results of this paper can be summarised by the following theorem.

Theorem 3.1: *Consider the nonlinear system (1) with observer (7). If the control law is chosen as (43), the intermediate virtual control signals α_i described as*

$$\alpha_i = -r_iz_i - \hat{\boldsymbol{\theta}}_i^T S_i(\boldsymbol{z}_i), i = 1,\dots,n-1 \tag{50}$$

and the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_i$ described as

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i\Gamma_iP_{m_i}(\boldsymbol{z}_n) - \eta_i\Gamma_i\hat{\boldsymbol{\theta}}_i, \quad i = 1,\dots,n \tag{51}$$

where constants r_i and η_i are designed parameters satisfy $r_i > \frac{3}{2}$ and $\eta_i > 0$, and matrices Γ_i satisfy $\Gamma_i = \Gamma_i^T > 0$, then for any initial condition, all the signals in the closed-loop system remain bounded and the tracking error converges to a small neighbourhood around the origin.

Proof: Choose the following Lyapunov function as

$$V = \tilde{\boldsymbol{x}}^TQ_1\tilde{\boldsymbol{x}} + \frac{1}{2}\sum_{i=1}^{n}z_i^2 + \frac{1}{2}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\boldsymbol{\theta}_i + \frac{1}{2}\sum_{i=2}^{n}\chi_i^2. \tag{52}$$

It follows from (49), we have

$$\dot{V} \le -\Xi||\tilde{\boldsymbol{x}}||^2 - \sum_{i=1}^{n}\gamma_iz_i^2 + \frac{1}{2}\sum_{i=1}^{n-1}\left(1-\frac{2}{\tau_{i+1}}+\xi_{i+1}^2\sigma_{i+1}^2\right)\chi_i^2 + \sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_iS_i(\boldsymbol{z}_i)-\Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i)+\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2+\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2}. \tag{53}$$

For the term $\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_iS_i(\boldsymbol{z}_i)-\Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i)$, according to (51), we have

$$\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_iS_i(\boldsymbol{z}_i)-\Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i) \le -\sum_{i=1}^{n}\left(\frac{\eta_i}{2\lambda_{\max}(\Gamma_i^{-1})}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i\right)+\sum_{i=1}^{n}\left(\frac{\eta_i}{2}||\boldsymbol{\theta}_i||^2\right). \tag{54}$$

Substituting (54) into (53), we have

$$V \le -\Xi||\tilde{\boldsymbol{x}}||^2 - \sum_{i=1}^{n}\gamma_iz_i^2 + \frac{1}{2}\sum_{i=1}^{n-1}\left(1-\frac{2}{\tau_{i+1}}+\xi_{i+1}^2\sigma_{i+1}^2\right)\chi_i^2 - \sum_{i=1}^{n}\left(\frac{\eta_i}{2\lambda_{\max}(\Gamma_i^{-1})}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i\right)+\frac{1}{2}\sum_{i=1}^{n}(\eta_i||\boldsymbol{\theta}_i||^2) + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2+\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2}. \tag{55}$$

Let

$$a_i = \min\left\{\frac{2\Xi}{\lambda_{\max}^2(Q_i)}, 2\gamma_i, \left(1-\frac{2}{\tau_{i+1}}+\xi_{i+1}^2\sigma_{i+1}^2\right), \frac{\eta_i}{\lambda_{\max}(\Gamma_i^{-1})}\right\},$$
$$a_0 = \min\{a_i,\dots,a_n\},$$
$$b_0 = \frac{1}{2}\sum_{i=1}^{n}(\eta_i||\boldsymbol{\theta}_i||^2)+\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2+\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2},$$

then, inequality (55) can be rewritten in the following form

$$V \le -a_0V + b_0. \tag{56}$$

Based on (56) and Lemma 1, using the similar arguments in (Wang, 2011; Wang & Huang, 2005; Wang, Liu, & Shi, 2011), it can be shown that all the signals of the closed system bounded in probability.

According to (56), we have

$$0 \le V(t) \le \left(V(0)-\frac{a_0}{b_0}\right)e^{-a_0t}+\frac{a_0}{b_0} \tag{57}$$

Meanwhile, note that

$$0 \le \frac{1}{2}z_1^2 \le V(t) \tag{58}$$

Combining (57) and (58), we have

$$\left|\frac{1}{2}z_1^2\right| \le \frac{a_0}{b_0}, \text{when } t \to +\infty \tag{59}$$

Inequality (59) means

$$\lim_{t\to+\infty}|e| = \lim_{t\to+\infty}|y-y_d| = \lim_{t\to+\infty}|z_1^2| \le \sqrt{\frac{2a_0}{b_0}} \tag{60}$$

This implies that the tracking error finally converges into a small region around the origin. That is to say, we can properly adjust the design parameters a_0 and b_0 to achieve the tracking error arbitrarily small.

The proof is completed. ■

Remark 3.4: According to Theorem 3.1, the satisfactory tracking performance can be achieved by properly choosing the design parameters r_j, τ_j, η_j, ε_j and ξ_j, matrices Γ_j and Q_1. Meanwhile, based on (60), the tracking error can be made arbitrarily small by reducing a_0 and increasing b_0. However, in practice, the design parameters should be selected appropriately to obtain a specific control objective.

4. Simulation examples

In this section, we will exploit three simulation examples to illustrate the effectiveness of the proposed adaptive control method.

Example 4.1: Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 - x_1, \\ \dot{x}_2 &= x_3 - x_1 - x_2^3, \\ \dot{x}_3 &= u - x_1 - x_3^3, \\ y &= x_1, \end{aligned} \tag{61}$$

with the initial states $x_1(0) = 0.1, x_2(0) = 0.1, x_3(0) = 0.1$. The reference signal is given as $y_d = 0.7 \sin t$.

According to (2) and (61), it can be easily get

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \; F = \begin{bmatrix} -x_1 \\ -x_1 - x_2^3 \\ -x_1 - x_3^3 \end{bmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2^3 \\ x_3^3 \end{pmatrix} = Hh(\boldsymbol{x}).$$

where $h(\boldsymbol{x})$ satisfies Assumption 2.1 and it's easy to verify that when

$$Q_1 = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, K = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, L = \begin{pmatrix} -1.5 \\ -5.5 \\ -0.5 \end{pmatrix}$$

LMI's (4) holds.

According to (7), the observer is designed as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - 1.5(\hat{x}_1 - x_1) - (\hat{x}_1 + (\hat{x}_1 - x_1)) \\ \dot{\hat{x}}_2 &= \hat{x}_2 - 5.5(\hat{x}_1 - x_1) - (\hat{x}_1 + (\hat{x}_1 - x_1)) \\ &\quad - (\hat{x}_2 + (\hat{x}_1 - x_1))^3 \\ \dot{\hat{x}}_3 &= u - 0.5(\hat{x}_1 - x_1) - (\hat{x}_1 + (\hat{x}_1 - x_1)) \\ &\quad - (\hat{x}_3 - (\hat{x}_1 - x_1))^3 \end{aligned} \tag{62}$$

According to Theorem 3.1, the virtual control laws α_i, the actual control law u, and the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_i$ are defined as

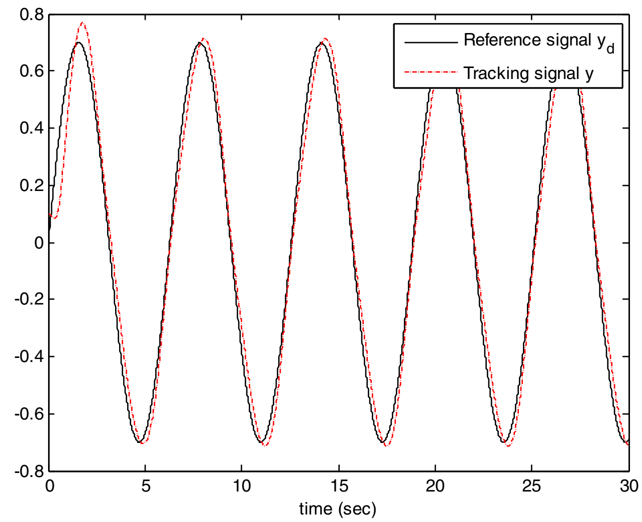

Figure 1. The trajectories of system output y and reference signal y_d of Example 4.1.

$$\alpha_i = -r_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} S_i(\boldsymbol{z}_i), \; i = 1, 2 \tag{63}$$

$$u = -r_3 z_3 - \hat{\boldsymbol{\theta}}_3^{\mathrm{T}} S_3(\boldsymbol{z}_3) \tag{64}$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i \Gamma_i P_{m_i}(\boldsymbol{z}_i) - \eta_i \Gamma_i \hat{\boldsymbol{\theta}}_i, \; i = 1, 2, 3 \tag{65}$$

where $z_1 = x_1 - y_d$, $z_2 = \hat{x}_2 - \alpha_{2,f}$, $z_3 = \hat{x}_3 - \alpha_{3,f}$ and $\boldsymbol{z}_1 = z_1, \boldsymbol{z}_2 = [z_1, z_2]^{\mathrm{T}}, \boldsymbol{z}_2 = [z_1, z_2, z_3]^{\mathrm{T}}$.

In the simulation, $\hat{\boldsymbol{\theta}}_1^{\mathrm{T}} S_1(\boldsymbol{z}_1)$ contains seven nodes with centre spaced evenly in the interval $[-3, 3]$ and widths being equal to 10; $\hat{\boldsymbol{\theta}}_2^{\mathrm{T}} S_2(\boldsymbol{z}_2)$ contains 11 nodes with centre spaced evenly in the interval $[-10, 10] \times [-10, 10]$ and widths being equal to 10; $\hat{\boldsymbol{\theta}}_3^{\mathrm{T}} S_3(\boldsymbol{z}_3)$ contains 11 nodes with centre spaced evenly in the interval $[-10, 10] \times [-10, 10] \times [-10, 10]$ and widths being equal to 10. The design parameters are chosen as: $k_1 = 16$, $k_2 = 15$, $k_3 = 15$, $\eta_1 = 0.1$, $\eta_2 = 2$, $\eta_3 = 0.2$, $\Gamma_1 = I_7$, $\Gamma_2 = I_{11}$, $\Gamma_3 = I_{11}$, $\tau_1 = 0.005$ and $\tau_2 = 0.005$.

The simulation results are illustrated in Figures 1234–5, respectively.

Figure 1 indicates the trajectories of system output y and the reference signal y_d, it can be seen that the good tracking performance has been achieved. Figure 2 displays the trajectory of the control input u. The estimation of states x_1, x_2 and x_3 are shown in Figures 3–5, respectively.

Remark 4.1: From Figures 1–5, we can see that the system output y, the control input u, the system states x_1, x_2 and x_3, and the observer states $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ are all bounded. The presented simulation results illustrate the

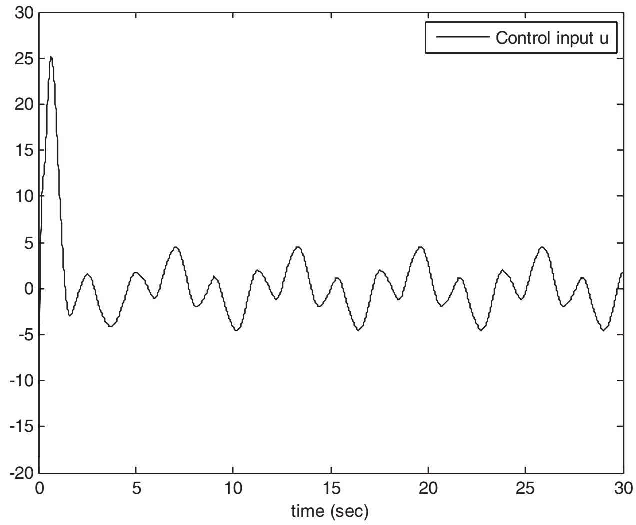

Figure 2. The trajectory of control input u of Example 4.1.

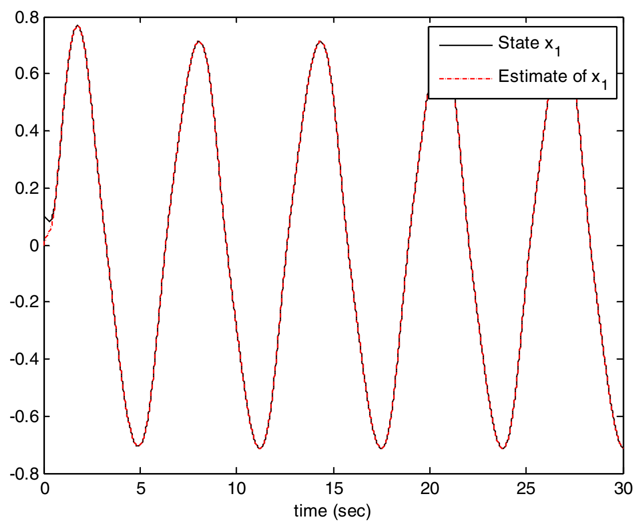

Figure 3. The trajectories of x_1 and $\hat{x}_1$ of Example 4.1.

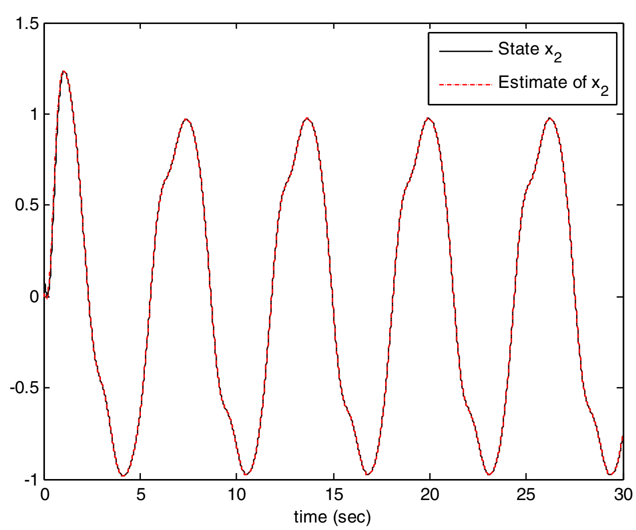

Figure 4. The trajectories of x_2 and $\hat{x}_2$ of Example 4.1.

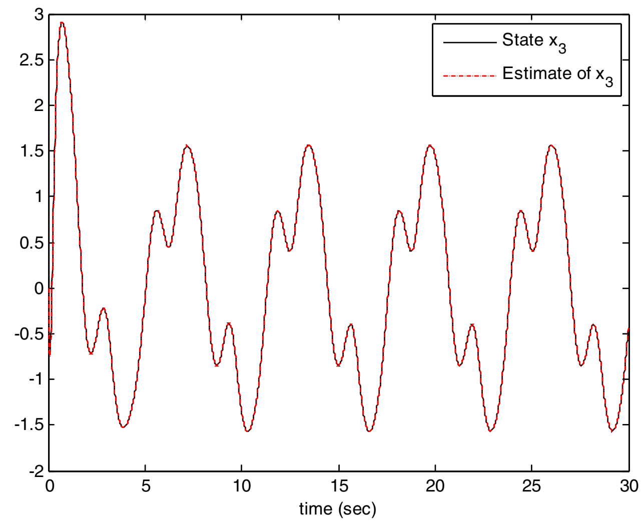

Figure 5. The trajectories of x_3 and $\hat{x}_3$ of Example 4.1.

effectiveness of the control approach proposed in this paper.

Example 4.2: Consider the following nonlinear system

$$\begin{aligned}
\dot{x}_1 &= x_2 - \frac{1}{3}x_1^3 \\
\dot{x}_2 &= x_3 - \frac{1}{3}x_1^3 - \frac{2}{5}x_2^5 \\
\dot{x}_3 &= u - \frac{1}{5}x_2^5 - \frac{1}{3}x_3^3 \\
y &= x_1
\end{aligned} \tag{66}$$

with the initial states $x_1(0) = 0.05, x_2(0) = 0.05, x_3(0) = 0.05$. The reference signal is given as $y_d = \sin t$.

Similarly, it can be easily get

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad F = \begin{bmatrix} -\frac{1}{3}x_1^3 \\ -\frac{1}{3}x_1^3 - \frac{2}{5}x_2^5 \\ -\frac{1}{5}x_2^5 - \frac{1}{3}x_3^3 \end{bmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ -1 & -2 & 0 \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{3}x_1^3 \\ \frac{1}{5}x_2^5 \\ \frac{1}{3}x_3^3 \end{pmatrix} = Hh(\boldsymbol{x}).$$

where $h(\boldsymbol{x})$ satisfies Assumption 2.1 and it's easy to verify that when

$$Q_1 = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}, \quad K = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \quad L = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

LMI's (4) holds.

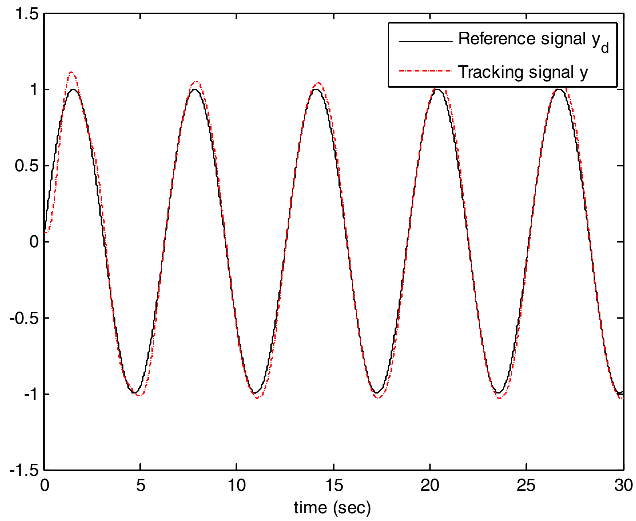

Figure 6. The trajectories of system output y and reference signal y_d of Example 4.2.

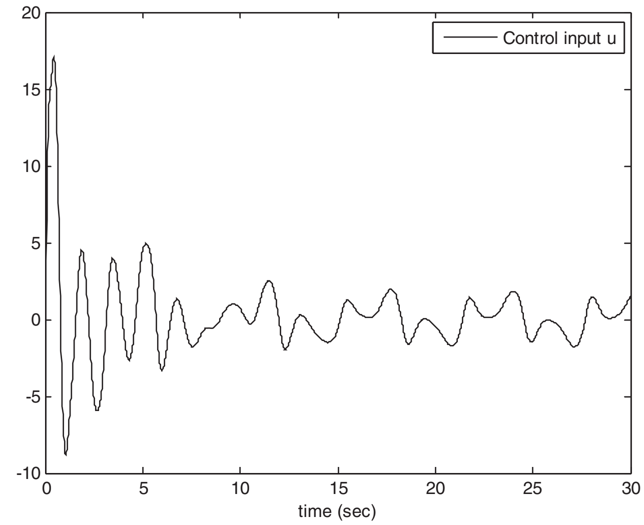

Figure 7. The trajectory of control input u of Example 4.2.

According to (7), the observer is designed as

$$
\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 - (\hat{x}_1 - x_1) - \frac{1}{3}(\hat{x}_1 - (\hat{x}_1 - x_1))^3 \\
\dot{\hat{x}}_2 &= \hat{x}_2 + (\hat{x}_1 - x_1) - \frac{1}{3}(\hat{x}_1 - (\hat{x}_1 - x_1))^3 \\
&\quad - \frac{2}{5}(\hat{x}_2 - (\hat{x}_1 - x_1))^5 \\
\dot{\hat{x}}_3 &= u - (\hat{x}_1 - x_1) - \frac{1}{5}(\hat{x}_2 - (\hat{x}_1 - x_1))^5 \\
&\quad - \frac{1}{3}(\hat{x}_3 + (\hat{x}_1 - x_1))^3
\end{aligned} \tag{67}
$$

Now, we further apply the control approach (63)–(65) to system (66), and the structure of RBFNN and all the parameters are kept as that in Example 1. The simulation results are shown in Figures 6–10. It can be seen that the tracking performance is still fairly satisfactory, which further verify the effectiveness of the control method designed in this paper.

Example 4.3: Consider a practical example to further illustrate the effectiveness of the proposed controller. Using the similar arguments in Barkhordari Yazdi and Jahed-Motlagh (2009); Niu et al. (2019), a type of closed, continuously stirred tank, chemical reactor with one mode of feed stream can be modelled as the following nonlinear system model:

$$
\begin{aligned}
\dot{x}_1 &= x_2 + \frac{1}{2}x_1 \\
\dot{x}_2 &= u \\
y &= x_1
\end{aligned} \tag{68}
$$

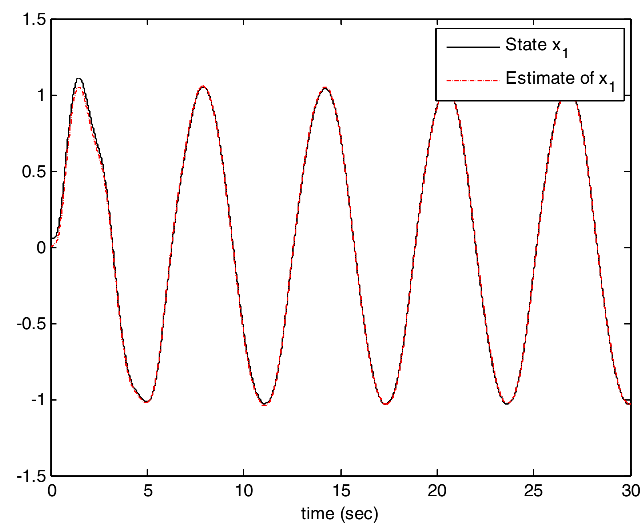

Figure 8. The trajectories of x_1 and $\hat{x}_1$ of Example 4.2.

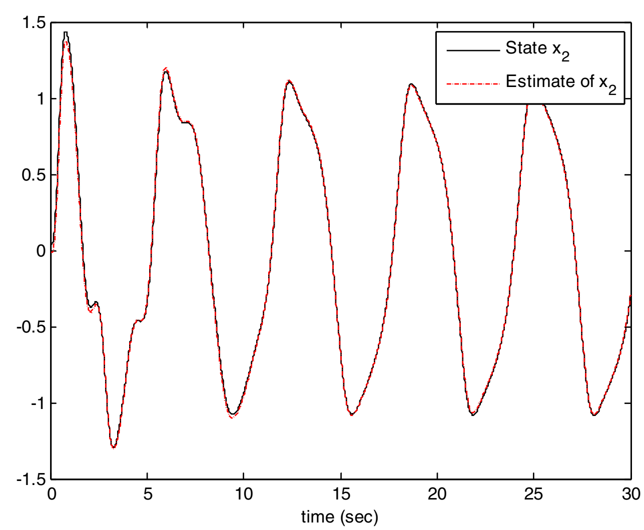

Figure 9. The trajectories of x_2 and $\hat{x}_2$ of Example 4.2.

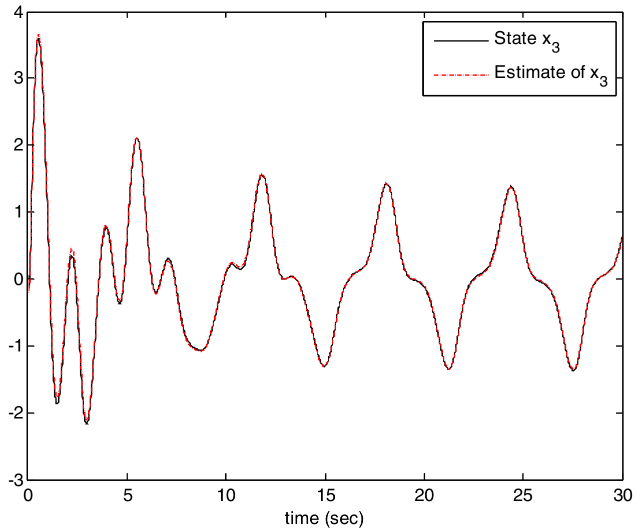

Figure 10. The trajectories of x_3 and $\hat{x}_3$ of Example 4.2.

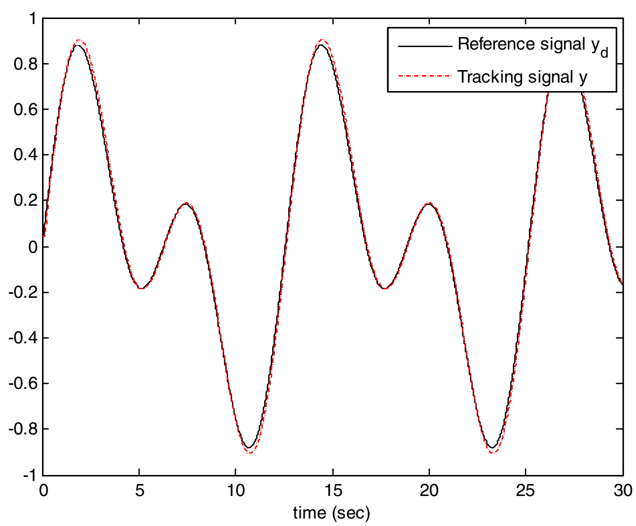

Figure 11. The trajectories of system output y and reference signal y_d of Example 4.3.

Similarly, it can be easily get

$$A=\begin{pmatrix}0&1\\0&0\end{pmatrix}, F=\begin{bmatrix}\frac{1}{2}x_1\\0\end{bmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}\frac{1}{2}x_1\\0\end{pmatrix}=Hh(\boldsymbol{x}).$$

where $h(\boldsymbol{x})$ satisfies Assumption 2.1 and it's easy to verify that when

$$Q_1=\begin{pmatrix}2&0\\0&1\end{pmatrix},\quad K=\begin{pmatrix}1\\0\end{pmatrix},\quad L=\begin{pmatrix}-1\\-1\end{pmatrix}$$

LMI's (4) holds.

According to (7), the observer is designed as

$$\begin{aligned}\dot{\hat{x}}_1&=\hat{x}_2-(\hat{x}_1-x_1)+\tfrac{1}{2}(\hat{x}_1+(\hat{x}_1-x_1))\\ \dot{\hat{x}}_2&=u-(\hat{x}_1-x_1)\end{aligned}\tag{69}$$

According to Theorem 3.1, the virtual control law α_1, the actual control law u, and the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_i$ are defined

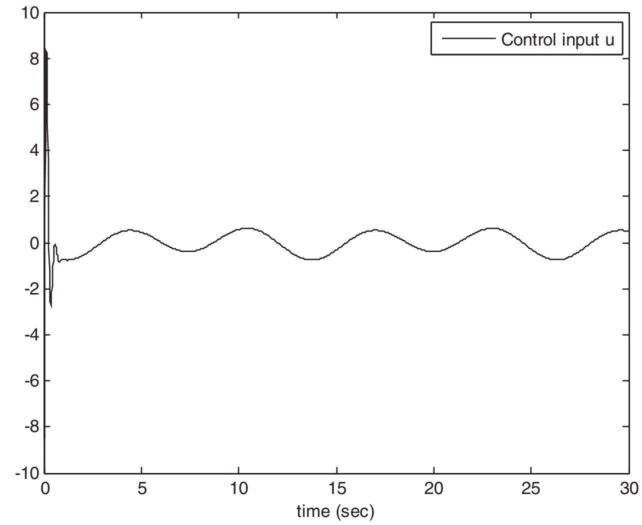

Figure 12. The trajectory of control input u of Example 4.3.

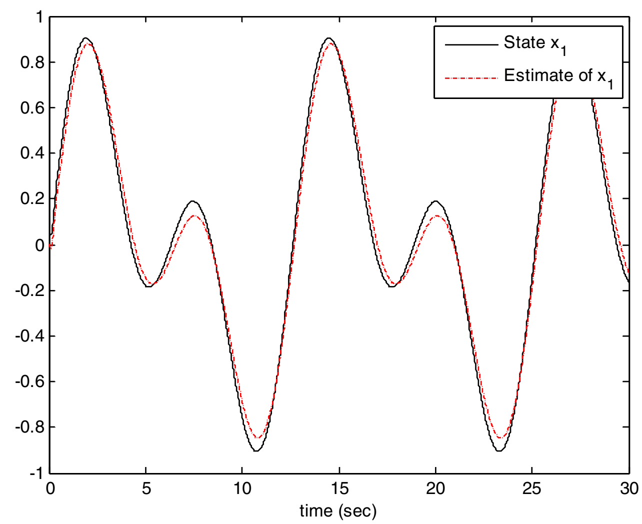

Figure 13. The trajectories of x_1 and $\hat{x}_1$ of Example 4.3.

as

$$\alpha_1=-r_1z_1-\hat{\boldsymbol{\theta}}_1^{\mathrm{T}}S_1(\boldsymbol{z}_1)\tag{70}$$

$$u=-r_2z_2-\hat{\boldsymbol{\theta}}_2^{\mathrm{T}}S_2(\boldsymbol{z}_2)\tag{71}$$

$$\dot{\hat{\boldsymbol{\theta}}}_i=z_i\Gamma_iP_{m_i}(\boldsymbol{z}_i)-\eta_i\Gamma_i\hat{\boldsymbol{\theta}}_i,\ i=1,2\tag{72}$$

where $z_1=x_1-y_d$, $z_2=\hat{x}_2-\alpha_{2,f}$, $\boldsymbol{z}_1=z_1$, $\boldsymbol{z}_2=[z_1,z_2]^{\mathrm{T}}$.

In the simulation, $\hat{\boldsymbol{\theta}}_1^{\mathrm{T}}S_1(\boldsymbol{z}_1)$ contains seven nodes with centre spaced evenly in the interval $[-3,3]$ and widths being equal to 10; $\hat{\boldsymbol{\theta}}_2^{\mathrm{T}}S_2(\boldsymbol{z}_2)$ contains 11 nodes with centre spaced evenly in the interval $[-10,10]\times[-10,10]$ and widths being equal to 10; The design parameters are chosen as: $k_1=16$, $k_2=15$, $\eta_1=0.1$, $\eta_2=2$, $\Gamma_1=I_7$,

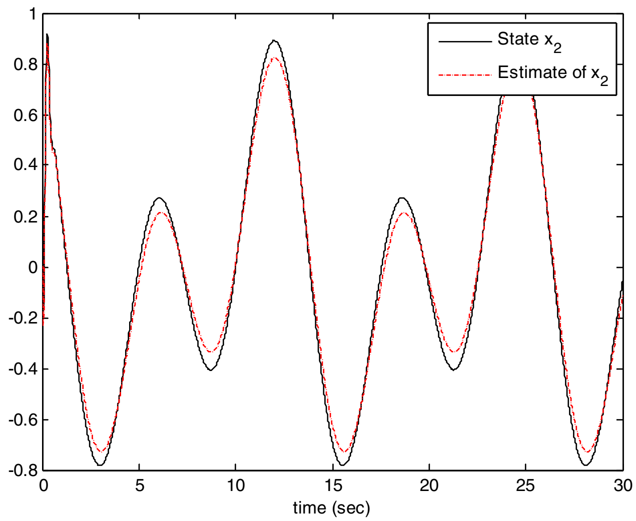

Figure 14. The trajectories of x_2 and $\hat{x}_2$ of Example 4.3.

$\Gamma_2 = I_{11}$ and $\tau_1 = 0.005$. Moreover, the reference signal is chosen as $y_d = 0.5(\sin t + \sin(0.5t))$ and the initial conditions are $x_1(0) = 0.05$, $x_2(0) = 0.05$. The simulation results are shown in Figures 11–14.

From Figures 11–14, we can conclude that a high quality tracking control performance is obtained under the control method designed in this paper. Meanwhile, all the signals in the closed-loop system are bounded.

5. Conclusions

In this paper, an adaptive neural output feedback control scheme is developed to solve the tracking control problem for a class of nonlinear system with unmeasurable states. Firstly, a state observer has been designed to estimate the unmeasured states. Secondly, RBFNNs are used to approximate the unknown nonlinear functions. Finally, in order to overcome the problem of explosion of complexity, a dynamic surface control method was introduced in the controller design. It is shown that the proposed controller ensures that all signals of the closed-loop system remain bounded and the tracking error converges to a small neighbourhood around the origin.

Our future work will be focused on extending the proposed methodology to other classes of control problems, such as finite-time control problems. It should be noted that the objective of this paper is accomplished if and only if time tends to infinity, that is to say, we focus on focus on infinite time stability issues. Recently, a growing attention has been paid on finite time (Wang, Chen, Sun, & Lin, 2019; Wang & Zhang, 2018) due to the fact that, in some engineering practice, system performances are required in finite time. The finite time control, as compared with the infinite time control, is characterised by better robustness and disturbance rejection properties (Ding & Li, 2011; Hong, Huang, & Xu, 2001), therefore, the investigation of the finite time control problems has important meanings in theory and practical engineering. How to extend the control approach designed in this paper to finite time control problems is a challengeable task.

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