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Adaptive decentralized tracking control of a class of large-scale nonlinear systems with unknown dead-zone inputs using neural network

Yu-Qun Han[1,2], Shan-Liang Zhu[1,2], De-Yu Duan[1,2], Lei Chu[1,2] and Shu-Guo Yang[1,2]

Abstract

In this paper, an adaptive decentralized control approach is proposed for a class of large-scale nonlinear systems with unknown dead-zone inputs using neural network. Firstly, the dead-zone outputs are firstly represented as simple linear systems with a static time-varying gain and bounded disturbance by introducing characteristic function. Secondly, in the controller design, neural networks are utilized to approximate the unknown nonlinear functions. Thirdly, an adaptive decentralized tracking control approach is constructed via backstepping design technique. It is shown that the proposed control approach can assure that all the signals of the closed-loop system semi-globally uniformly ultimately bounded and the tracking errors finally converge to a small domain around the origin. The proposed method can get precise tracking results with low computational cost, and have a good real-time performance and convergence. Finally, two examples are given to demonstrate the effectiveness of the proposed control scheme.

Keywords

Decentralized, tracking control, large-scale nonlinear system, dead-zone input, neural network

Introduction

In general, large-scale systems are often considered as a set of interconnected subsystems, which exist in many practical systems, such as electric power systems, aerospace systems, and multi-agent systems, the wheeled mobile manipulator model (Yin et al., 2017). Compared with single-input single-output (SISO) systems, the controller design for large-scale systems is more difficult to be implemented due to the complexity of control systems and the information exchange among subsystems. Therefore, investigations on large-scale nonlinear systems have received considerable attention in the past years. Existing literature on controller design for large-scale nonlinear systems can be divided in two general categories: centralized control and decentralized control. Compared with the former control, the merits of the latter lie in that it can reduce the computational burden, enhance the robustness and reliability against interacting operation failures (Tong et al., 2013). Therefore, the decentralized adaptive control has become a powerful design approach for large-scale nonlinear systems. Wen (1994) first presented an adaptive backstepping decentralized control scheme for a class of large-scale systems without satisfying the matching condition. Based on this, many interesting results have been achieved for different large-scale nonlinear systems, such as large-scale non-linear systems (Jiang and Repperger, 2001; Zhang and Lin, 2014), large-scale nonlinear systems with actuator faults and unknown dead-zones (Li and Yang, 2017b), large-scale nonlinear systems with unmodeled dynamics (Liu and Li, 2002), large-scale nonlinear systems with systems with hysteresis (Wen and Zhou, 2007), large-scale nonlinear systems with a neurally stable uncertain exosystem (Ye and Huang, 2003), large-scale stochastic nonlinear systems (Arslan and Basar, 2003), large-scale feedforward nonlinear time-delay systems (Zhang et al., 2014). In actual operations, many large-scale nonlinear systems fail to preserve stability under complexity information exchange among subsystems. Therefore, it is necessary to study the stability analysis of large-scale nonlinear systems.

On the other hand, due to the excellent approximation characteristics of neural network (NN) or multi-dimensional Taylor network (MTN) or fuzzy logic systems (FLSs), the method of approximation-based NN or FLS control has received considerable attention, for example, the works (Chen and Ge, 2013; Ge et al., 2003; Han, 2018; Liu et al., 2011; Yin et al., 2017) discussed the adaptive control problem for some

[1]School of Mathematics and Physics, Qingdao University of Science and Technology, China
[2]Research Center for Date Science and Information Technology, Qingdao University of Science and Technology, China

Corresponding author:
Shu-Guo Yang, School of Mathematics and Physics, Qingdao University of Science and Technology, No.99 Songling Rd, Qingdao, Shandong 266061, China.
Email: ysg_2005@163.com

classes of nonlinear systems based on NN, the works (Han et al., 2019; Yan et al., 2018) discussed the adaptive control problem for some classes of nonlinear systems using MTN-based approaches, the works (Yin et al., 2016) discussed the adaptive control problem for some classes of nonlinear systems based on FLSs. However, most of the results focused on SISO systems. Recently, a growing attention has been paid on large-scale systems (Chen and Li, 2008; Duan et al., 2017; Tong et al., 2011a, 2011b; Wang and Yang, 2016). Among them, combining approximation-based method along with the adaptive control technique, many interesting results have been reported, such as unknown nonlinear interconnected systems (Chen and Li, 2009; Hou et al., 2007; Li and Yang, 2017a; Zhai et al., 2012), uncertain large-scale nonlinear systems with actuator faults (Yang and Yue, 2016), linearly parametrized large-scale nonlinear systems (Jiang, 2000), large-scale nonlinear systems with input delays (Baigzadehnoe et al., 2017), interconnected large-scale uncertain nonlinear time-delay systems with input saturation (Li et al., 2011). For the reason that NNs or MTN or FLSs are universal approximators, these design ideas do not require nonlinear systems to be known exactly or the unknown nonlinear functions to be linearly parameterized. Meanwhile, adaptive controllers are constructed recursively in the framework of backstepping, and NNs or FLSs are used to approximate the unknown smooth functions. Owing to the complexity of control synthesis and the physical restrictions on information exchange among subsystems, the decentralized control scheme designing with NNs or FLSs is still a challenging problem.

On the other hand, dead-zone is one of the most important non-smooth nonlinearity in many industrial processes, which can severely limit system performance, and its study has been drawing much interest in the control community for a long time (Selmic and Lewis, 2000; Tao and Kokotovic, 1992; Taware and Tao, 2003; Wang et al., 2004). Wang et al. (2004) proposed a robust adaptive control scheme without constructing the dead-zone inverse. And then, this method has been successfully extended to the case of non-symmetric dead-zones by Ibrir et al. (2007). So far, many interesting results have been obtained for control design of nonlinear systems with both unknown functions and unknown dead-zone input. However, most of the results focused on SISO systems. Recently, a growing attention has been paid on multiple-input multiple-output (MIMO) nonlinear systems. For example, Zhang and Ge (2007, 2009) investigated the problem of adaptive neural control scheme for MIMO nonlinear systems with unknown dead-zones. Chen and Tao (2016) proposed an adaptive neural fault-tolerant control scheme for large-scale nonlinear systems with unknown dead-zone and external disturbances. An approximation-based adaptive tracking control approach was proposed by Zhou et al. (2015) for a class of MIMO nonlinear systems with input saturation using NN. Although the adaptive NN backstepping control design has achieved great progress, most the aforementioned control approaches achieved good control performance at the cost of increasing the number of nodes. In fact, it is still a challenging task to construct a simple but effective control algorithm for the large-scale nonlinear systems with unknown dead-zone, which motivates our research.

Motivated by the aforementioned discussion, an adaptive decentralized tracking control approach is proposed for large-scale nonlinear systems with unknown nonsymmetric dead-zone inputs using NN. The dead-zone output is represented as a simple linear system with a static time-varying gain and bounded disturbance by introducing characteristic function by using methods of Zhang and Ge (2007, 2008), based on the proposed novel description of dead-zone model, an adaptive decentralized neural tracking control approach is developed without necessarily constructing the dead-zone inverse and knowing some parameter bounds of dead-zones. During the controller design, radial basis function neural networks (RBFNNs) are used to approximate unknown nonlinearities, and then an adaptive neural decentralized neural tracking control scheme is constructed via backstepping. The main contributions of this paper are listed as follows:

(1) A novel adaptive NN-backstepping-based controller design technique for large-scale nonlinear systems with the unknown non-symmetric dead-zone inputs is established. Moreover, the studied system is the large-scale nonlinear systems with unknown control directions, which means its system structure is more general than some existing systems, such as those (Chen and Tao, 2016; Wei et al., 2017).
(2) In the control design, instead of using RBFNN to approximate each unknown function, we lump all unknown functions into a suitable unknown function that is approximated only by a RBFNN in each step of the backstepping. Thus, the computation burden of the design controller is significantly decreased.
(3) The control strategy proposed in this paper can get precise tracking results with low computational cost, and have a good real-time performance and convergence.

The remainder of this paper is organized as follows. The preliminaries and problem formulation are given in Section 2. Adaptive decentralized neural tracking control scheme and stability analysis are presented in Section 3. The simulation results are given in Section 4. Finally, the conclusion is given in Section 5.

Preliminaries and problem formulation

Throughout this paper, the following notations will be used. R_+ denotes the set of all nonnegative real numbers. R^n denotes the real n-dimensional space. For a given vector or matrix X, X^T denotes its transpose; $|X|$ denotes the Euclidean norm of a vector X and the corresponding induced norm for matrices is denoted by $\|X\|$; $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ denotes the minimal eigenvalue and maximal eigenvalue of real matrix X, respectively. I_n denotes the $n \times n$ identity matrix.

Problem description

Consider the following large-scale nonlinear system with N subsystems, and the behavior of the i-th ($i = 1, \cdots, N$) subsystem is governed by the following differential equations

$$\begin{cases} \dot{x}_{i,j} = g_{i,j}(\bar{\mathbf{x}}_{i,j})x_{i,j+1} + f_{i,j}(\bar{\mathbf{x}}_{i,j}) + h_{i,j}(\bar{\mathbf{y}}) \\ \dot{x}_{i,n_i} = g_{i,n_i}(\bar{\mathbf{x}}_{i,n_i})u_i + f_{i,n_i}(\bar{\mathbf{x}}_{i,n_i}) + h_{i,n_i}(\bar{\mathbf{y}}) \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, N$, $\bar{\mathbf{x}}_{i,j} = [x_{i,1}, x_{i,2}, \cdots, x_{i,j}]^T \in R^j$ and $\bar{\mathbf{y}} = [y_1, y_2, \cdots, y_N]^T \in R^N$. $\mathbf{x}_i = [x_{i,1}, x_{i,2}, \cdots, x_{i,n_i}]^T \in R^{n_i}$ and $y_i \in R$ are the state vector and the scalar output of the i-th nonlinear subsystem, respectively; $f_{i,j}(\cdot) : R^j \to R$ and $g_{i,j}(\cdot) : R^j \to R$ are unknown smooth nonlinear functions with $f_{i,j}(\mathbf{0}) = 0$. $h_{i,j}(\cdot) : R^N \to R$ is unknown smooth interconnections between the i-th subsystem and other subsystems with $h_{i,j}(\mathbf{0}) = 0$. Moreover, $u_i \in R$ denotes the output of the i-th dead-zone, which can be expressed as the following form

$$u_i = D_i(v_i) = \begin{cases} g_{i,r}(v_i), & v_i \geq b_{i,r} \\ 0, & b_{i,l} \leq v_i \leq b_{i,r} \\ g_{i,l}(v_i), & v_i \leq b_{i,l} \end{cases} \tag{2}$$

where v_i is the input to the i-th dead-zone, $b_{i,r}$ and $b_{i,l}$ are the unknown parameters of the i-th dead-zone.

The objective of this paper is to design an adaptive neural decentralized controller such that the system output y_i follows the given reference signal $y_{d,i}$ $(i = 1, \cdots, N)$, respectively.

To facilitate control system design, we need the following assumptions.

Assumption 1: For each $i \in \{1, \cdots, N\}$, the dead-zone output u_i is not available.

Assumption 2: (Zhang and Ge, 2007). For each $i \in \{1, \cdots, N\}$, the dead-zone parameters $b_{i,r}$ and $b_{i,l}$ are unknown bounded constants, but their signs are known, i.e. $b_{i,r} > 0$ and $b_{i,l} < 0$.

Assumption 3: (Zhang and Ge, 2007). For each $i \in \{1, \cdots, N\}$, $g_{i,r}(v_i)$ and $g_{i,l}(v_i)$ are smooth functions, there exist unknown positive constants $k_{i,l0}$, $k_{i,l1}$, $k_{i,r0}$ and $k_{i,r1}$, such that

$$\begin{aligned} 0 < k_{i,l0} \leq g'_{i,l}(v_i) \leq k_{i,l1}, \quad \forall v_i \in (-\infty, b_{i,l}] \\ 0 < k_{i,r0} \leq g'_{i,r}(v_i) \leq k_{i,r1}, \quad \forall v_i \in [b_{i,r}, +\infty) \end{aligned}$$

where $g'_{i,l}(v_i) = \frac{dg_{i,l}(\xi)}{d\xi}\Big|_{\xi = v_i}$ and $g'_{i,r}(v_i) = \frac{dg_{i,r}(\xi)}{d\xi}\Big|_{\xi = v_i}$. In addition, define a known positive constant $\beta_{i,0}$ satisfies $\beta_{i,0} \leq \min\{k_{i,l0}, k_{i,r0}\}$.

Remark 1: For large-scale nonlinear systems with unknown dead-zone inputs, Assumptions 1–3 are the commonly assumptions, and moreover, these assumptions assure the unknown dead-zone inputs of the large-scale nonlinear systems can be handled in the controller design.

Assumption 4: For each $j \in \{1, 2, \cdots, n_i\}$ and $i \in \{1, 2, \cdots, N\}$, the sign of function $g_{i,j}$ does not change, and there exist constants $\underset{\sim}{b}$ and $\bar{b}$ such that

$$0 < \underset{\sim}{b} \leq |g_{i,j}(\bar{\mathbf{x}}_{i,j})| \leq \bar{b} < \infty.$$

Remark 2: In Assumption 4, the true value of the constant $\bar{b}$ is not necessary to be known due to the fact that it not used for controller design.

Assumption 5: For each $j \in \{1, 2, \cdots, n_i\}$ and $i \in \{1, 2, \cdots, N\}$, there exist an unknown smooth function $h_{i,j,l}(y_l)$ with $h_{i,j,l}(0) = 0$, such that

$$|h_{i,j}(\bar{\mathbf{y}})|^2 \leq \sum_{l=1}^{N} h_{i,j,l}^2(y_l).$$

Remark 3: Assumption 5 assures the unknown smooth interconnections between the i-th subsystem and other subsystems can be handled in the backstepping design. In addition, although Assumptions 4 and 5 seem to be too restrictive for large-scale nonlinear systems, there still exist many physical systems to satisfy it, such as two inverted pendulums that are connected by a spring (Spooner and Passino, 1999; Tong et al., 2014).

Dead-zone transformation (Zhang and Ge, 2007, 2008)

Based on Assumption 3 and using the same arguments as Zhang and Ge (2007, 2008), the i-th dead-zone (2) can be rewritten as

$$u_i = D_i(v_i) = K_i^T(t)\Phi_i(t)v_i + d_i(v_i) \tag{3}$$

where

$$\Phi_i(t) = [\varphi_{i,r}(t), \varphi_{i,l}(t)]^T,$$

with $\varphi_{i,r}(t) = \begin{cases} 1, & v_i(t) > b_{i,l} \\ 0, & v_i(t) \leq b_{i,l} \end{cases}$, $\varphi_{i,l}(t) = \begin{cases} 1, & v_i(t) < b_{i,r} \\ 0, & v_i(t) \geq b_{i,r} \end{cases}$

$$K_i(t) = [K_{i,r}(v_i(t)), K_{i,l}(v_i(t))]^T \tag{4}$$

with $K_{i,r}(v(t)) = \begin{cases} 0, & v_i(t) \leq b_{i,l} \\ g'_{i,r}(\xi_{i,r}(v_i(t))), & b_{i,l} < v_i(t) < +\infty \end{cases}$

$$K_{i,l}(v_i(t)) = \begin{cases} g'_{i,l}(\xi_{i,l}(v_i(t))), & -\infty < v_i(t) < b_{i,r} \\ 0, & v_i(t) \geq b_{i,r} \end{cases}$$

$$d_i(v_i) = \begin{cases} -g'_{i,r}(\xi_{i,r}(v_i))b_{i,r}, & v_i \geq b_{i,r} \\ -[g'_{i,l}(\xi_{i,l}(v_i)) + g'_{i,r}(\xi_{i,r}(v))]v_i, & b_{i,l} < v_i < b_{i,r} \\ -g'_{i,l}(\xi_{i,l}(v_i))b_{i,l}, & v_i \leq b_{i,l} \end{cases} \tag{5}$$

where

$$\xi_{i,l}(v_i) \in \begin{cases} (v_i, b_{i,l}), & \text{if } v_i < b_{i,l} \\ (b_{i,l}, v_i), & \text{if } b_{i,l} \leq v_i < b_{i,r} \end{cases}$$

$$\xi_{i,r}(v_i) \in \begin{cases} (b_{i,r}, v_i), & \text{if } b_{i,r} < v_i \\ (v_i, b_{i,r}), & \text{if } b_{i,l} < v_i \leq b_{i,r} \end{cases}$$

and $|d_i(v_i)| \leq \hat{k}_i$, $\hat{k}_i$ is an unknown positive constant with $\hat{k}_i = (k_{i,l1} + k_{i,r1})\max\{b_{i,r}, -b_{i,l}\}$.

Remark 4: According to Assumptions 3 and 4, and (4) and (5), we can easily get

$$\beta_{i,0} \leqslant K_i^T(t)\Phi_i(t) \leqslant k_{i,l1} + k_{i,r1} < \infty.$$

Remark 5: Assumption 4 implies that the functions $g_{i,j}(\bar{x}_{i,j})$ are strictly either positive or negative, without loss of generality, it is further assumed that $0 < \underset{\sim}{b} \leqslant g_{i,j}(\bar{x}_{i,j}) \leqslant \bar{b} < \infty$. In addition, considering that Assumptions 3 and Remark 4, it can be further assumed that

$$0 < \underline{b} \leqslant g_{i,j} \leqslant \bar{b} < \infty,$$
$$0 < \underline{b} \leqslant g_{i,j} K_i^T(t)\Phi_i(t),$$

where $\underline{b} = \min\left\{ \underset{\sim}{b}, \underset{\sim}{b}, \beta_{i,0} \right\}$ is a known constant.

RBFNN

In this paper, the RBFNN will be used to approximate the continuous functions $f(Z): R^n \rightarrow R$, that is

$$f(Z) = \boldsymbol{\theta}^T \boldsymbol{S}(Z)$$

where $Z \in \Omega_z \subset R^q$ is the input vector with q being the NN input dimension, weight vector $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_l]^T \in R^l$, $l > 1$ is the NN's node number, and $\boldsymbol{S}(Z) = [s_1(Z), \cdots, s_l(Z)]^T$ means the basis function vector with $s_i(Z)$ being chosen as the Gaussian function of the form $s_i(Z) = \exp\left[-(Z - \boldsymbol{\mu}_i)^T (Z - \boldsymbol{\mu}_i) \big/ b_i^2\right]$, $i = 1, 2, \cdots, l$, where $\boldsymbol{\mu}_i = \left[\mu_{i,1}, \cdots, \mu_{i,q}\right]^T$ is the center of the receptive field, and b_i is the width of the Gaussian function.

Then, the following lemma is stated for function approximation.

Lemma 1: Assume that $f(z)$ is a continuous function defined on a compact set Ω_z. Then, for any given desired level of accuracy $\varepsilon > 0$, there exists a RBFNN, such that

$$f(z) = \boldsymbol{\theta}^{*T} \boldsymbol{S}(z) + \delta(z), \quad |\delta(z)| \leqslant \varepsilon$$

where $\boldsymbol{\theta}^*$ is the optimal weight vector and defined as

$$\boldsymbol{\theta}^* = \arg\min_{W \in R^l} \left\{ \sup_{Z \in \Omega_Z} \left| f(z) - \boldsymbol{\theta}^T \boldsymbol{S}(z) \right| \right\}$$

where $\delta(z)$ is the approximation error and satisfies $|\delta(z)| \leqslant \varepsilon$.

Main results

In this section, a systemic control design and stability analysis procedure will be presented using adaptive backstepping technique.

To develop the backstepping design, define the following coordinate transformation

$$z_{i,j} = x_{i,j} - \alpha_{i,j-1}, \; i = 1, \cdots, N; j = 1, \cdots, n_i \tag{6}$$

where $\alpha_{i,0} = y_{d,i}$.

It follows from (1) and (6) that

$$\begin{cases} \dot{z}_{i,j} = g_{i,j}(\bar{\boldsymbol{x}}_{i,j}) x_{i,j+1} + f_{i,j}(\bar{\boldsymbol{x}}_{i,j}) + h_{i,j}(\bar{\boldsymbol{y}}) - \dot{\alpha}_{i,j-1} \\ \dot{z}_{i,n_i} = g_{i,n_i}(\bar{\boldsymbol{x}}_{i,n_i}) u_i + f_{i,n_i}(\bar{\boldsymbol{x}}_{i,n_i}) + h_{i,n_i}(\bar{\boldsymbol{y}}) - \dot{\alpha}_{i,n_i-1} \end{cases} \tag{7}$$

where $i = 1, \cdots, N$, $j = 1, \cdots, n_{i-1}$, and

$$\dot{\alpha}_{i,0} = \dot{y}_{d,i}, \; \dot{\alpha}_{i,j-1} = \sum_{l=1}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,l}} (g_{i,l} x_{i,l+1} + f_{i,l} + h_{i,l}) + \frac{\partial \alpha_{i,j-1}}{\partial \hat{\boldsymbol{\theta}}_{i,j-1}} \dot{\hat{\boldsymbol{\theta}}}_{i,j-1}$$

Adaptive NN control design

In the following, for simplicity, the time variable t will be omitted from the corresponding functions. $\boldsymbol{S}_{i,j}(\boldsymbol{z}_{i,j})$, $g_{i,j}(\bar{\boldsymbol{x}}_{i,j})$, $f_{i,j}(\bar{\boldsymbol{x}}_{i,j})$ and $h_{i,j}(\bar{\boldsymbol{y}})$ are abbreviated as $\boldsymbol{S}_{i,j}$, $g_{i,j}$, $f_{i,j}$ and $h_{i,j}$, respectively.

Step i, **1:** Consider the Lyapunov function candidate as follows

$$V_{i,1} = \frac{1}{2} z_{i,1}^2 + \frac{1}{2} \tilde{\boldsymbol{\theta}}_{i,1}^T \boldsymbol{\Gamma}_{i,1}^{-1} \tilde{\boldsymbol{\theta}}_{i,1},$$

where $\tilde{\boldsymbol{\theta}}_{i,1} = \boldsymbol{\theta}_{i,1} - \hat{\boldsymbol{\theta}}_{i,1}$ is the parameter estimate error, and matrix $\boldsymbol{\Gamma}_{i,1} = \boldsymbol{\Gamma}_{i,1}^T > 0$.

Then, the time derivative of $V_{i,1}$ along (7) with $j = 1$, we have

$$\begin{aligned} \dot{V}_{i,1} &= z_{i,1}(g_{i,1} x_{i,2} + \bar{f}_{i,1} + h_{i,1}) \\ &\quad - z_{i,1}^2 - z_{i,1}\lambda_{i,1}(z_{i,1}) - \tilde{\boldsymbol{\theta}}_{i,1}^T \boldsymbol{\Gamma}_{i,1}^{-1} \dot{\hat{\boldsymbol{\theta}}}_{i,1} \end{aligned} \tag{8}$$

where $\bar{f}_{i,1} = f_{i,1} - \dot{y}_{d,i} + z_{i,1} + \lambda_{i,1}(z_{i,1})$, $\lambda_{i,1}(z_{i,1})$ is a smooth nonnegative function and its true value is not necessary to be known because it is not used for controller design.

Obviously, unknown function $\bar{f}_{i,1}$ cannot be directly used to construct virtual control signal $\alpha_{i,1}$. According to the Lemma 1, for any given constant $\varepsilon_{i,1} > 0$, there exists a NN as $\boldsymbol{\theta}_{i,1}^T \boldsymbol{S}_{i,1}$ such that

$$\bar{f}_{i,1} = \boldsymbol{\theta}_{i,1}^T \boldsymbol{S}_{i,1} + \delta_{i,1}(\boldsymbol{z}_{i,1}), \; |\delta_{i,1}(\boldsymbol{z}_{i,1})| \leqslant \varepsilon_{i,1},$$

where $\delta_{i,1}$ is the approximation error.

According to Young's inequality, we have

$$z_{i,1}\bar{f}_{i,1} = z_{i,1}\boldsymbol{\theta}_{i,1}^T \boldsymbol{S}_{i,1} + z_{i,1}\delta_{i,1} \leqslant z_{i,1}\boldsymbol{\theta}_{i,1}^T \boldsymbol{S}_{i,1} + \frac{1}{2} z_{i,1}^2 + \frac{1}{2} \varepsilon_{i,1}^2 \tag{9}$$

$$z_{i,1} h_{i,1} \leqslant \frac{1}{2} z_{i,1}^2 + \frac{1}{2} h_{i,1}^2 \leqslant \frac{1}{2} z_{i,1}^2 + \frac{1}{2} \sum_{l=1}^{N} h_{i,1,l}^2(y_l) \tag{10}$$

Considering that $x_{i,2} = z_{i,2} + \alpha_{i,1}$ and substituting (9) and (10) into (8) gives

$$\dot{V}_{i,1} \leqslant z_{i,1}(g_{i,1}z_{i,2} + g_{i,1}\alpha_{i,1}) + \frac{1}{2}\sum_{l=1}^{N} h_{i,1,l}^2(y_l) - z_{i,1}\lambda_{i,1}(z_{i,1}) + \frac{1}{2}\varepsilon_{i,1}^2 + z_{i,1}\boldsymbol{\theta}_{i,1}^T\boldsymbol{S}_{i,1} - \tilde{\boldsymbol{\theta}}_{i,1}^T\Gamma_{i,1}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,1}. \tag{11}$$

Applying Young's inequality, we have

$$z_{i,1}g_{i,1}z_{i,2} \leqslant \frac{1}{2}g_{i,1}z_{i,1}^2 + \frac{1}{2}g_{i,1}z_{i,2}^2 \tag{12}$$

Choosing the virtual control law $\alpha_{i,1}$ as follows

$$\alpha_{i,1} = -\frac{1}{\underline{b}}\left(r_{i,1}|z_{i,1}| + \left|\hat{\boldsymbol{\theta}}_{i,1}^T\boldsymbol{S}_{i,1}\right|\right)\operatorname{sgn}(z_{i,1})$$

where $r_{i,1}$ is positive design parameters.

Then, the following inequality can be obtained

$$z_{i,1}g_{i,1}\alpha_{i,1} \leqslant - r_{i,1}z_{i,1}^2 - \left|z_{i,1}\hat{\boldsymbol{\theta}}_{i,1}^T\boldsymbol{S}_{i,1}\right|. \tag{13}$$

Substituting (12) and (13) into (11) gives

$$\begin{aligned}\dot{V}_{i,1} \leqslant& -\left(r_{i,1} - \frac{1}{2}g_{1,k}\right)z_{i,1}^2 + \frac{1}{2}g_{i,1}z_{i,2}^2 + \frac{1}{2}\varepsilon_{i,1}^2 + \frac{1}{2}\sum_{l=1}^{N}h_{i,1,l}^2(y_l) \\ & - z_{i,1}\lambda_{i,1}(z_{i,1}) - \left|z_{i,1}\hat{\boldsymbol{\theta}}_{i,1}^T\boldsymbol{S}_{i,1}\right| + z_{i,1}\left(\tilde{\boldsymbol{\theta}}_{i,1}^T + \hat{\boldsymbol{\theta}}_{i,1}^T\right)S_{i,1} - \tilde{\boldsymbol{\theta}}_{i,1}^T\Gamma_{i,1}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,1} \\ \leqslant& -\left(r_{i,1} - \frac{1}{2}g_{i,1}\right)z_{i,1}^2 + \frac{1}{2}g_{i,1}z_{i,2}^2 + \tilde{\boldsymbol{\theta}}_{i,1}^T\left(z_{i,1}\boldsymbol{S}_{i,1} - \Gamma_{i,1}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,1}\right) \\ & + \frac{1}{2}\varepsilon_{i,1}^2 + \frac{1}{2}\sum_{l=1}^{N}h_{i,1,l}^2(y_l) - z_{i,1}\lambda_{i,1}(z_{i,1}).\end{aligned} \tag{14}$$

Step i, 2**:** Consider the Lyapunov function candidate as follows

$$V_{i,2} = V_{i,1} + \frac{1}{2}z_{i,2}^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,2}^T\Gamma_{i,2}^{-1}\tilde{\boldsymbol{\theta}}_{i,2},$$

where $\tilde{\boldsymbol{\theta}}_{i,2} = \boldsymbol{\theta}_{i,2} - \hat{\boldsymbol{\theta}}_{i,2}$ is the parameter estimate error, and matrix $\Gamma_{i,2} = \Gamma_{i,2}^T > 0$.

Then, the time derivative of $V_{i,2}$ along (7) with $j = 2$, we have

$$\begin{aligned}\dot{V}_{i,2} =& \dot{V}_{i,1} + z_{i,2}\left(g_{i,2}x_{i,3} + \bar{f}_{i,2} + h_{i,2}\right) \\ & - z_{i,2}^2 - z_{i,2}\lambda_{i,2}(z_{i,1}) - \tilde{\boldsymbol{\theta}}_{i,2}^T\Gamma_{i,2}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,2}\end{aligned} \tag{15}$$

where $\bar{f}_{i,2} = f_{i,2} - \dot{\alpha}_{i,1} + z_{i,2} + \lambda_{i,2}(z_{i,1})$, $\lambda_{i,2}(z_{i,1})$ is a smooth nonnegative function and its true value is not necessary to be known because it is not used for controller design.

Obviously, unknown function $\bar{f}_{i,2}$ cannot be directly used to construct virtual control signal $\alpha_{i,2}$. According to Lemma 1, for any given constant $\varepsilon_{i,2} > 0$, there exists a NN as $\boldsymbol{\theta}_{i,2}^T\boldsymbol{S}_{i,2}$ such that

$$\bar{f}_{i,2} = \boldsymbol{\theta}_{i,2}^T\boldsymbol{S}_{i,2} + \delta_{i,2},\ |\delta_{i,2}| \leqslant \varepsilon_{i,2},$$

where $\delta_{i,2}$ is the approximation error.

According to Young's inequality, we have

$$z_{i,2}\bar{f}_{i,2} = z_{i,2}\boldsymbol{\theta}_{i,2}^T\boldsymbol{S}_{i,2} + z_{i,2}\delta_{i,2} \leqslant z_{i,2}\boldsymbol{\theta}_{i,2}^T\boldsymbol{S}_{i,2} + \frac{1}{2}z_{i,2}^2 + \frac{1}{2}\varepsilon_{i,2}^2 \tag{16}$$

$$z_{i,2}h_{i,2} \leqslant \frac{1}{2}z_{i,2}^2 + \frac{1}{2}h_{i,2}^2 \leqslant \frac{1}{2}z_{i,2}^2 + \frac{1}{2}\sum_{l=1}^{N}h_{i,2,l}^2(y_l) \tag{17}$$

Considering that $x_{i,3} = z_{i,3} + \alpha_{i,2}$ and substituting (16) and (17) into (15) gives

$$\begin{aligned}\dot{V}_{i,2} \leqslant& \dot{V}_{i,1} + z_{i,2}(g_{i,2}z_{i,3} + g_{i,2}\alpha_{i,2}) + \frac{1}{2}\sum_{l=1}^{N}h_{i,2,l}^2(y_l) \\ & -z_{i,2}\lambda_{i,2}(z_{i,1}) + \frac{1}{2}\varepsilon_{i,2}^2 + z_{i,2}\boldsymbol{\theta}_{i,2}^T\boldsymbol{S}_{i,2} - \tilde{\boldsymbol{\theta}}_{i,2}^T\Gamma_{i,2}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,2}.\end{aligned} \tag{18}$$

Applying Young's inequality, we have

$$z_{i,2}g_{i,2}z_{i,3} \leqslant \frac{1}{2}g_{i,2}z_{i,2}^2 + \frac{1}{2}g_{i,2}z_{i,3}^2 \tag{19}$$

Choosing the virtual control law $\alpha_{i,2}$ as follows

$$\alpha_{i,2} = -\frac{1}{\underline{b}}\left(r_{i,2}|z_{i,2}| + \left|\hat{\boldsymbol{\theta}}_{i,2}^T\boldsymbol{S}_{i,2}\right|\right)\operatorname{sgn}(z_{i,2})$$

where $r_{i,2}$ is positive design parameters.

Then, the following inequality can be obtained

$$z_{i,2}g_{i,2}\alpha_{i,2} \leqslant - r_{i,2}z_{i,2}^2 - \left|z_{i,2}\hat{\boldsymbol{\theta}}_{i,2}^T\boldsymbol{S}_{i,2}\right|. \tag{20}$$

Substituting (14), (19) and (20) into (18) gives

$$\begin{aligned}\dot{V}_{i,2} \leqslant& -\sum_{j=1}^{2}\left(r_{i,j} - \frac{1}{2}g_{i,j}\right)z_{i,j}^2 + \frac{1}{2}\sum_{j=1}^{2}g_{i,j}z_{i,j+1}^2 + \frac{1}{2}\sum_{j=1}^{2}\varepsilon_{i,j}^2 \\ & + \frac{1}{2}\sum_{j=1}^{2}\sum_{l=1}^{N}h_{i,j,l}^2(y_l) - \sum_{j=1}^{2}z_{i,j}\lambda_{i,j}(z_{i,1}) + \sum_{j=1}^{2}\tilde{\boldsymbol{\theta}}_{i,j}^T\left(z_{i,j}\boldsymbol{S}_{i,j} - \Gamma_{i,2}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,j}\right).\end{aligned} \tag{21}$$

Step i, π $(2 \leqslant \pi \leqslant n_i - 1)$**:** Consider the Lyapunov function candidate as follows

$$V_{i,\pi} = V_{i,\pi-1} + \frac{1}{2}z_{i,\pi}^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,\pi}^T\Gamma_{i,\pi}^{-1}\tilde{\boldsymbol{\theta}}_{i,\pi}$$

where $\tilde{\boldsymbol{\theta}}_{i,\pi} = \boldsymbol{\theta}_{i,\pi} - \hat{\boldsymbol{\theta}}_{i,\pi}$ is the parameter estimate error, and matrix $\Gamma_{i,\pi} = \Gamma_{i,\pi}^T > 0$.

According to *Mathematical Induction*, we have

$$\begin{aligned}\dot{V}_{i,\pi-1} \leqslant& -\sum_{j=1}^{\pi-1}\left(r_{i,j} - \frac{1}{2}g_{i,j}\right)z_{i,j}^2 + \frac{1}{2}\sum_{j=1}^{\pi-1}g_{i,j}z_{i,j+1}^2 + \frac{1}{2}\sum_{j=1}^{\pi-1}\varepsilon_{i,j}^2 \\ & + \frac{1}{2}\sum_{j=1}^{\pi-1}\sum_{l=1}^{N}h_{i,j,l}^2(y_l) - \sum_{j=1}^{\pi-1}z_{i,j}\lambda_{i,j}(z_{i,1}) \\ & + \sum_{j=1}^{\pi-1}\tilde{\boldsymbol{\theta}}_{i,j}^T\left(z_{i,j}\boldsymbol{S}_{i,j} - \Gamma_{i,j}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,j}\right).\end{aligned} \tag{22}$$

Then, the time derivative of $V_{i,\pi}$ along (7) with $j=\pi$, we have

$$\begin{aligned}\dot{V}_{i,\pi} &= \dot{V}_{i,\pi-1} + z_{i,\pi}\left(g_{i,\pi}x_{i,\pi+1} + \bar{f}_{i,\pi} + h_{i,\pi}\right) \\ &\quad - z_{i,\pi}^2 - z_{i,\pi}\lambda_{i,\pi}(z_{i,1}) - \tilde{\theta}_{i,\pi}^T\Gamma_{i,\pi}^{-1}\dot{\hat{\theta}}_{i,\pi}\end{aligned} \tag{23}$$

where $\bar{f}_{i,\pi} = f_{i,\pi} - \dot{\alpha}_{i,\pi-1} + z_{i,\pi} + \lambda_{i,\pi}(z_{i,1})$, $\lambda_{i,\pi}(z_{i,1})$ is a smooth nonnegative function and its true value is not necessary to be known because it is not used for controller design.

Obviously, unknown function $\bar{f}_{i,\pi}$ cannot be directly used to construct virtual control signal $\alpha_{i,\pi}$. According to Lemma 1, for any given constant $\varepsilon_{i,\pi}>0$, there exists a NN as $\theta_{i,\pi}^T S_{i,\pi}$ such that

$$\bar{f}_{i,\pi} = \boldsymbol{\theta}_{i,\pi}^T \boldsymbol{S}_{i,\pi} + \delta_{i,\pi},\ |\delta_{i,\pi}| \le \varepsilon_{i,\pi},$$

where $\delta_{i,\pi}$ is the approximation error.

According to Young's inequality, we have

$$z_{i,\pi}\bar{f}_{i,\pi} = z_{i,\pi}\boldsymbol{\theta}_{i,\pi}^T\boldsymbol{S}_{i,\pi} + z_{i,\pi}\delta_{i,\pi} \le z_{i,\pi}\boldsymbol{\theta}_{i,\pi}^T\boldsymbol{S}_{i,\pi} + \frac{1}{2}z_{i,\pi}^2 + \frac{1}{2}\varepsilon_{i,\pi}^2 \tag{24}$$

$$z_{i,\pi}h_{i,\pi} \le \frac{1}{2}z_{i,\pi}^2 + \frac{1}{2}h_{i,\pi}^2 \le \frac{1}{2}z_{i,\pi}^2 + \frac{1}{2}\sum_{l=1}^{N} h_{i,\pi,l}^2(y_l) \tag{25}$$

Considering that $x_{i,\pi+1} = z_{i,\pi+1} + \alpha_{i,\pi}$ and substituting (24) and (25) into (23) gives

$$\begin{aligned}\dot{V}_{i,\pi} &\le \dot{V}_{i,\pi-1} + z_{i,\pi}(g_{i,\pi}z_{i,\pi+1} + g_{i,\pi}\alpha_{i,\pi}) + \frac{1}{2}\sum_{l=1}^{N} h_{i,\pi,l}^2(y_l) \\ &\quad - z_{i,\pi}\lambda_{i,\pi}(z_{i,1}) + \frac{1}{2}\varepsilon_{i,\pi}^2 + z_{i,\pi}\boldsymbol{\theta}_{i,\pi}^T\boldsymbol{S}_{i,\pi} - \tilde{\boldsymbol{\theta}}_{i,\pi}^T\boldsymbol{\Gamma}_{i,\pi}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,\pi}.\end{aligned}$$

Applying Young's inequality, we have

$$z_{i,\pi}g_{i,\pi}z_{i,\pi+1} \le \frac{1}{2}g_{i,\pi}z_{i,\pi}^2 + \frac{1}{2}g_{i,\pi}z_{i,\pi+1}^2 \tag{26}$$

Choosing the virtual control law $\alpha_{i,\pi}$ as follows

$$\alpha_{i,\pi} = -\frac{1}{\underline{b}}\left(r_{i,\pi}|z_{i,\pi}| + \left|\hat{\boldsymbol{\theta}}_{i,\pi}^T\boldsymbol{S}_{i,\pi}\right|\right)\mathrm{sgn}(z_{i,\pi})$$

where $r_{i,\pi}$ is positive design parameters.

Then, the following inequality can be obtained:

$$z_{i,\pi}g_{i,\pi}\alpha_{i,\pi} \le -r_{i,\pi}z_{i,\pi}^2 - \left|z_{i,\pi}\hat{\boldsymbol{\theta}}_{i,\pi}^T\boldsymbol{S}_{i,\pi}\right|. \tag{27}$$

Substituting (22), (26) and (27) into (23) gives

$$\begin{aligned}\dot{V}_{i,\pi} &\le -\sum_{j=1}^{\pi}\left(r_{i,j} - \frac{1}{2}g_{i,j}\right)z_{i,j}^2 + \frac{1}{2}\sum_{j=1}^{\pi} g_{i,j}z_{i,j+1}^2 + \frac{1}{2}\sum_{j=1}^{\pi}\varepsilon_{i,j}^2 \\ &\quad + \frac{1}{2}\sum_{j=1}^{\pi}\sum_{l=1}^{N} h_{i,j,l}^2(y_l) - \sum_{j=1}^{\pi} z_{i,j}\lambda_{i,j}(z_{i,1}) \\ &\quad + \sum_{j=1}^{\pi}\tilde{\theta}_{i,j}^T\left(z_{i,j}S_{i,j} - \Gamma_{i,\pi}^{-1}\dot{\hat{\theta}}_{i,j}\right).\end{aligned} \tag{28}$$

Step i, n_i**:** Consider the Lyapunov function candidate as follows

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{2}z_{i,n_i}^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,n_i}^T\boldsymbol{\Gamma}_{i,n_i}^{-1}\tilde{\boldsymbol{\theta}}_{i,n_i},$$

where $\tilde{\boldsymbol{\theta}}_{i,n_i} = \boldsymbol{\theta}_{i,n_i} - \hat{\boldsymbol{\theta}}_{i,n_i}$ is the parameter estimate error, and matrix $\boldsymbol{\Gamma}_{i,n_i} = \boldsymbol{\Gamma}_{i,n_i}^T > 0$.

Similar to (22), we have

$$\begin{aligned}\dot{V}_{i,n_i-1} &\le -\sum_{j=1}^{n_i-1}\left(r_{i,j} - \frac{1}{2}g_{i,j}\right)z_{i,j}^2 \\ &\quad + \frac{1}{2}\sum_{j=1}^{n_i-1} g_{i,j}z_{i,j+1}^2 + \frac{1}{2}\sum_{j=1}^{n_i-1}\varepsilon_{i,j}^2 \\ &\quad + \frac{1}{2}\sum_{j=1}^{n_i-1}\sum_{l=1}^{N} h_{i,j,l}^2(y_l) - \sum_{j=1}^{n_i-1} z_{i,j}\lambda_{i,j}(z_{i,1}) \\ &\quad + \sum_{j=1}^{n_i-1}\tilde{\boldsymbol{\theta}}_{i,j}^T\left(z_{i,j}\boldsymbol{S}_{i,j} - \boldsymbol{\Gamma}_{i,j}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,j}\right).\end{aligned} \tag{29}$$

Then the time derivative of V_{i,n_i} along (7) with $j=n_i$, we have

$$\begin{aligned}\dot{V}_{i,n_i} &= \dot{V}_{i,n_i-1} + z_{i,n_i}\left(g_{i,n_i}u_i + \bar{f}_{i,n_i} + h_{i,n_i}\right) \\ &\quad - z_{i,n_i}^2 - z_{i,n_i}\lambda_{i,n_i}(z_{i,1}) - \tilde{\boldsymbol{\theta}}_{i,n_i}^T\boldsymbol{\Gamma}_{i,n_i}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,n_i}\end{aligned} \tag{30}$$

where $\bar{f}_{i,n_i} = f_{i,n_i} - \dot{\alpha}_{i,n_i-1} + z_{i,n_i} + \lambda_{i,n_i}(z_{i,1})$, $\lambda_{i,n_i}(z_{i,1})$ is a smooth nonnegative function and its true value is not necessary to be known because it is not used for controller design.

Obviously, unknown function $\bar{f}_{i,n_i}$ cannot be directly used to construct actual control input $\boldsymbol{v}_i$. According to Lemma 1, for any given constant $\varepsilon_{i,n_i}>0$, there exists a NN as $\boldsymbol{\theta}_{i,n_i}^T\boldsymbol{S}_{i,n_i}$ such that

$$\bar{f}_{i,n_i} = \boldsymbol{\theta}_{i,n_i}^T\boldsymbol{S}_{i,n_i} + \delta_{i,n_i},\ |\delta_{i,n_i}| \le \varepsilon_{i,n_i},$$

where δ_{i,n_i} is the approximation error.

According to Young's inequality, we have

$$\begin{aligned}z_{i,n_i}\bar{f}_{i,n_i} &= z_{i,n_i}\boldsymbol{\theta}_{i,n_i}^T\boldsymbol{S}_{i,n_i} + z_{i,n_i}\delta_{i,n_i} \\ &\le z_{i,n_i}\boldsymbol{\theta}_{i,n_i}^T\boldsymbol{S}_{i,n_i} + \frac{1}{2}z_{i,n_i}^2 + \frac{1}{2}\varepsilon_{i,n_i}^2\end{aligned} \tag{31}$$

$$z_{i,n_i}h_{i,n_i} \le \frac{1}{2}z_{i,n_i}^2 + \frac{1}{2}h_{i,n_i}^2 \le \frac{1}{2}z_{i,n_i}^2 + \frac{1}{2}\sum_{l=1}^{N} h_{i,n_i,l}^2(y_l) \tag{32}$$

According to (3) and considering that $|d_i(\boldsymbol{v}_i)| \le \hat{k}_i$, we have

$$z_{i,n_i}g_{i,n_i}u_i \le z_{i,n_i}g_{i,n_i}K_i^T(t)\boldsymbol{\Phi}_i(t)\boldsymbol{v}_i + \frac{1}{2}g_{i,n_i}z_{i,n_i}^2 + \frac{1}{2}g_{i,n_i}\hat{k}_i^2 \tag{33}$$

Choosing the actual control law $\boldsymbol{v}_i$ as follows

$$\boldsymbol{v}_i = -\frac{1}{\underline{b}}\left(r_{i,n_i}|z_{i,n_i}| + \left|\hat{\boldsymbol{\theta}}_{i,n_i}^T\boldsymbol{S}_{i,n_i}\right|\right)\mathrm{sgn}(z_{i,n_i}) \tag{34}$$

where r_{i,n_i} is positive design parameters.

According to Remarks 4 and 5, and (34), the following inequality can be obtained

$$z_{i,n_i}g_{i,n_i}K_i^T(t)\boldsymbol{\Phi}_i(t)\boldsymbol{v}_i \leqslant -r_{i,n_i}z_{i,n_i}^2 - \left|z_{i,n_i}\hat{\boldsymbol{\theta}}_{i,n_i}^T\boldsymbol{S}_{i,n_i}\right| \tag{35}$$

Substituting (29), (31), (32), (33) and (35) into (30) gives

$$\begin{aligned}\dot{V}_{i,n_i} \leqslant &-\sum_{j=1}^{n_i}\left(r_{i,j}-\frac{1}{2}g_{i,j}\right)z_{i,j}^2 + \frac{1}{2}\sum_{j=1}^{n_i-1}g_{i,j}z_{i,j+1}^2 \\ &+ \frac{1}{2}g_{i,n_i}\hat{k}_i^2 + \frac{1}{2}\sum_{j=1}^{n_i}\varepsilon_{i,j}^2 \\ &+ \frac{1}{2}\sum_{j=1}^{n_i}\sum_{l=1}^{N}h_{i,j,l}^2(y_l) - \sum_{j=1}^{n_i}z_{i,j}\lambda_{i,j}(z_{i,1}) \\ &+ \sum_{j=1}^{n_i}\tilde{\boldsymbol{\theta}}_{i,j}^T\left(z_{i,j}\boldsymbol{S}_{i,j}-\boldsymbol{\Gamma}_{i,j}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,j}\right).\end{aligned} \tag{36}$$

So far, the adaptive control design has been completed.

Remark 6: It should be noticed the most difficulty work in the controller design is to handle the unknown functions $f_{i,j}(\bar{\boldsymbol{x}}_{i,j})$ and $h_{i,j}(\bar{y})$, $i = 1, \cdots, N; j = 1, \cdots, n_i$. Therefore, we lump all unknown functions into a suitable unknown function in each step of backstepping, that is $\bar{f}_{i,j}(\cdot)$, $i = 1, \cdots, N$; $j = 1, \cdots, n_i$, which can be approximated by a RBFNN. Thus, we may conclude that the computation burden is reduced.

Remark 7: Based on (3), a simple control structure (34) is presented for a class of large-scale nonlinear systems with unknown dead-zone inputs by introducing sign function.

Stability analysis

Theorem 1: Consider the large-scale nonlinear system (1) with unknown dead-zone (2) under Assumptions 1–5, if the control laws are chosen as (34) and the intermediate virtual control signals $\alpha_{i,j}(i = 1, 2, \cdots, N; j = 1, 2, \cdots, n_i - 1)$ described as

$$\alpha_{i,j} = -\frac{1}{\underline{b}}\left(r_{i,j}|z_{i,j}| + \left|\hat{\boldsymbol{\theta}}_{i,j}^T\boldsymbol{S}_{i,j}\right|\right)\operatorname{sgn}(z_{i,j}) \tag{37}$$

and the adaptive laws defined as

$$\dot{\hat{\boldsymbol{\theta}}}_{i,j} = z_{i,j}\boldsymbol{\Gamma}_{i,j}\boldsymbol{S}_{i,j} - \eta_{i,j}\boldsymbol{\Gamma}_{i,j}\hat{\boldsymbol{\theta}}_{i,j},\ i = 1, 2, \cdots, N; j = 1, 2, \cdots, n_i, \tag{38}$$

where $r_{i,j}$ and $\eta_{i,j}$ are positive design constants, and $\boldsymbol{\Gamma}_{i,j} = \boldsymbol{\Gamma}_{i,j}^T$ are positive definite matrixes, then under the bounded initial conditions, all the signals of the closed-loop system can be guaranteed to be semi-globally bounded and the tracking errors finally converge to a small domain around the origin.

Proof: Consider the following Lyapunov function for the whole systems

$$V = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}z_{i,j}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\tilde{\boldsymbol{\theta}}_{i,j}^T\boldsymbol{\Gamma}_{i,j}^{-1}\tilde{\boldsymbol{\theta}}_{i,j}.$$

From the previous discussions, we have

$$\begin{aligned}\dot{V} \leqslant &-\sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(r_{i,j}-\frac{1}{2}g_{i,j}\right)z_{i,j}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i-1}g_{i,j}z_{i,j+1}^2 \\ &+ \frac{1}{2}\sum_{i=1}^{N}g_{i,n_i}\hat{k}_i^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\varepsilon_{i,j}^2 \\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{l=1}^{N}h_{i,j,l}^2(y_l) - \sum_{i=1}^{N}\sum_{j=1}^{n_i}z_{i,j}\lambda_{i,j}(z_{i,1}) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{n_i}\tilde{\boldsymbol{\theta}}_{i,j}^T\left(z_{i,j}\boldsymbol{S}_{i,j}-\boldsymbol{\Gamma}_{i,j}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{i,j}\right).\end{aligned} \tag{39}$$

Substituting (38) into (39) gives

$$\begin{aligned}\dot{V} \leqslant &-\sum_{i=1}^{N}\sum_{j=1}^{n_i}c_{i,j}z_{i,j}^2 + \frac{1}{2}\sum_{i=1}^{N}g_{i,n_i}\hat{k}_i^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\varepsilon_{i,j}^2 \\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{l=1}^{N}h_{i,j,l}^2(y_l) - \sum_{i=1}^{N}\sum_{j=1}^{n_i}z_{i,j}\lambda_{i,j}(z_{i,1}) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{n_i}\eta_{i,j}\tilde{\boldsymbol{\theta}}_{i,j}^T\hat{\boldsymbol{\theta}}_{i,j}.\end{aligned} \tag{40}$$

where $c_j = r_{i,j} - \frac{1}{2}g_{i,j} > 0$

For the term $\sum_{i=1}^{N}\sum_{j=1}^{n_i}\eta_{i,j}\tilde{\boldsymbol{\theta}}_{i,j}^T\hat{\boldsymbol{\theta}}_{i,j}$, we have the following inequality

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\eta_{i,j}\tilde{\boldsymbol{\theta}}_{i,j}^T\hat{\boldsymbol{\theta}}_{i,j} &\leqslant -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\eta_{i,j}\tilde{\boldsymbol{\theta}}_{i,j}^T\tilde{\boldsymbol{\theta}}_{i,j} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\eta_{i,j}\left\|\boldsymbol{\theta}_{i,j}\right\|^2 \\ &\leqslant -\frac{1}{2}\underline{\eta}_{i,j}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\tilde{\boldsymbol{\theta}}_{i,j}^T\boldsymbol{\Gamma}_{i,j}^{-1}\tilde{\boldsymbol{\theta}}_{i,j} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\eta_{i,j}\left\|\tilde{\boldsymbol{\theta}}_{i,j}\right\|^2,\end{aligned} \tag{41}$$

where $\underline{\eta}_{i,j} = \min\left\{\dfrac{\eta_{i,j}}{\lambda_{\max}\left(\boldsymbol{\Gamma}_{i,j}^{-1}\right)} : i = 1, \cdots, N;\ j = 1, 2, \cdots, n_i\right\}$

For the term $\sum_{i=1}^{N}g_{i,n_i}\hat{k}_i^2$, we have the following inequality

$$\sum_{i=1}^{N}g_{i,n_i}\hat{k}_i^2 \leqslant \frac{1}{2}\sum_{i=1}^{N}\left(g_{i,n_i}^2 + \hat{k}_i^4\right) \leqslant \frac{1}{2}\sum_{i=1}^{N}\left(\bar{b}^2 + \hat{k}_i^4\right). \tag{42}$$

Choose the smooth nonnegative functions $\lambda_{i,j}(z_{i,1})$ such that

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\sum_{l=1}^{N}h_{i,j,l}^2(y_l) - \sum_{i=1}^{N}\sum_{j=1}^{n_i}z_{i,j}\lambda_{i,j}(z_{i,1}) \leqslant 0. \tag{43}$$

Substituting (41), (42) and (43) into (40) gives

$$\begin{aligned}\dot{V} \leqslant &-\sum_{i=1}^{N}\sum_{j=1}^{n_i}c_{i,j}z_{i,j}^2 - \frac{1}{2}\underline{\eta}_{i,j}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\tilde{\boldsymbol{\theta}}_{i,j}^T\boldsymbol{\Gamma}_{i,j}^{-1}\tilde{\boldsymbol{\theta}}_{i,j} \\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\eta_{i,j}\left\|\tilde{\boldsymbol{\theta}}_{i,j}\right\|^2 + \frac{1}{4}\sum_{i=1}^{N}\left(\bar{b}^2 + \hat{k}_i^4\right) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\varepsilon_{i,j}^2.\end{aligned} \tag{44}$$

Let

$$a_0 = \min\left\{2c_{i,j}, \underline{\eta}_{i,j} : i = 1, \cdots, N; j = 1, \cdots, n_i\right\},$$

$$b_0 = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\eta_{i,j}\|\tilde{\boldsymbol{\theta}}_{i,j}\|^2 + \frac{1}{4}\sum_{i=1}^{N}\left(\bar{b}^2 + \hat{k}_i^4\right) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\varepsilon_{i,j}^2$$

Then, (44) can be rewritten as

$$\dot{V} \leqslant -a_0 V + b_0. \tag{45}$$

According (45), we have

$$V(t) \leqslant \left(V(0) - \frac{b_0}{a_0}\right)e^{-a_0 t} + \frac{b_0}{a_0}, \ \forall t > 0.$$

Therefore, using similar arguments to Tong et al. (2011, 2014), it can be seen that all signals of the closed-loop system are semi-globally uniformly ultimately bounded. Furthermore, the tracking errors finally converge to a small domain around the origin.

Remark 8: Theoretically speaking, based on Theorem 1, a prominent tracking control performance can be achieved with appropriate choice parameters $r_{i,j}$, $\eta_{i,j}$ and $\Gamma_{i,j} = \Gamma_{i,j}^T$. However, these design parameters should be adjusted carefully to achieve a suitable transient performance and control action in practical applications.

Simulation results

In this section, two numerical simulation examples are given to demonstrate the effectiveness of the proposed approach.

Example 1: Consider the following large-scale nonlinear systems with dead-zone input

$$\begin{cases} \dot{x}_{1,1} = g_{1,1}(\bar{x}_{1,1})x_{1,2} + f_{1,1}(\bar{x}_{1,1}) + h_{1,1}(\bar{y}) \\ \dot{x}_{1,2} = g_{1,2}(\bar{x}_{1,2})u_1 + f_{1,2}(\bar{x}_{1,2}) + h_{1,2}(\bar{y}) \\ \dot{x}_{2,1} = g_{2,1}(\bar{x}_{2,1})x_{2,2} + f_{2,1}(\bar{x}_{2,1}) + h_{2,1}(\bar{y}) \\ \dot{x}_{2,2} = g_{2,2}(\bar{x}_{2,2})u_2 + f_{2,2}(\bar{x}_{2,2}) + h_{2,2}(\bar{y}) \\ y_1 = x_{1,1} \\ y_2 = x_{2,1} \end{cases} \tag{46}$$

where $g_{1,1} = 2 + \sin\left(x_{1,1}^2\right)$, $g_{1,2} = 2 + \sin(x_{1,1}x_{1,2})$, $g_{2,1} = 2 + \sin(x_{2,1})$, $g_{2,2} = 2 + \sin(x_{2,1}x_{2,2})$, $f_{1,1} = -2x_{1,1}e^{-0.5x_{1,1}}$, $f_{1,2} = -x_{1,1}^2 x_{1,2}$, $f_{2,1} = -2x_{2,1}e^{-x_{2,1}}$, $f_{2,2} = x_{2,2}^2 + x_{2,1}e^{0.5x_{2,2}}$, $h_{1,1} = -x_{1,1}x_{2,1} + x_{2,1}$, $h_{1,2} = x_{1,1}^2 x_{2,1}$, $h_{2,1} = x_{1,1} - x_{1,1}x_{2,1}$ and $h_{2,2} = e^{-x_{1,1}x_{2,1}}$

The dead-zone $D_i(v_i)$ defined as

$$u_1 = D_1(v_1) = \begin{cases} (1 - 0.5\sin v_1)(v_1 - 3), & v_1 \geqslant 3 \\ 0, & -2 \leqslant v_1 \leqslant 3 \\ (0.8 - 0.1\cos v_1)(v_1 + 2), & v_1 \leqslant -2 \end{cases}$$

$$u_2 = D_2(v_2) = \begin{cases} (1 - 0.2\sin v_2)(v_2 - 1), & v_2 \geqslant 1 \\ 0, & -1.5 \leqslant v_2 \leqslant 1 \\ (1 - 0.1\cos v_2)(v_2 + 1.5), & v_2 \leqslant -1.5 \end{cases}$$

According to Theorem 1, the virtual control signals $\alpha_{1,1}$, $\alpha_{2,1}$ and the actual control laws v_1, v_2 are chosen respectively as

$$\alpha_{1,1} = -\frac{1}{\underline{b}}\left(r_{1,1}|z_{1,1}| + \left|\hat{\boldsymbol{\theta}}_{1,1}^T \boldsymbol{S}_{1,1}(z_{1,1})\right|\right)\text{sgn}(z_{1,1}),$$

$$v_1 = -\frac{1}{\underline{b}}\left(r_{1,2}|z_{1,2}| + \left|\hat{\boldsymbol{\theta}}_{1,2}^T \boldsymbol{S}_{1,2}(z_{1,2})\right|\right)\text{sgn}(z_{1,2}),$$

$$\alpha_{2,1} = -\frac{1}{\underline{b}}\left(r_{2,1}|z_{2,1}| + \left|\hat{\boldsymbol{\theta}}_{2,1}^T \boldsymbol{S}_{2,1}(z_{2,1})\right|\right)\text{sgn}(z_{2,1}),$$

$$v_2 = -\frac{1}{\underline{b}}\left(r_{2,2}|z_{2,2}| + \left|\hat{\boldsymbol{\theta}}_{2,2}^T \boldsymbol{S}_{2,2}(z_{2,2})\right|\right)\text{sgn}(z_{2,2}),$$

where $z_{1,1} = x_{1,1} - y_{d,1}$, $z_{1,2} = x_{1,2} - \alpha_{1,1}$, $z_{2,1} = x_{2,1} - y_{d,2}$, $z_{2,2} = x_{2,2} - \alpha_{2,1}$ and $\boldsymbol{z}_{1,1} = z_{1,1}$, $\boldsymbol{z}_{1,2} = [z_{1,1}, z_{1,2}]^T$, $\boldsymbol{z}_{2,1} = z_{2,1}$, $\boldsymbol{z}_{2,2} = [z_{2,1}, z_{2,2}]^T$.

The adaptive laws are given as

$$\dot{\hat{\boldsymbol{\theta}}}_{i,j} = z_{i,j}\Gamma_{i,j}\boldsymbol{S}_{i,j} - \eta_{i,j}\Gamma_{i,j}\hat{\boldsymbol{\theta}}_{i,j}, \ i = 1,2; j = 1,2$$

In the simulation, the RBFNNs are chosen in the following way. $\boldsymbol{\theta}_{i,1}^T \boldsymbol{S}_{i,1}(\boldsymbol{z}_{i,1})$, $i = 1,2$ contains 7 nodes with centers spaced evenly in the interval $[-3,3]$ and widths being equal to 10. $\boldsymbol{\theta}_{i,2}^T \boldsymbol{S}_{i,2}(\boldsymbol{z}_{i,2})$, $i = 1,2$ contains 9 nodes with centers spaced evenly in the interval $[-9,9] \times [-9,9]$ and widths being equal to 10. The design parameters are chosen as follows: $\eta_{1,1} = 5$, $\Gamma_{1,1} = \text{I}_7$, $r_{1,1} = 2$, $\eta_{1,1} = 5$, $\Gamma_{1,2} = \text{I}_7$, $r_{1,2} = 0.5$, $\eta_{2,1} = 10$, $\Gamma_{2,1} = 10\text{I}_9$, $r_{2,1} = 10$, $\eta_{2,2} = 2$, $\Gamma_{2,2} = 20\text{I}_9$, $r_{2,2} = 2$, $\underline{b} = 0.1$.

The simulation is run with the initial conditions $[x_{1,1}(0), x_{1,2}(0), x_{2,1}(0), x_{2,2}(0)]^T = [0,0,0,0]^T$, and the reference signals are chosen follows: $y_{d,1} = 0.5(\sin t + \sin(0.4t))$ and $y_{d,2} = 0.5(0.5\sin t + \sin(0.4t))$.

The simulation results are shown in Figures 1–5.

Figure 1 shows the system output y_1 and the reference signal $y_{d,1}$. Figure 2 shows the system output y_2 and the reference signal $y_{d,2}$. From Figures 1–2, it can be seen that good tracking performances have been achieved. Figure 3 displays the control signals v_1 and v_2. Figure 4 shows that the state variables $x_{1,2}$ and $x_{2,2}$. From Figures 1–4, it can be seen that

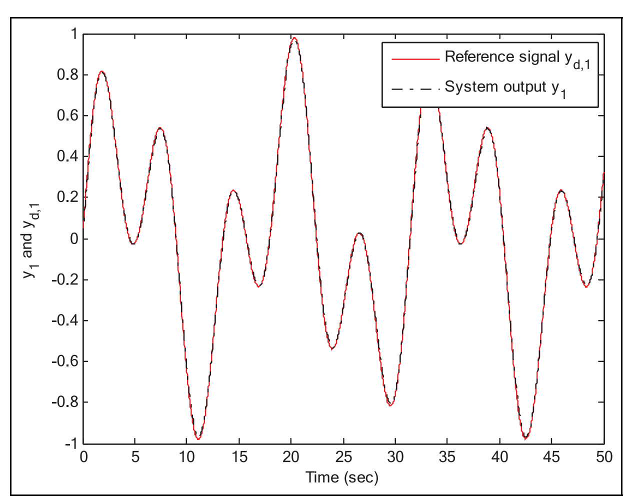

Figure 1. Output y_1 (dot line) and the tracking signal $y_{d,1}$ of Example 1.

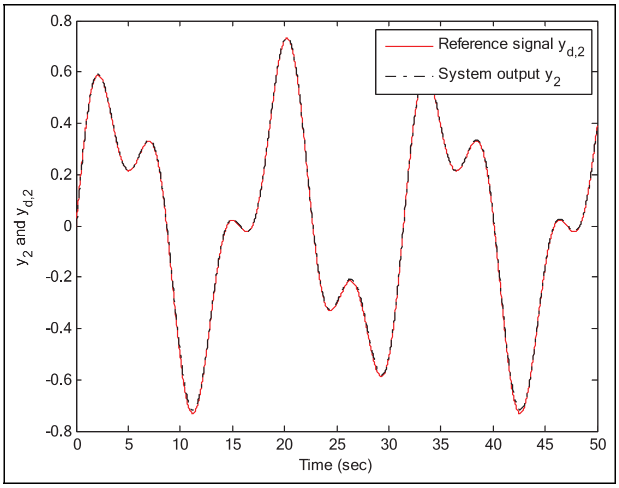

Figure 2. Output y_2 (dot line) and the tracking signal $y_{d,2}$ of Example 1.

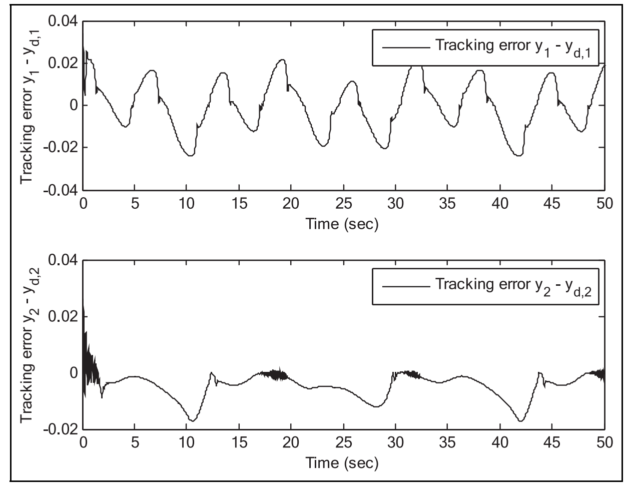

Figure 5. Tracking errors $y_1 - y_{d,1}$ and $y_2 - y_{d,2}$ of Example 1.

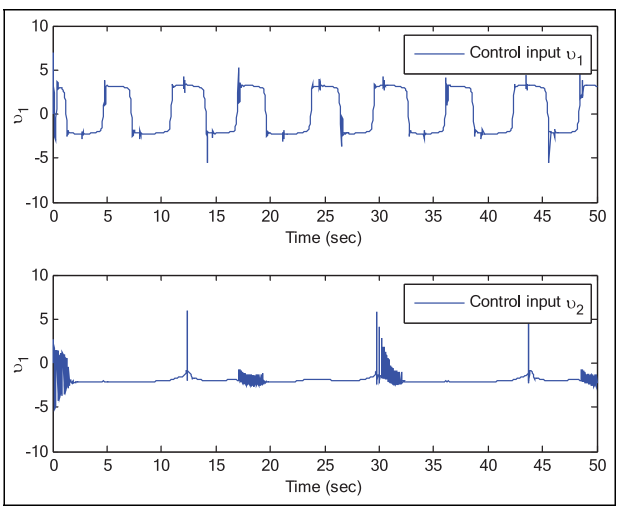

Figure 3. Control signals υ_1 and υ_2 of Example 1.

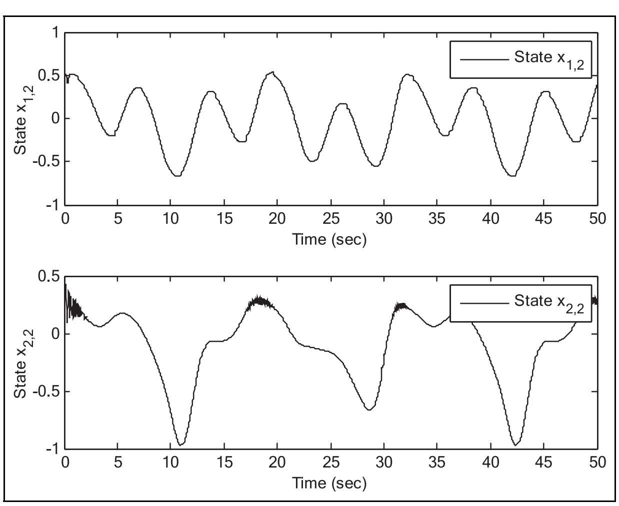

Figure 4. State variables $x_{1,2}$ and $x_{2,2}$ of Example 1.

all the signals of the closed-loop system are bounded. Figure 5 depicts that the tracking errors finally converge to a small domain around the origin. From the above simulation results, we can conclude that even though the nonlinear functions in the large-scale system is not available, and the system affected by unknown dead-zone, the proposed adaptive decentralized tracking control approach guarantees the stability of the closed-loop adaptive control system and achieves good tracking performance.

Example 2: Consider two inverted pendulums that are connected by a spring. According to (Spooner and Passino, 1999; Tong et al., 2014), the equations that describe the motion of the pendulums are defined by

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} \\ \dot{x}_{1,2} = \left(\frac{m_1 gr}{J_1} - \frac{kr^2}{4J_1}\right)\sin x_{1,1} + \frac{kr}{2J_1}(l-d) + \frac{u_1}{J_1} + \frac{kr^2}{4J_1}\sin x_{2,1} \\ y_1 = x_{1,1} \end{cases}$$
$$\begin{cases} \dot{x}_{2,1} = x_{2,2} \\ \dot{x}_{2,2} = \left(\frac{m_2 gr}{J_2} - \frac{kr^2}{4J_2}\right)\sin x_{2,1} + \frac{kr}{2J_2}(l-d) + \frac{u_2}{J_2} + \frac{kr^2}{4J_2}\sin x_{1,2} \\ y_2 = x_{2,1} \end{cases} \tag{47}$$

The parameters $m_1 = 2\ kg$ and $m_2 = 2\ kg$ are the pendulum end masses, $J_1 = 1\ kg$ and $J_2 = 1\ kg$ are the moments of inertia, $k = 10\ N/m$ is the spring constant of the connecting spring, $r = 0.1\ m$ is the pendulum height, $l = 0.4\ m$ is the natural length of the spring, the distance between the pendulum hinges is defined as $d = 0.4\ m$. $g = 9.81\ m/s^2$ is the gravitational acceleration.

In the simulation, the parameters of dead-zone, virtual control functions, the true control laws and the adaptive laws are the same as in Experiment 1, and the reference signals are $y_{d,1} = 0.5(\sin t + \sin(0.4t))$, $y_{d,2} = 0.5(0.5\cos t + \cos(0.4t))$. The simulation results are shown in Figures 6–10.

Figures 1 and 2 show the system output and the reference signal trajectories. From Figures 6 and 7, it can be seen that good tracking performances have been achieved. Figure 8

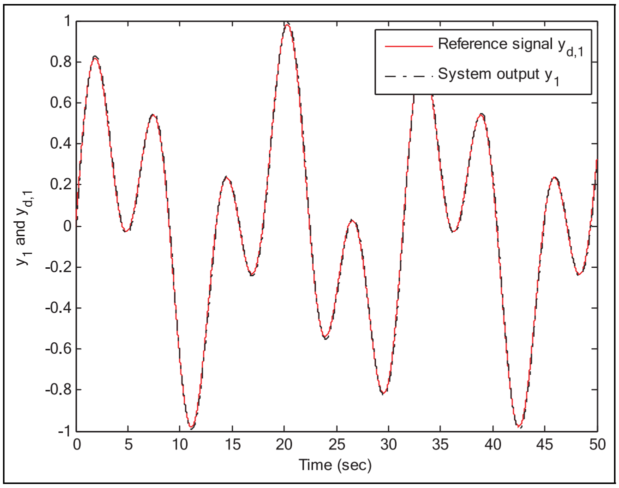

Figure 6. Output y_1 (dot line) and the tracking signal $y_{d,1}$ of Example 2.

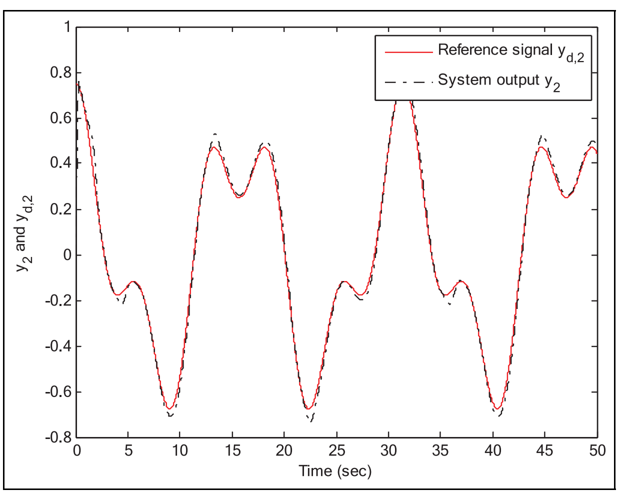

Figure 7. Output y_2 (dot line) and the tracking signal $y_{d,2}$ of Example 2.

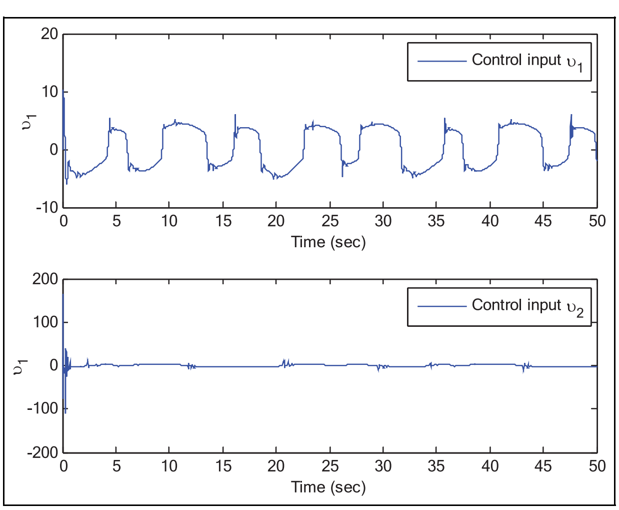

Figure 8. Control signals υ_1 and υ_2 of Example 2.

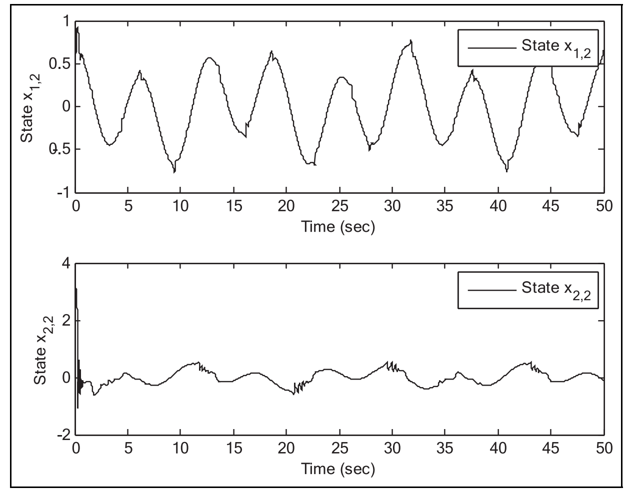

Figure 9. State variables $x_{1,2}$ and $x_{2,2}$ of Example 2.

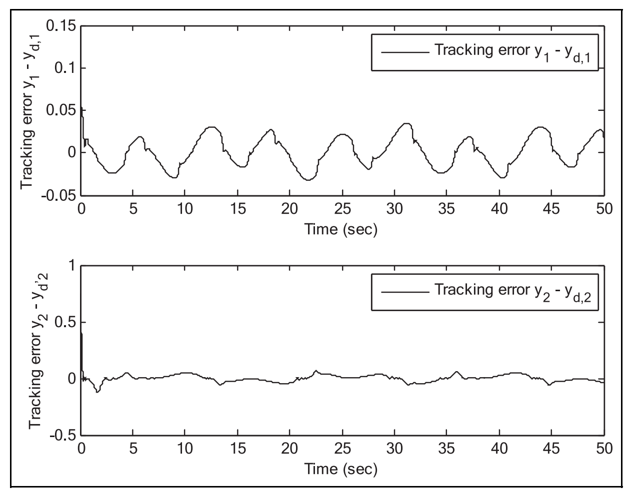

Figure 10. Tracking errors $y_1 - y_{d,1}$ and $y_2 - y_{d,2}$ of Example 2.

displays the control signals υ_1 and υ_2. Figure 9 shows that the state variables $x_{1,2}$ and $x_{2,2}$. Figure 10 depicts that the tracking errors finally converge to a small domain around the origin. From Figures 6–10, it can be seen that all the signals of the closed-loop system are bounded. From the above simulation results, we can conclude that the control performance for practical system is still fairly satisfactory, which further show that the presented control strategy in this paper is effective.

Conclusions

In this paper, an adaptive decentralized tracking control approach is proposed for large-scale nonlinear systems with unknown nonsymmetric dead-zone inputs using NN. The proposed approach has the following three advantages: (1) A novel adaptive NN-backstepping-based controller design technique for a class of more general large-scale nonlinear systems with the unknown non-symmetric dead-zone inputs. (2) The computation burden of the designed controller is significantly decreased. (3) The control strategy can get precise tracking results with low computational cost, and have a good real-time performance and convergence. Future research will be focused on adaptive neural controller design for large-scale stochastic nonlinear systems with unknown dead-zone inputs using NN.

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ORCID iD

Yu-Qun Han https://orcid.org/0000-0002-9055-2954

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