Article - Measurement

TRANSACTIONS OF THE INSTITUTE OF MEASUREMENT & CONTROL

Transactions of the Institute of Measurement and Control
1–10

DOI: 10.1177/0142331220953355
journals.sagepub.com/home/tim
SAGE

Multi-dimensional Taylor network-based adaptive control for nonlinear systems with unknown parameters

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Abstract

This paper presents an adaptive multi-dimensional Taylor network (MTN) control approach for a class of nonlinear systems with unknown parameters. MTN is employed to identify unknown nonlinear characteristics existing in the system, and then a novel adaptive MTN tracking control method is proposed, via backstepping technique. In the controller design, double adaptive laws are designed and appropriate Lyapunov functions are chosen to overcome the difficulties caused by the unknown parameters. The designed controller can guarantee that all the variables in the closed-loop systems are bounded and the tracking error can be arbitrarily small. Finally, simulation results are presented to verify the effectiveness of the proposed approach.

Keywords

Multi-dimensional Taylor network, nonlinear systems, adaptive control, backstepping technique, unknown parameters

Introduction

During the past decades, since nonlinear functions inevitably appear in the field of practical engineering, the stability problem for nonlinear systems has attracted much attention and various effective nonlinear control methods have been proposed such as adaptive control (Han and Narendra, 2012), variable structure control (Aghababa, 2013), backstepping control (Yu et al., 2018), neural network control (Liu et al., 2014) and fuzzy control (Zhao et al., 2018). To mention a few, an adaptive MDADT method was addressed in Niu et al. (2020a) for a class of switched uncertain nonlinear systems. In Sun and Wu (2014), the motion/force control of nonholonomic mechanical systems with affine constraints was investigated, and an adaptive integral feedback compensation strategy were applied to identify the dynamic uncertainty. Recently, in order to account for unknown dynamics without linear condition, the neural networks (NNs) and the fuzzy logic systems (FLSs) have been incorporated into an adaptive control design owing to their excellent approximation ability. Based on the approximation-based adaptive control technique, the tracking problem for several classes of nonlinear systems has been addressed in Niu et al. (2019, 2020b); Tong et al. (2010, 2014); Wang et al. (2012); Zhou et al. (2011). For nonstrict-feedback nonlinear switched systems with completely unknown nonlinearities, adaptive neural control methods were given in Niu et al. (2018). Appling fuzzy logic system to approximate unknown nonlinearities, the high-order nonlinear systems with input and output constrained (Sun et al., 2019) and the nonlinear systems with prescribed performance (Sun et al., 2020) were studied, respectively. Nevertheless, although many interesting results have been achieved, there are common defects exist in the previous works. For example: (a) the controller is complex and the calculation is complication; (b) the real-time performance of the controller is poor, which is not suitable for practical application. Therefore, it is still a meaningful task to find a simple and effective adaptive control algorithm, and then a new control method based on multi-dimensional Taylor network (MTN) is proposed.

MTN is a network structure connected by polynomials. It has three layers, including the input layer, the intermediate layer and the output layer. In view of its excellent approximation performance, MTN was initially used to solve the identification and prediction problems of nonlinear systems (Zhou and Yan, 2014). Later, it was found that MTN has strong ability to learn and generalize, so it was successfully applied to the control of various kinds of nonlinear systems (Han, 2018; Han and Yan, 2018; Sun and Yan, 2014, 2015). For instance, single input single output (SISO) nonlinear systems (Sun and Yan, 2014), time-varying delay nonlinear systems (Han and Yan, 2018), large-scale stochastic nonlinear systems

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(Han, 2018) and multiple input single output (MISO) nonlinear systems (Sun and Yan, 2015). However, although many important achievements have been made in the study of MTN-based nonlinear system control problems, it is still in its infancy, and many problems need to be further studied. It is worth noting that the parameters uncertainty is omitted in these works, which is a factor of degrading system performance.

From a practical point of view, most physical systems suffer from the effect of unknown parameters, such as the inertia wheel pendulum system (Wang et al., 2015), the ironless linear permanent-magnet synchronous motor system (Yao et al., 2011) and the projective synchronization of three memristor chaotic system (Wang et al., 2018). With the research focus has become the unknown parameters, some appropriate control methods (Chakrabortty and Scholtz, 2011; Qi et al., 2012) have been proposed to deal with the stability problem of the nonlinear systems with them. To eliminate the effect of parameters uncertainly, the method of defining bounds of unknown parameters was proposed in Liu et al. (2015) and Tsinias (2000) and became an important tool in recent years. As we all know, the bounded parameter is a special case of the parameter uncertainty and the practical applications of the results in the nonlinear systems with unknown boundary parameters are limited. To break this limitation, the novel adaptive form has been redesigned in Liu and Liu (2011) and Wang et al. (2017) for the nonlinear high-order multi-agent systems and the chaotic complex nonlinear systems, respectively. It is worth noting that the problem of large amount of calculation caused by too many adaptive parameters has not been solved in these works. Therefore, it is of great significance to propose an effective method to solve the problems of unknown parameters and high computational complexity of nonlinear systems at the same time, which motivates our study.

Based on the above discussion, this paper focuses on the problem of adaptive MTN control for nonlinear system with unknown parameters. By approximating the unknown nonlinear functions with MTNs, a novel adaptive MTN control scheme is developed based on backstepping technique. The main innovations of this paper include:

(1) This paper investigates the tracking control problem for nonlinear systems with unknown parameters, and an adaptive backstepping controller with double adaptive laws is put forward innovatively. The developed adaptive controller does not need the assumptions that the unknown parameters are bounded in Liu et al. (2015) and Tsinias (2000). In addition, the reasonable Lyapunov functions are selected to guarantee all the signals in the closed-loop systems are bounded.
(2) The MTN-based technology is successfully applied to the nonlinear system with unknown parameters and a simple structure adaptive MTN tracking control strategy is proposed. Compared with the existing control results in Liu (2011) and Wang et al. (2017), the proposed control scheme in this paper overcome the problems of too many adaptive parameters and heavy computing burden.
(3) With the adaptive control method based on MTN, the designed controller has simple structure, low computational complexity and good real-time performance.

Problem formulation and prelimaries

Problem description

Consider the following nonlinear system with unknown parameters

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + \vartheta_i \varphi_i(\bar{x}_i) \\ \qquad i = 1, \cdots, n-1 \\ \dot{x}_n = u + f_n(\bar{x}_n) + \vartheta_n \varphi_n(\bar{x}_n) \\ \qquad y = x_1 \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the system output, $\bar{\boldsymbol{x}}_i = [x_1, \cdots, x_i]^{\mathrm{T}} \in \mathbb{R}^i$, $i = 1, 2, \cdots, n$. $f(\cdot): \mathbb{R}^i \to \mathbb{R}$ are unknown smooth functions with $f_i(\mathbf{0}) = 0$. $\varphi_i(\bar{x}_i)$ are known functions and ϑ_i are unknown parameters.

Remark 1: It should be noted that system (1) is a class of standard nonlinear system with unknown parameters. When $\vartheta_i = 0$, $i = 1, \ldots, n$, system (1) can be converted to strict-feedback nonlinear system, which was studied in Sun et al. (2020). In addition, actuators are frequently involved with several restrictions. such as input and output constrained, and many works have been focused on nonlinear systems with input and output constrained (Han, 2020; Sun et al., 2019).

The objective of this paper is to design an adaptive MTN controller for system (1), such that the system output y tracks the given reference signal y_d, and all the signals in the closed-loop systems remain bounded.

The main results of this paper are based on the following Assumptions and Lemmas.

Assumption 1: The state variables of the nonlinear system (1) are measurable.

Assumption 2: The given reference signal y_d and its time derivatives up to the n-th order are continuous and bounded.

Lemma 1: (Deng and Krstic, 1997) For $\forall (x, y) \in R^2$ and $\varepsilon > 0$, the following inequality holds

$$xy \leqslant \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q \tag{2}$$

where p and q are constants and satisfy $p > 1$, $q > 1$ and $(p-1)(q-1) = 1$.

MTN

MTN is composed of some polynomials. Its structure consists of three layers: input layer, middle layer and output layer. $z_1, \cdots, z_n$ are the input layer of MTN, and the middle layer is the polynomial combination of the input layer, it maps the input space to a new space and its mathematical expression is as follows

$$f_{MTN}(z) = \boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{S}_{m_n}(z)$$

where n denotes the $n-$th state input and m represents the highest power of a polynomial. $\boldsymbol{z} = [z_1, \cdots, z_n]^{\mathrm{T}} \subset \mathbb{R}^n$ denotes input, $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_n]^{\mathrm{T}} \subset \mathbb{R}^l$ represents the weight vector of MTN and $\boldsymbol{S}_{m_n}(\boldsymbol{z})$ means $\prod_{i,j=1}^{n} s_i^{\sigma_i} s_j^{\sigma_j}$, where σ_i and σ_j are non-negative numbers and satisfy $1 \leqslant \sigma_i + \sigma_j \leqslant m$, the form of vector $\boldsymbol{S}_{m_n}(\boldsymbol{z})$ is as follows

$$S_{m_n}(z) = [\underbrace{z_1, \cdots, z_n}_{1term}, \underbrace{z_1^2, \cdots, z_n^2}_{2term}, \cdots, \underbrace{z_1^m, \cdots, z_n^m}_{mterm}]^{\mathrm{T}} \subset \mathbb{R}^l$$

In this paper, MTN is used to approximate the unknown functions encountered in controller design and stability analysis. To this end, there is the following Lemma.

Lemma 2: (Han and Yan, 2018) Assume that $f(x) : \mathbb{R}^m \to \mathbb{R}^n$ is a continuous function defined on a compact set Ω_Z. Then, for any given desired level of accuracy $\varepsilon > 0$, there exists a MTN, such that

$$f(Z) = \boldsymbol{\theta}^{*\mathrm{T}} S(Z) + \sigma(Z),\ \forall Z \in \Omega_Z$$

where $\boldsymbol{\theta}^*$ is the ideal constant weight vector and defined as

$$\boldsymbol{\theta}^* = \arg\min_{\theta \in \mathbb{R}^l} \left\{ \sup \left| \varphi(z) - \boldsymbol{\theta}^{\mathrm{T}} S_{m_n}(z) \right| \right\}$$

where $\sigma(Z)$ denotes the approximation error and satisfies $|\sigma(Z)| \leqslant \varepsilon$.

Output feedback tracking control based on adaptive MTN

MTN controller design

This paper combines the MTN approximation method to design the controller. The design process consists of n steps. Virtual control $\alpha_i (i = 1, 2 \cdots, n-1)$ is designed in the first $n-1$ steps, and adaptive controller u is designed in the second n step.

Now, we introduce the following coordinate changes

$$z_1 = x_1 - y_d$$
$$z_i = x_i - \alpha_{i-1},\ i = 2, \cdots, n$$

where, α_{i-1} $(i = 2, \cdots, n)$ are the intermediate control signals introduced in the backstepping process.

Step 1: According to coordinate transformation with $i = 1$, we have

$$\dot{z}_1 = x_2 + f_1 + \vartheta_1 \varphi_1 - \dot{y}_d \tag{3}$$

Consider the following Lyapunov function

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_1 + \frac{1}{2} \tilde{\vartheta}_1^2 \tag{4}$$

where $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$, $\tilde{\vartheta}_1 = \vartheta_1 - \hat{\vartheta}_1$ are the parameter error, and the time derivative of V_1 is

$$\dot{V}_1 = z_1(x_2 + \tilde{f}_1) + \tilde{\vartheta}_1(z_1 \varphi_1 - \dot{\hat{\vartheta}}_1) + z_1 \hat{\vartheta}_1 \varphi_1 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} \dot{\hat{\boldsymbol{\theta}}}_1 \tag{5}$$

where $\tilde{f}_1 = f_1 - \dot{y}_d$.

According to the approximate properties of MTN, for any given $\varepsilon_1 > 0$, there exists $\boldsymbol{\theta}_1{}^{\mathrm{T}} S_1(z_1)$ such that

$$\tilde{f}_1 = \boldsymbol{\theta}_1^{\mathrm{T}} S_1 + \sigma_1(z_1), |\sigma_1(z_1)| \leqslant \varepsilon_1 \tag{6}$$

where $\sigma_1(\boldsymbol{z}_1)$ is approximation error.

Taking the virtual control signal α_1 as

$$\alpha_1 = -k_1 z_1 - \hat{\boldsymbol{\theta}}_1{}^{\mathrm{T}} S_1(z_1) - \hat{\vartheta}_1 \varphi_1(x_1) \tag{7}$$

where $k_1 > 0$ is a design parameter.

Form (6) and (7), we have

$$z_1(x_2 + \tilde{f}_1) = z_1(x_2 + \tilde{\boldsymbol{\theta}}_1{}^{\mathrm{T}} S_1 - \alpha_1 - k_1 z_1 + \sigma_1 - \hat{\vartheta}_1 \varphi_1(x_1)) \tag{8}$$

By the Lemma 1 and (6), we can obtain

$$z_1 \sigma_1 \leqslant \frac{1}{2} z_1^2 + \frac{1}{2} \sigma_1^2 \leqslant \frac{1}{2} z_1^2 + \frac{1}{2} \varepsilon_1^2 \tag{9}$$

Then, substituting (8) and (9) into (5), and combining with $z_2 = x_2 - \alpha_1$ gives

$$\dot{V}_1 \leqslant z_1 z_2 - k_1 z_1^2 + \frac{1}{2} z_1^2 + \frac{1}{2} \varepsilon_1^2 + \tilde{\vartheta}_1(z_1 \varphi_1 - \dot{\hat{\vartheta}}_1) + \tilde{\boldsymbol{\theta}}_1{}^{\mathrm{T}}(z_1 S_1 - \dot{\hat{\boldsymbol{\theta}}}_1) \tag{10}$$

Step 2: According to coordinate transformation with $i = 2$, we have

$$\dot{z}_2 = x_3 + f_2 + \vartheta_2 \varphi_2 - \dot{\alpha}_1 \tag{11}$$

Consider the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2} \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_2 + \frac{1}{2} \tilde{\vartheta}_2^2 \tag{12}$$

where $\tilde{\boldsymbol{\theta}}_2 = \boldsymbol{\theta}_2 - \hat{\boldsymbol{\theta}}_2$, $\tilde{\vartheta}_2 = \vartheta_2 - \hat{\vartheta}_2$ are the parameter error, and the time derivative of V_2 is

$$\dot{V}_2 = \dot{V}_1 + z_2(x_3 + \tilde{f}_2) + \tilde{\vartheta}_2(z_2 \varphi_2 - \dot{\hat{\vartheta}}_2) + z_2 \hat{\vartheta}_2 \varphi_2 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}} \dot{\hat{\boldsymbol{\theta}}}_2 \tag{13}$$

where $\tilde{f}_2 = f_2 - \dot{\alpha}_1$, $\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1} - \frac{\partial \alpha_1}{\partial \hat{\boldsymbol{\theta}}_1} \dot{\hat{\boldsymbol{\theta}}}_1 - \frac{\partial \alpha_1}{\partial \hat{\vartheta}_1} \dot{\hat{\vartheta}}_1 - \frac{\partial \alpha_1}{\partial y_d} \dot{y}_d$.

Similarly, we employ the MTN $\boldsymbol{\theta}_2^{\mathrm{T}} S_2(z_2)$ to approximate unknown function $\tilde{f}_2$, for any given $\varepsilon_2 > 0$, we have

$$\tilde{f}_2 = \boldsymbol{\theta}_2^{\mathrm{T}} S_2 + \sigma_2(z_2), |\sigma_2(z_2)| \leqslant \varepsilon_2 \tag{14}$$

where $z_2 = [z_1, z_2]^{\mathrm{T}}$, and $\sigma_2(z_2)$ is approximation error.

Taking the virtual control signal α_2 as

$$\alpha_2 = -k_2 z_2 - \hat{\boldsymbol{\theta}}_2^{\mathrm{T}} S_2(z_2) - \hat{\vartheta}_2 \varphi_2(\bar{x}_2), (k_2 > 0) \tag{15}$$

From (14) and (15), and combining with $z_3 = x_3 - \alpha_2$ gives

$$\begin{aligned}\dot{V}_2 \leqslant \dot{V}_1 + z_2 z_3 + z_2 \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}} S_2 - k_2 z_2^2 + \frac{1}{2} z_2^2 \\ + \frac{1}{2}\varepsilon_2^2 + \tilde{\vartheta}_2(z_2\varphi_2 - \dot{\hat{\vartheta}}_2) - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}} \dot{\hat{\boldsymbol{\theta}}}_2\end{aligned} \tag{16}$$

Step i ($3 \leqslant i \leqslant n-1$): According to coordinate transformation with i, we have

$$\dot{z}_i = x_{i+1} + f_i + \vartheta_i \varphi_i - \dot{\alpha}_{i-1} \tag{17}$$

Consider the following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_i + \frac{1}{2}\tilde{\vartheta}_i^2 \tag{18}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$, $\tilde{\vartheta}_i = \vartheta_i - \hat{\vartheta}_i$ are the parameter error, and the time derivative of V_i is

$$\dot{V}_i = \dot{V}_{i-1} + (x_{i+1} + \tilde{f}_i) + \tilde{\vartheta}_i(z_i\varphi_i - \dot{\hat{\vartheta}}_i) + z_i \hat{\vartheta}_i \varphi_i - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \dot{\hat{\boldsymbol{\theta}}}_i \tag{19}$$

where $\tilde{f}_i = f_i - \dot{\alpha}_{i-1}$,

$$\dot{\alpha}_{i-1} = \sum_{k=1}^{i-1} \frac{\partial \alpha_k}{\partial x_k} - \sum_{k=1}^{i-1} \frac{\partial \alpha_k}{\partial \hat{\boldsymbol{\theta}}_k} \dot{\hat{\boldsymbol{\theta}}}_k - \frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}_{i-1}} \dot{\hat{\vartheta}}_{i-1} - \frac{\partial \alpha_{i-1}}{\partial y_d} \dot{y}_d.$$

Similarly, we employ the MTN $\boldsymbol{\theta}_i^{\mathrm{T}} S_i(\boldsymbol{z}_i)$ to approximate unknown function $\tilde{f}_i$, for any given $\varepsilon_i > 0$, we have

$$\tilde{f}_i = \boldsymbol{\theta}_i^{\mathrm{T}} S_i + \sigma_i(z_i), |\sigma_i(z_i)| \leqslant \varepsilon_i \tag{20}$$

where $\boldsymbol{z}_i = [z_1, z_2, \cdots, z_i]^{\mathrm{T}}$, and $\sigma_i(z_i)$ is approximation error.

Taking the virtual control signal α_i as

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} S_i(\boldsymbol{z}_i) - \hat{\vartheta}_i \varphi_i(x_i) \tag{21}$$

where $k_i > 0$ is a design parameter.

From (20) and (21), and combining with $z_{i+1} = x_{i+1} - \alpha_i$ gives

$$\begin{aligned}\dot{V}_i \leqslant \sum_{i=1}^{i} z_i z_{i+1} - \sum_{i=1}^{i} k_i z_i^2 + \sum_{i=1}^{i} \frac{1}{2} z_i^2 \\ + \sum_{i=1}^{i} \frac{1}{2}\varepsilon_i^2 + \sum_{i=1}^{i} \tilde{\vartheta}_i(z_i\varphi_i - \dot{\hat{\vartheta}}_i) + \sum_{i=1}^{i} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_i S_i - \dot{\hat{\boldsymbol{\theta}}}_i)\end{aligned} \tag{22}$$

Step n: Consider the following Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_n + \frac{1}{2}\tilde{\vartheta}_n^2 \tag{23}$$

where $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n - \hat{\boldsymbol{\theta}}_n$, $\tilde{\vartheta}_n = \vartheta_n - \hat{\vartheta}_n$ are the parameter error, and the time derivative of V_n is

$$\dot{V}_n \leqslant \dot{V}_{n-1} + z_n(u + \tilde{f}_n) + \tilde{\vartheta}_n(z_n\varphi_n - \dot{\hat{\vartheta}}_n) + z_n \hat{\vartheta}_n \varphi_n - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} \dot{\hat{\boldsymbol{\theta}}}_n \tag{24}$$

where $\tilde{f}_n = f_n - \dot{\alpha}_{n-1}$, $\dot{\alpha}_{n-1} = \sum_{k=1}^{n-1} \frac{\partial \alpha_k}{\partial x_k} - \sum_{k=1}^{n-1} \frac{\partial \alpha_k}{\partial \hat{\boldsymbol{\theta}}_k} \dot{\hat{\boldsymbol{\theta}}}_k - \frac{\partial \alpha_{n-1}}{\partial \hat{\vartheta}_{n-1}} \dot{\hat{\vartheta}}_{n-1} - \frac{\partial \alpha_{n-1}}{\partial y_d} \dot{y}_d$.

Similarly, we employ the MTN $\boldsymbol{\theta}_n^{\mathrm{T}} S_n(z_n)$ to approximate unknown function $\tilde{f}_n$, for any given $\varepsilon_n > 0$, we have

$$\tilde{f}_n = \boldsymbol{\theta}_n^{\mathrm{T}} S_n + \sigma_n(\boldsymbol{z}_n), |\sigma_n(\boldsymbol{z}_n)| \leqslant \varepsilon_n \tag{25}$$

where $\boldsymbol{z}_n = [z_1, \cdots, z_n]^{\mathrm{T}}$, and $\sigma_n(\boldsymbol{z}_n)$ is approximation error.

Taking the controller u as

$$u = -k_n z_n - \hat{\boldsymbol{\theta}}_n^{\mathrm{T}} S_n(\boldsymbol{z}_n) - \hat{\vartheta}_n \varphi_n(x_n) \tag{26}$$

where $k_n > 0$ is a design parameter.

According to Lemma 1, the following inequality is obtained

$$\begin{aligned} z_n(u + \tilde{f}_n) &= z_n(-k_n z_n - \hat{\vartheta}_n \varphi_n + \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} S_n + \sigma_n) \\ &\leqslant z_n \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} S_n - k_n z + \frac{1}{2} z_n^2 + \frac{1}{2}\varepsilon_n^2 - z_n \hat{\vartheta}_n \varphi_n \end{aligned} \tag{27}$$

Substituting (27) into (24) gives

$$\begin{aligned}\dot{V}_n \leqslant \sum_{i=1}^{n} z_i z_{i+1} - \sum_{i=1}^{n} k_i z_i^2 + \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \sum_{i=1}^{n} \frac{1}{2}\varepsilon_i^2 \\ + \sum_{i=1}^{n} \tilde{\vartheta}_i(z_i\varphi_i - \dot{\hat{\vartheta}}_i) + \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_i S_i - \dot{\hat{\boldsymbol{\theta}}}_i)\end{aligned} \tag{28}$$

Due to Lemma 1, we have

$$\sum_{i=1}^{n-1} z_i z_{i+1} \leqslant \sum_{i=1}^{n-1} \left(\frac{1}{2} z_i^2 + \frac{1}{2} z_{i+1}^2\right) \leqslant \sum_{i=1}^{n} z_i^2 \tag{29}$$

Substituting (29) into (28) gives

$$\begin{aligned}\dot{V}_n \leqslant -\left(k_i - \frac{3}{2}\right) \sum_{i=1}^{n} z_i^2 + \sum_{i=1}^{n} \frac{1}{2}\varepsilon_i^2 + \sum_{i=1}^{n} \tilde{\vartheta}_i(z_i\varphi_i - \dot{\hat{\vartheta}}_i) \\ + \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_i S_i - \dot{\hat{\boldsymbol{\theta}}}_i)\end{aligned} \tag{30}$$

Stability analysis

Theorem 1: Considering the nonlinear system (1), if a control law u is chosen as (26), with the intermediate virtual control signals $\alpha_i (i = 1, \cdots, n-1)$ described as (21) and the adaptive laws defined as

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i S_{m_i}(z_n) - \eta_i \hat{\boldsymbol{\theta}}_i (i = 1, 2, \cdots n) \tag{31}$$

$$\dot{\hat{\vartheta}}_i = z_i \varphi_i(x_i) - \gamma_i \hat{\vartheta}_i (i = 1, 2, \cdots n) \tag{32}$$

where constants $\eta_i > 0$ and $\gamma_i > 0$ are designed parameters. Then, under bounded initial conditions, all the signals in the closed-loop system are bounded, and the tracking error converges to a small neighborhood of the origin.

Proof: For the stability analysis of the closed-loop system, we choose the following Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}\sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i + \frac{1}{2}\sum_{i-1}^{n} \tilde{\vartheta}_i^2 \tag{33}$$

According to (30), we have

$$\dot{V} \leqslant -\left(k_i - \frac{3}{2}\right)\sum_{i=1}^{n} z_i^2 + \sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^2 + \sum_{i=1}^{n}\gamma_i\tilde{\vartheta}_i\hat{\vartheta}_i + \sum_{i=1}^{n}\eta_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i \tag{34}$$

By the Lemma 1, we can obtain

$$\eta_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i \leqslant -\frac{\eta_i}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i + \frac{\eta_i}{2}\|\boldsymbol{\theta}_i\|^2 \tag{35}$$

$$\gamma_i\tilde{\vartheta}_i\hat{\vartheta}_i \leqslant -\frac{\gamma_i}{2}\tilde{\boldsymbol{\theta}}_i^2 + \frac{\gamma_i}{2}\|\vartheta_i\|^2 \tag{36}$$

Substituting (35) and (36) into (34) gives

$$\begin{aligned}\dot{V} \leqslant &-\left(k_i - \frac{3}{2}\right)\sum_{i=1}^{n} z_i^2 + \sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^2 - \sum_{i=1}^{n}\frac{\eta_i}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i \\ &+ \sum_{i=1}^{n}\frac{\eta_i}{2}\|\boldsymbol{\theta}_i\|^2 - \sum_{i=1}^{n}\frac{\gamma_i}{2}\tilde{\vartheta}_i^{\mathrm{T}}\tilde{\vartheta}_i + \sum_{i=1}^{n}\frac{\gamma_i}{2}\|\vartheta_i\|^2\end{aligned} \tag{37}$$

Let

$$\begin{aligned} a_i &= \min\left\{2\left(k_i - \frac{3}{2}\right), \eta_i, \gamma_i\right\} \\ a_0 &= \min\{a_1, \cdots, a_n\} \\ b_0 &= \sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^2 + \sum_{i=1}^{n}\frac{\eta_i}{2}\|\boldsymbol{\theta}_i\|^2 + \sum_{i=1}^{n}\frac{\gamma_i}{2}\|\vartheta_i\|^2 \end{aligned}$$

Then the inequality (37) can be rewritten in the following form

$$\dot{V} \leqslant -a_0 V + b_0 \tag{38}$$

Therefore, we can get the following inequalities

$$0 \leqslant V(t) \leqslant \left(V(0) - \frac{b_0}{a_0}\right)e^{-a_0 t} + \frac{b_0}{a_0}, \forall t \geqslant 0 \tag{39}$$

Let $k_i - \frac{3}{2} > 0$, the inequality (39) indicates that: $V(t)$ is bounded and the upper bound is b_0/a_0, meanwhile all the signals $x_i, \hat{\theta}, \hat{\vartheta}, \alpha_i, u$ of the closed-loop system are uniformly bounded. By choosing the appropriate design parameter η_i, γ_i and ε_i, $\tilde{\theta}_i, \tilde{\vartheta}_i$ can be small enough, and b_0/a_0 can be arbitrarily small, so that the tracking error converges to the small neighborhood of the origin.

Remark 2: Recalling (33) and (39), we have

$$\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i \leqslant 2V(0)e^{-a_0 t} + 2\frac{b_0}{a_0} \tag{40}$$

$$\sum_{i=1}^{n}\tilde{\vartheta}_i^2 \leqslant 2V(0)e^{-a_0 t} + 2\frac{b_0}{a_0} \tag{41}$$

Thus, for given $\varpi_1, \varpi_2 > 2b_0/a_0$, there exists a time T, for all $t \geqslant T$, such that

$$\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i \leqslant \varpi_1 \tag{42}$$

$$\sum_{i=1}^{n}\tilde{\vartheta}_i^2 \leqslant \varpi_2 \tag{43}$$

which means that $\|\tilde{\boldsymbol{\theta}}_i\|$ and $\|\tilde{\vartheta}_i\|$ converge to zero by properly adjusting the parameters, such as $\eta_i, \gamma_i, \varepsilon_i$.

Simulation results

This section provides three examples to illustrate the effectiveness of the proposed approach in this paper.

Example 1: A third-order system with unknown parameters is considered as follows

$$\begin{cases} \dot{x}_1 = x_2 - 2x_1^2 + \vartheta_1 x_1 \\ \dot{x}_2 = x_3 + x_1\cos x_2^2 + \vartheta_2 x_2 \sin x_1 \\ \dot{x}_3 = u + x_2 x_3^2 + \vartheta_3 \sin x_3 \\ y = x_1 \end{cases} \tag{44}$$

with the initial conditions $x_1 = 0.01$, $x_2 = 0.01$, and $x_3 = 0.01$. The reference signal is defined as $y_d = 0.5\sin t$.

According to Theorem 2, the intermediate control signals, the actual control law, and the adaptive laws are defined as

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} S_i(z_i) - \hat{\vartheta}_i\varphi_i(x_i) \; i = 1, 2$$

$$u = -k_3 z_3 - \hat{\boldsymbol{\theta}}_3^{\mathrm{T}} S_3(z_3) - \hat{\vartheta}_3\varphi_3(x_3)$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i S_{m_i}(z_n) - \eta_i\hat{\boldsymbol{\theta}}_i \; i = 1, 2, 3$$

$$\dot{\hat{\vartheta}}_i = z_i\varphi_i(x_i) - \gamma_i\hat{\vartheta}_i \; i = 1, 2, 3$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $z_3 = x_3 - \alpha_2$ and $z_1 = [z_1]^{\mathrm{T}}$, $z_2 = [z_1, z_2]^{\mathrm{T}}$, $z_2 = [z_1, z_2, z_3]^{\mathrm{T}}$.

The designed parameters are selected as $k_1 = 34, k_2 = 16, k_3 = 60$, $\eta_1 = 6, \eta_2 = 2, \eta_3 = 0.1$, $\gamma_1 = 2$, $\gamma_2 = 1, \gamma_3 = 1$. The highest power of the polynomial $S_{m_i}(z_i)$ of MTN is that $m_1 = 2, m_2 = 3, m_3 = 3$. The simulation results are shown in Figures 1–6, respectively. Figure 1 describes the trajectories of system output y and reference signal y_d. Figure 2 shows the trajectory of tracking error $y - y_d$. Figure 3 depicts the trajectory of the control signal u and Figure 4 illustrates the state variables x_2 and x_3 are bounded. Figures 5–6 depicts that the adaptive parameters $\|\hat{\theta}_i\|, i = 1, 2, 3$ and $\left\|\hat{\vartheta}_i\right\|, i = 1, 2, 3$ are bounded. The above simulations results conclude that the control approach proposed in this paper is effective.

Example 2: To further illustrate the effectiveness of the proposed method, another nonlinear system with unknown parameters is considered as follows

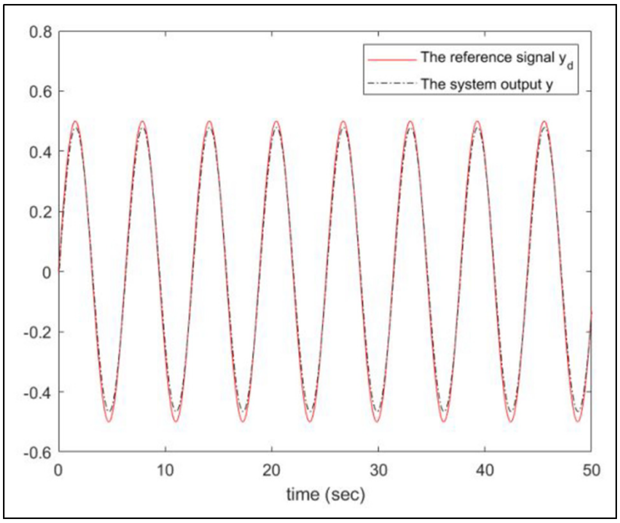

Figure 1. The trajectories of system output y and reference signal y_d.

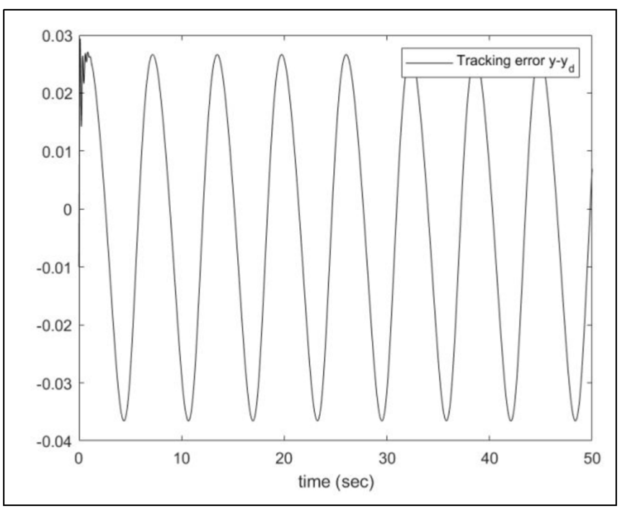

Figure 2. The trajectory of tracking error $y - y_d$.

$$\begin{cases} \dot{x}_1 = x_2 + x_1 \sin x_1 + 0.3\vartheta_1 \sin x_1 x_2 \\ \dot{x}_2 = u + x_2^3 + 0.2\vartheta_2 x_1 x_2 \\ y = x_1 \end{cases} \tag{45}$$

with the initial conditions $x_1 = 0.01$ and $x_2 = 0.01$. The reference signal is defined as $y_d = 0.5(\sin t + \sin 0.5t)$.

Similarly, according to Theorem 2, the intermediate control signals, the actual control law, and the adaptive laws are defined as

$$\alpha_1 = -k_1 z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}} S_1(z_1) - \hat{\vartheta}_1 \varphi_1(x_1)$$

$$u = -k_2 z_2 - \hat{\boldsymbol{\theta}}_2^{\mathrm{T}} S_2(z_2) - \hat{\vartheta}_2 \varphi_2(x_2)$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i S_{m_i}(z_n) - \eta_i \hat{\boldsymbol{\theta}}_i \ i = 1, 2$$

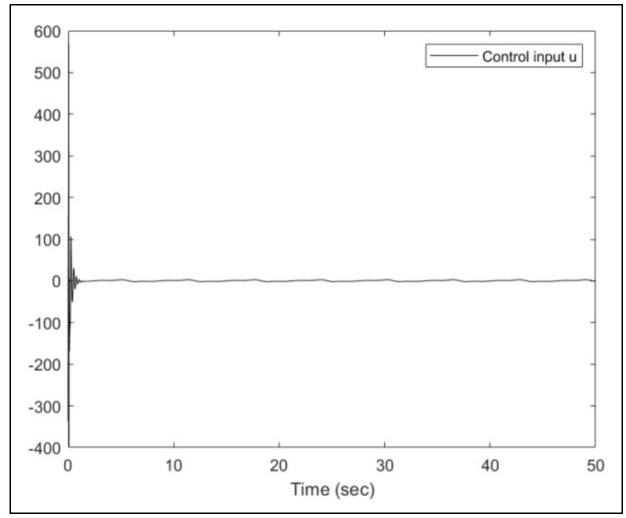

Figure 3. The trajectory of control signal u.

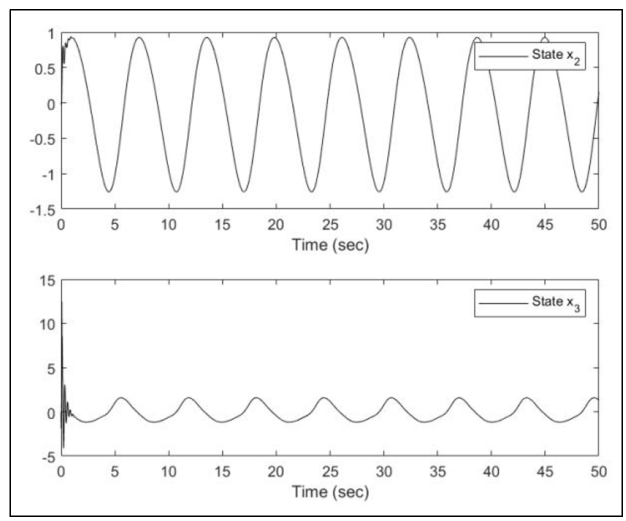

Figure 4. The trajectories of state variables x_2 and x_3.

$$\dot{\hat{\vartheta}}_i = z_i \varphi_i(x_i) - \gamma_i \hat{\vartheta}_i \ i = 1, 2$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$ and $z_1 = [z_1]^{\mathrm{T}}, z_2 = [z_1, z_2]^{\mathrm{T}}$.

The designed parameters are selected as $k_1 = 40, k_2 = 40$, $\eta_1 = 10, \eta_2 = 6, \ \gamma_1 = 2, \gamma_2 = 2$. The highest power of the polynomial $S_{m_i}(z_i)$ of MTN is that $m_1 = 2, m_2 = 3$. Figures 7–12 provide the simulation results. It can be observed that the output y can well track the reference signal y_d form Figure 7 and Figure 8. In Figure 9, it can be seen that the control signal u is smooth and reasonable. Figures 10–12 indicate that the state x_2 and the adaptive parameters $\|\hat{\theta}_i\|, i = 1, 2, \|\hat{\vartheta}_i\|, i = 1, 2$ are bounded.

Example 3: To illustrate the superiority of the proposed method, a contrasting experiment has been done in this section. Then, the system and all the control parameters are kept as that in Example 1. Moreover, the radial basis function

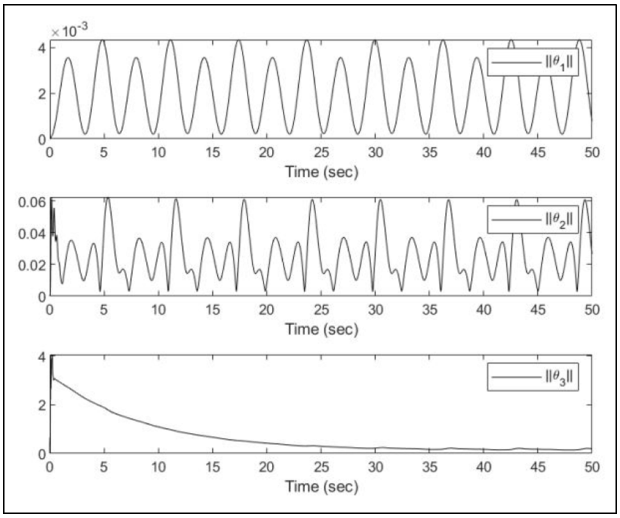

Figure 5. The trajectories of adaptive parameters $\|\hat{\theta}_1\|$, $\|\hat{\theta}_2\|$ and $\|\hat{\theta}_3\|$.

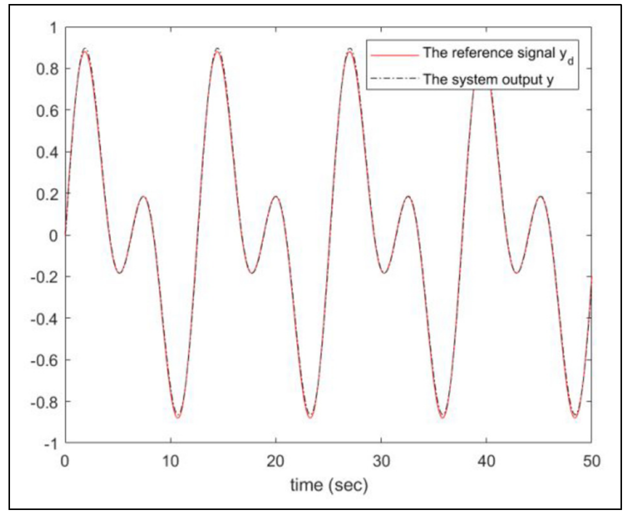

Figure 7. The trajectories of system output y and reference signal y_d.

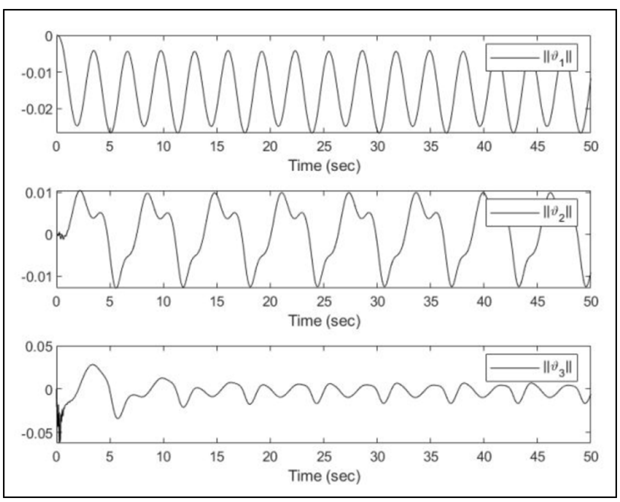

Figure 6. The trajectories of adaptive parameters $\|\hat{\vartheta}_1\|$, $\|\hat{\vartheta}_2\|$ and $\|\hat{\vartheta}_3\|$.

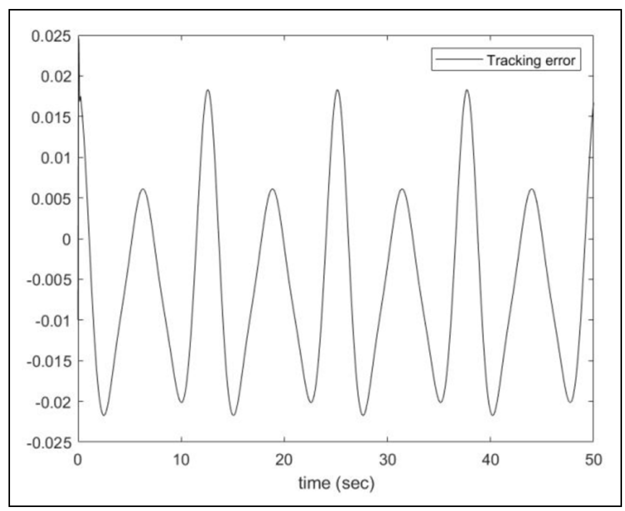

Figure 8. The trajectory of tracking error $y - y_d$.

neural network (RBFNN) has been utilized instead of MTN as approximation function in the simulation.

Figure 13 shows the tracking performance of the two control strategies, where y_d represents the given reference signal, y_{MTN} means the control output under the MTN control method proposed in this paper and y_{RBFNN} is the control output under RBFNN control method. Figure 14 displays absolute value of convergence errors of two control strategies. We can see that the tracking error of our proposed method is slightly smaller than the traditional RBF control method in this system. Therefore, we can draw the following conclusion that though both MTN-based controller and RBFNN-based controller can well realize the tracking control, the former promises more satisfactory performance than the latter in this system. Contrasting experiment results further verify the effectiveness of our proposed method.

Based on the simulation results in Example 1, Example 2 and Example 3, we can observe that the good tracking performances can be achieved by suitably choosing the design parameters. Generally speaking, in the presence of unknown parameters of nonlinear system, the accurate tracking performance can be guaranteed by the adaptive MTN control proposed in this paper.

Conclusion

By employing MTN and backstepping technology, an adaptive tracking control scheme has been proposed in this paper. Additionally, through introducing two adaptive laws, a MTN-based adaptive controller has been constructed. The method proposed in this paper can be applied to a class of nonlinear systems with unknown parameters. As MTN has a

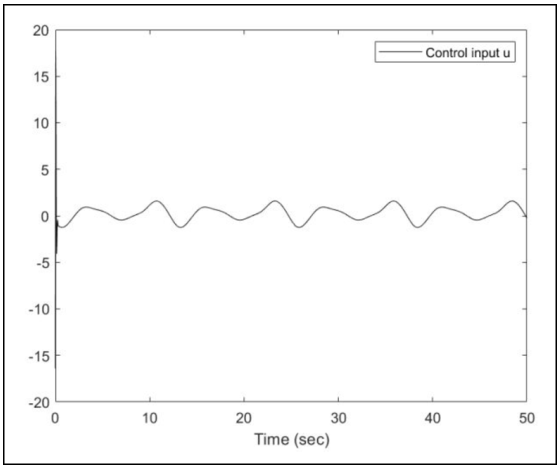

Figure 9. The trajectory of control signal u.

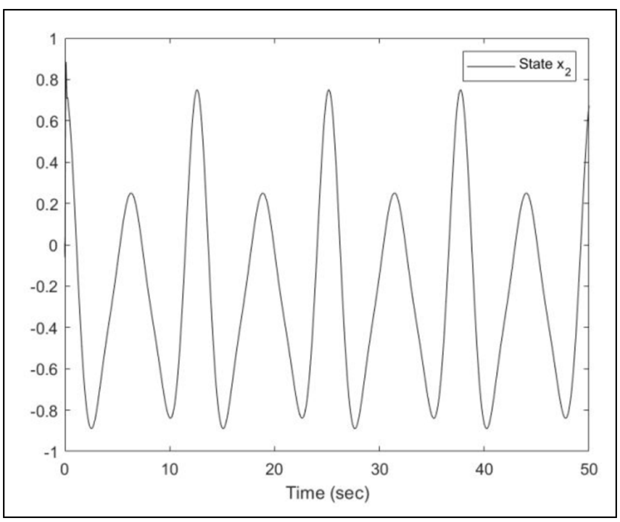

Figure 10. The trajectory of state variable x_2

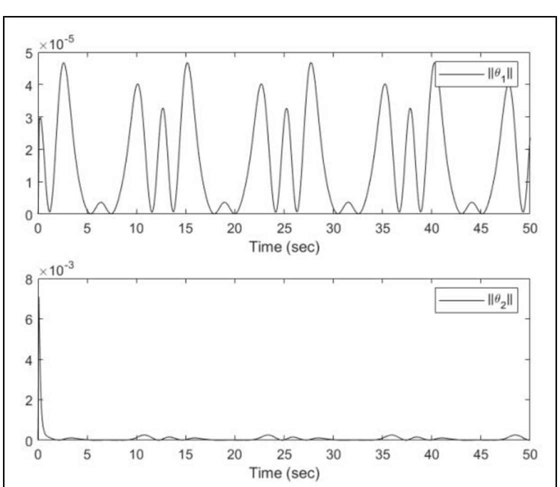

Figure 11. The trajectories of adaptive parameters $\|\hat{\theta}_1\|$ and $\|\hat{\theta}_2\|$.

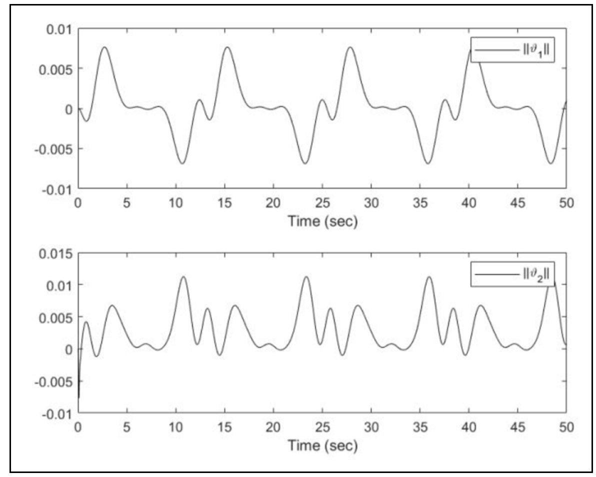

Figure 12. The trajectories of adaptive parameters $\|\hat{\vartheta}_1\|$ and $\|\hat{\vartheta}_2\|$.

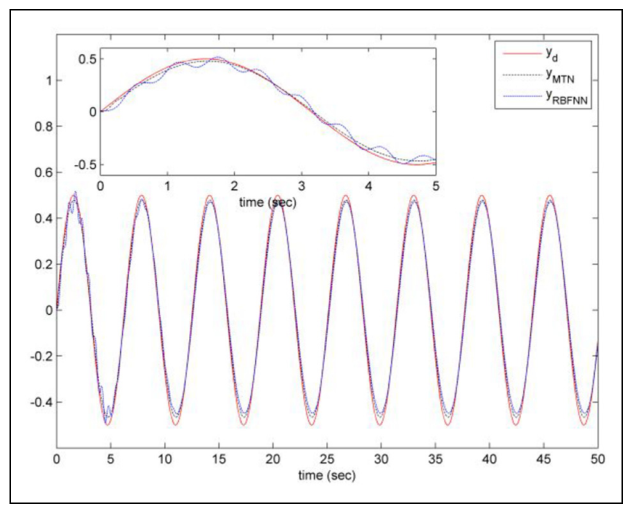

Figure 13. The comparison trajectories of two control strategies.

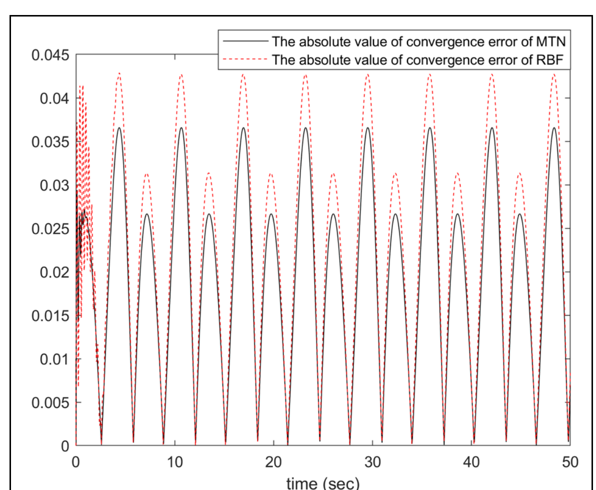

Figure 14. The absolute value of convergence errors of two control strategies.

simple structure, the computation burden can be reduced accordingly; therefore, it is convenient to implement this algorithm in practical systems. Finally, three simulation examples illustrate the effectiveness of the proposed approach. Furthermore, the offered method can also be used to solve switching systems and MIMO systems with unknown parameters.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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