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Adaptive Tracking Control of a Class of Nonlinear Systems with Input Delay and Dynamic Uncertainties Using Multi-dimensional Taylor Network

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Abstract: This paper concerns on the control problem of a class of nonlinear systems with input delay and dynamic uncertainties using multi-dimensional Taylor Network (MTN) control method. Firstly, a new variable is introduced to eliminate the effect of input delay by combining Padé approximation with Laplace transformation. Secondly, a MTN-backstepping-based control strategy is constructively designed by introducing a new coordinate transformation, and the proposed controller has the advantages of simple structure and good real-time performance. Finally, the effectiveness of the proposed MTN-based control approach is demonstrated by three examples.

Keywords: Adaptive control, dynamic uncertainties, input delay, multi-dimensional Taylor network, nonlinear systems.

1. INTRODUCTION

It is well known that the control of nonlinear systems is an important branch of cybernetics, and many controlling methodologies have been developed and applied widely, such as feedback linearization [1–4], backstepping [5], sliding mode control [6–9], and adaptive control [10], linear matrix inequality (LMI) [11,12] and so on. Over the past two decades, some intelligent techniques have been proposed to meet the demands of high control accuracy, for example, neural networks (NNs) [13,14], fuzzy logic systems (FLSs) [15–17] and multi-dimensional Taylor networks (MTNs) [18–22]. Among these control methods, MTN-based control approach has received much attention, and gained many significant results. Especially, combining MTN with adaptive control technique, Han et al. [23] first proposed a MTN-based control strategy to solve the tracking control problem of nonlinear systems. And then, this control method was generalized to structure tracking control schemes for stochastic nonlinear systems [24–27], which has yielded a good result. Furthermore, the results mentioned above indicated that MTN-based controller has the advantages of simple structure (the information processing layer is composed of polynomials), good real-time control capability, and strong study ability, which is better for practical use.

On the one hand, unmodelled dynamics frequently exist in the actual systems, and their existence causes a serious threat to the stability of system [28]. Therefore, it is a very meaningful topic to discuss the stable problem of nonlinear system with unmodelled dynamics. Hu [29] developed a tracking control strategy for a class of multi-agent systems with unmodelled dynamics. Authors in [30] presented a novel NN-based control method for uncertain non-affine nonlinear systems with unmodelled dynamics. Authors in [31,32] studied the control problem for time-varying delays nonlinear systems with unmodelled dynamics. Authors in [33] proposed a FLSs-based control strategy to solve the problem of decentralized fault-tolerant control for a class of nonlinear large-scale systems with unmodelled dynamics and actuator failures. Authors in [34] proposed an observer-NN-based adaptive control scheme to address the tracking control problem for a class of nonlinear systems with dead-zone and unmodelled dynamics. Although many important results for nonlinear systems with unmodelled dynamics have been obtained, there are still many problems that require further study and exploration. For example, i) reducing the complexity, and then improving the real-time performance of controller; ii) propose a novel control strategy with the advantage of simply and effective. These inspire our research.

On the other hand, recently, nonlinear systems with input constraints have received increasing attentions, such as nonlinear systems with input dead-zone [35,36], non-

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linear systems with input saturation [37–39]. Specially, more and more attention has been focused on nonlinear systems with input delay since the fact that delay often occurs [40,41]. To the author's knowledge, there few efforts have been committed to the tracking control for nonlinear systems with dynamic uncertainties and input delay. Therefore, this paper devotes to study the control problem for nonlinear systems with input delay and dynamic uncertainties in a unified framework, which is a task full of applied meaning and practical value.

Based on the aforementioned discussion, this paper tries to discuss the control problem for a class of nonlinear system with input delay and dynamic uncertainties. With the help of approximation performance of MTN, a new adaptive MTN controller is proposed. It is shown that the control method proposed in this paper is feasible, namely, the system output tracks the given reference signal and all the signals in the closed-loop system are bounded. The main contributions of this paper can be summarised as follows:i) MTN control technique is first applied to the problem of nonlinear systems with input delay and dynamic uncertainties, and a novel MTN-based controller design approach is developed with the help of backstepping technique. Although in [23,42], adaptive MTN-based approaches for nonlinear systems have been proposed, they are only investigated the nonlinear systems without considering the existence of input delay. Authors in [43] only addressed the tracking control problem for nonlinear systems with input delay without dynamic uncertainties. The mentioned above adaptive control approaches cannot solve the input delay and dynamic uncertainties problems simultaneously. ii) For a class of nonlinear systems, the results of this paper take into consideration of input delay and dynamic uncertainties. Specifically, on the one hand, utilizing the techniques of Padé approximation and Laplace transformation, the input delay of nonlinear system is eliminated by introducing a new variable. On the other hand, a novel coordinate transformation is adopted in the process of controller design. iii) In view of the excellent characteristics of MTN, such as simple structure and small amount of calculation, the control scheme designed in this paper has the advantages of simple structure, good tracking performance and real-time performance, and then has broad application prospect.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. System statements

In this paper, the following nonlinear systems with dynamic uncertainties and input delay:

$$\begin{cases}\dot{\xi}=q(\xi,x),\\ \dot{x}_i=x_{i+1}+f_i(\bar{x}_i)+\Delta_i(x,\xi),\ 1\le i\le n-1,\\ \dot{x}_n=u(t-\tau)+f_n(\bar{x}_n)+\Delta_n(x,\xi),\\ y=x_1,\end{cases}\tag{1}$$

where $\xi\in\mathbb{R}^{n_0}$ denotes the unmeasured portion of the state and the ξ-dynamics in (1) is unmodelled dynamics. $u\in\mathbb{R}$ is the control input , $y\in\mathbb{R}$ is the system output, and $x=[x_1,\cdots,x_n]^{\mathrm{T}}\in\mathbb{R}^n$ is the system state vector with $\bar{x}_i=[x_1,\cdots,x_i]^{\mathrm{T}}\in\mathbb{R}^i$. $\Delta_i(x,\xi)$ denotes the dynamic disturbances, and $\Delta_i(x,\xi)$ and $q(\xi,x)$ are assumed to be uncertain Lipschitz continuous functions. τ denotes the input delay. $f_i(\cdot)$ are unknown smooth functions with $f_i(0)=0$.

For a given reference signal y_d, this paper tries to design an MTN-based controller for system (1), which can realize the objectives of y follows y_d and all the signals in the closed-loop are bounded as well.

2.2. The transform of input delay

According to [40], the following lemma is holds.

Lemma 1: For $u(t-\tau)$, there exist a new variable x_{n+1} and a constant $\gamma=\frac{2}{\tau}$, such that

$$\dot{x}_{n+1}=-\gamma x_{n+1}+2\gamma u(t).\tag{2}$$

Proof: The idea of proof for Lemma 1 can be found in [40], also as follows:

$$\mathcal{L}\{u(t-\tau)\}=\frac{\exp(-\tau v/2)}{\exp(\tau v/2)}\mathcal{L}\{u(t)\},\tag{3}$$

where v denotes the Laplace variable and $\mathcal{L}\{u(t)\}$ denotes the Laplace transformation of $u(t)$. According to Padé approximation, we have

$$\frac{\exp(-\tau v/2)}{\exp(\tau v/2)}\mathcal{L}\{u(t)\}\approx\frac{1-\tau v/2}{1+\tau v/2}\mathcal{L}\{u(t)\}.\tag{4}$$

Next, introduced a new variable, denote as x_{n+1}, and it satisfies the following equation

$$\frac{1-\tau v/2}{1+\tau v/2}\mathcal{L}\{u(t)\}=\mathcal{L}\{x_{n+1}(t)\}-\mathcal{L}\{u(t)\}.\tag{5}$$

According to (5), the following equation holds

$$u(t)-\frac{\tau\dot{u}(t)}{2}=x_{n+1}+\frac{\tau\dot{x}_{n+1}}{2}-u(t)-\frac{\tau\dot{u}(t)}{2}.\tag{6}$$

Let $\gamma=\frac{2}{\tau}$, (6) can be rewritten as (2). □

Remark 1: Combining (3) with (5), one has

$$x_{n+1}(t)-u(t)=u(t-\tau).\tag{7}$$

Remark 2: It should be noted that the premise of Lemma 1 is the input delay τ is relatively small. The main reason for this lies in Padé approximation has some limitations in handling large delay [40].

Remark 3: Substituting (7) into system (1) and taking (2) into account, system (1) can be expressed as follows:

$$\begin{cases} \dot{\xi} = q(\xi, x) \\ \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + \Delta_i(x,\xi), 1 \le i \le n-1 \\ \dot{x}_n = x_{n+1} - u + f_n(\bar{x}_n) + \Delta_n(x,\xi) \\ \dot{x}_{n+1} = -\gamma x_{n+1} + 2\gamma u \\ y = x_1 \end{cases} \tag{8}$$

Based on the new transformed system (8), a new MTN-based control strategy for system (1) will be given in the following section.

2.3. Hypothesis and lemmas

Following are some of the Assumptions and Lemmas used in this article.

Assumption 1: The reference signal y_d and $y_d^{(i)}$ are continuous and bounded, where $y_d^{(i)}$ is the ith derivative of y_d with $i = 1, \cdots, n$.

Assumption 2 [44]**:** In the system (1), $q(\xi, x)$ and $\Delta_i(x, \xi)$ are uncertain Lipschitz continuous functions.

Assumption 3: For each $i = 1, \cdots, n$, the dynamic disturbance $\Delta_i(x, \xi)$ satisfies the following inequality

$$|\Delta_i(x,\xi)| \le \varphi_{i,1}(|\bar{x}_i|) + \varphi_{i,2}(|\xi|), \tag{9}$$

where $\varphi_{i,1}(\cdot)$ and $\varphi_{i,2}(\cdot)$ are unknown non-negative increasing smooth functions and $\varphi_{i,2}(\cdot)$ satisfies $\varphi_{i,2}(0) = 0$.

Assumption 4 [28,44]**:** For the $\dot{\xi} = q(\xi, x)$, there exist three κ_∞ functions κ_1, κ_2 and κ_3, and an exponentially input-to-state practically stable (exp-ISpS) Lyapunov function $V(\xi)$, such that

$$\begin{cases} \kappa_1(|\xi|) \le V(\xi) \le \kappa_2(|\xi|), \\ \dfrac{\partial V(\xi)}{\partial \xi} q(\xi, x) \le -c_0 V(\xi) + \kappa_3(|x|) + d_0, \end{cases} \tag{10}$$

where $c_0 > 0$ and $d_0 > 0$ are known constants.

Lemma 2 [28,44]**:** For the $\dot{\xi} = q(\xi, x)$ and the initial value $\xi_0 = \xi(0)$, there exist an exp-ISpS Lyapunov function $V(\xi)$, a function $\bar{\kappa}_3(\cdot)$, a limited time $T_0 = T_0(\bar{c}, \zeta_0, \xi_0)$, and a signal ζ expressed by

$$\dot{\zeta} = -\bar{c}\zeta + \bar{\kappa}_3(x_1) + d_0, \ \zeta(0) = \zeta_0, \tag{11}$$

such that

$$V(\xi(t)) \le \zeta(t) + H(t), \tag{12}$$

where $H(t)$, $t \ge 0$ is a nonnegative function and satisfies $H(t) = 0$, $\forall t \ge T_0$, $\bar{\kappa}_3(\cdot)$ satisfies $\bar{\kappa}_3(x_1) \ge \kappa_3(|x_1|)$, constants ζ_0 and c_0 satisfy $\zeta_0 > 0$ and $\bar{c} \in (0, c_0)$.

Assumption 5: Assume function $\kappa(\cdot)$ is non-negative smooth and $\bar{\kappa}_3(\cdot)$ satisfies $\bar{\kappa}_3(s) = (s - y_d)^2 \kappa(s^2)$.

Remark 4: According to Assumption 5, $\bar{\kappa}_3(\cdot)$ is a smooth function and satisfies $\bar{\kappa}_3(0) \ge 0$. In addition, according to Lemma 2 and Assumption 5, we have

$$\dot{\zeta} = -\bar{c}\zeta + (x_1 - y_d)^2 \kappa\left(|x_1|^2\right) + d_0, \ \zeta(0) = \zeta_0. \tag{13}$$

Lemma 3 [28]**:** For any given $x \in \mathbb{R}$ and $\tilde{\varepsilon} > 0$, there exists a function $f_x(x)$ satisfies $|x| \le x f_x(x) + \tilde{\varepsilon}$, where $f_x : \mathbb{R} \to \mathbb{R}$ is a smooth function with $f_x(0) = 0$.

Lemma 4 [28]**:** For any continuous function $f(\cdot) : \mathbb{R} \to \mathbb{R}$ satisfies $f(0) = 0$, the following inequality holds:

$$|f(x)| \le \hat{f}(x) + \hat{\varepsilon}, \ \ \forall x \in \mathbb{R}, \tag{14}$$

where $\hat{\varepsilon} > 0$ is a constant and $\hat{f}(\cdot)$ is a nonnegative smooth function satisfies $\hat{f}(0) = 0$ and $\left.\frac{\partial \hat{f}}{\partial x}\right|_{x=0} = 0$.

In this paper, MTN is used to approach the unknown function of nonlinear system, the details of MTN were introduced in [25,27], no more expatiations it here except to give the following Lemma.

Lemma 5 [25,27]**:** A continuous function $f(s)$ over on a compact set Ω_s can be approximated by a MTN with the form of $\theta^{\mathrm{T}} P_{m_n}(s)$ by any precision, namely,

$$f(s) = \theta^{\mathrm{T}} P_{m_n}(s) + \delta(s) \tag{15}$$

where n and m are the input number and the highest power of the middle layer of MTN, respectively. $s = [s_1, \cdots, s_n]^{\mathrm{T}} \in \Omega_s \subset \mathbb{R}^n$ and $\theta = [\theta_1, \cdots, \theta_l]^{\mathrm{T}} \in \mathbb{R}^l$ are the input vector and weight vector of MTN, respectively. $P_{m_n}(s) = \left[s_1, \cdots, s_n, s_1^2, \cdots, s_n^2, \cdots, s_1^m, \cdots, s_n^m\right]^{\mathrm{T}} \in \mathbb{R}^l$, and $\delta(s)$ is estimated error and satisfies $|\delta(s)| \le \varepsilon$, for $\forall \varepsilon > 0$.

3. MAIN RESULTS

First of all, define the coordinate transformation as following

$$\begin{aligned} z_1 &= x_1 - y_d, \\ z_i &= x_i - \alpha_{i-1}, \ \ 2 \le i \le n-1, \\ z_n &= x_n - \alpha_{n-1} + \frac{1}{\gamma} x_{n+1}. \end{aligned} \tag{16}$$

Substituting (16) into (8), we have

$$\begin{cases} \dot{\xi} = q(\xi, x), \\ \dot{z}_1 = x_2 + f_1(\bar{x}_1) - \dot{y}_d + \Delta_1(x,\xi), \\ \dot{z}_i = x_{i+1} + f_i(\bar{x}_i) - \dot{\alpha}_{i-1} + \Delta_i(x,\xi), \\ \qquad 2 \le i \le n-1, \\ \dot{z}_n = u + f_n(\bar{x}_n) + \Delta_n(x,\xi) - \dot{\alpha}_{n-1}. \end{cases} \tag{17}$$

Remark 5: In the following, for the sake of convenience in writing, $f_i(\bar{x}_i)$ is abbreviated as f_i, $\Delta_i(x,\xi)$ is abbreviated as Δ_i, $P_{m_i}(z_i)$ is abbreviated as P_{m_i}.

3.1. Controller design

Step 1: According to (17), we have

$$\begin{aligned}\dot{\xi} &= q(\xi, x), \\ \dot{z}_1 &= x_2 + f_1 + \Delta_1 - \dot{y}_d.\end{aligned} \tag{18}$$

Choose the Lyapunov function as

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{\lambda_0}\zeta + \frac{1}{2}\tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_1, \tag{19}$$

where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ is the parameter error, $\lambda_0 > 0$ and $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$ are design parameter and constant matrix, respectively.

Combining (18) and (19), according to Assumption 3 and Lemma 2, one has

$$\begin{aligned}\dot{V}_1 \le & z_1(x_2 + f_1 - \dot{y}_d) + |z_1|\varphi_{1,1}(|\bar{x}_1|) \\ & + |z_1|\varphi_{1,2}(|\xi|) + \frac{1}{\lambda_0}\left(z_1^2\kappa\left(|x_1|^2\right) + d_0\right) \\ & - \frac{\bar{c}}{\lambda_0}\zeta - \tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\theta}}_1,\end{aligned} \tag{20}$$

where $\varphi_{1,1}$ and $\varphi_{1,2}$ are non-negative increasing smooth functions.

On the one hand, for the term $|z_1|\varphi_{1,1}(|\bar{x}_1|)$, according to Lemmas 3 and 4, for any given $\varepsilon_{1,1} > 0$, the following inequality can be obtained

$$|z_1|\varphi_{1,1}(|\bar{x}_1|) \le z_1\tilde{\varphi}_{1,1}(\bar{x}_1) + \varepsilon_{1,1}, \tag{21}$$

where $\tilde{\varphi}_{1,1}(\cdot)$ is a smooth function satisfies $\tilde{\varphi}_{1,1}(0) = 0$.

On the other hand, for the term $|z_1|\varphi_{1,2}(|\xi|)$, using the same argument in [28], for any given $\varepsilon_{1,2} > 0$, the following inequality can be obtained

$$|z_1|\varphi_{1,2}(|\xi|) \le z_1\tilde{\varphi}_{1,2}(z_1,\zeta) + 2\varepsilon_{1,2} + \frac{z_1^2}{4} + h_1(t), \tag{22}$$

with $\tilde{\varphi}_{1,2}(z_1,\zeta)$ is a smooth function satisfies $\tilde{\varphi}_{1,2}(z_1,\zeta) = \hat{\varphi}_{1,2}(\zeta)\hat{\varphi}_{1,3}(z_1,\zeta) + \hat{\varphi}_{1,4}(z_1)$, $h_1(t) = \left(\varphi_{1,2}\circ\kappa_1^{-1}(2H(t))\right)^2$, where $\hat{\varphi}_{1,2}(\cdot)$, $\hat{\varphi}_{1,3}(\cdot)$ and $\hat{\varphi}_{1,4}(\cdot)$ are smooth functions satisfy $\hat{\varphi}_{1,2}(0) = 0$, $\hat{\varphi}_{1,3}(0) = 0$ and $\hat{\varphi}_{1,4}(0) = 0$. Meanwhile, for all $t \ge 0$, $h_1(t) \ge 0$ and when $t \ge T_0$, $h_1(t) = 0$.

Substituting (21) and (22) into (20) gives

$$\begin{aligned}\dot{V}_1 \le & z_1\left(x_2 + \bar{f}_1\right) - z_1^2 + h_1(t) - \frac{\bar{c}}{\lambda_0}\zeta \\ & - \tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\theta}}_1 + \bar{\Xi}_1,\end{aligned} \tag{23}$$

where $\bar{\Xi}_1 = \varepsilon_{1,1} + 2\varepsilon_{1,2} + \frac{1}{\lambda_0}d_0$ and $\bar{f}_1 = f_1 - \dot{y}_d + \tilde{\varphi}_{1,1}(\bar{x}_1) + \tilde{\varphi}_{1,2} + \frac{1}{4}z_1 + \frac{1}{\lambda_0}z_1\kappa\left(|x_1|^2\right) + z_1$.

Obviously, unknown functions $\bar{f}_1$ cannot be used directly for controller design, according to Lemma 5, for any given constant $\varsigma_1 > 0$, there exists a MTN $\theta_1^{\mathrm{T}}P_{m_1}(\mathbf{z}_1)$ such that

$$\bar{f}_1 = \theta_1^{\mathrm{T}}P_{m_1}(\mathbf{z}_1) + \delta_1(\mathbf{z}_1), \quad |\delta_1(\mathbf{z}_1)| \le \varsigma_1, \tag{24}$$

where $\mathbf{z}_1 = [z_1]^{\mathrm{T}}$, and $\delta_1(\mathbf{z}_1)$ denotes the estimate error.

Considering that $x_2 = z_2 + \alpha_1$, based on (23) and (24), by Young's inequality, one has

$$\begin{aligned}\dot{V}_1 \le & z_1\alpha_1 + \frac{1}{2}z_2^2 + z_1\theta_1^{\mathrm{T}}P_{m_1} + h_1(t) \\ & - \frac{\bar{c}}{\lambda_0}\zeta - \tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\theta}}_1 + \Xi_1,\end{aligned} \tag{25}$$

where $\Xi_1 = \bar{\Xi}_1 + \frac{1}{2}\varsigma_1^2$.

Choosing the virtual control law α_1 as follows:

$$\alpha_1 = -r_1z_1 - \hat{\theta}_1^{\mathrm{T}}P_{m_1}, \tag{26}$$

where $r_1 > 0$ denotes a design constant.

Combining (25) and (26), one has

$$\begin{aligned}\dot{V}_1 \le & -r_1z_1^2 + \frac{1}{2}z_2^2 - \frac{\bar{c}}{\lambda_0}\zeta + h_1(t) \\ & + \tilde{\theta}_1^{\mathrm{T}}\left(z_1P_{m_1} - \Gamma_1^{-1}\dot{\hat{\theta}}_1\right) + \Xi_1.\end{aligned} \tag{27}$$

Step 2: According to (17), one has $\dot{z}_2 = x_3 + f_2 + \Delta_2 - \dot{\alpha}_1$, where $\dot{\alpha}_1 = \nabla\alpha_{x_1}^{[1]}(x_2 + f_1 + \Delta_1) + \nabla\alpha_{\hat{\theta}_1}^{[1]}\dot{\hat{\theta}}_1 + \sum_{j=0}^{1}\nabla\alpha_{y_d^{(j)}}^{[1]}y_d^{(j+1)} + \nabla\alpha_{\zeta}^{[1]}\dot{\zeta}$, $\nabla\alpha_{x_1}^{[1]} = \frac{\partial\alpha_1}{\partial x_1}$, $\nabla\alpha_{\hat{\theta}_1}^{[1]} = \frac{\partial\alpha_1}{\partial\hat{\theta}_1}$, $\nabla\alpha_{y_d^{(1)}}^{[1]} = \frac{\partial\alpha_1}{\partial y_d^{(1)}}$ and $\nabla\alpha_{\zeta}^{[1]} = \frac{\partial\alpha_1}{\partial\zeta}$.

Choosing the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\theta}_2^{\mathrm{T}}\Gamma_2^{-1}\tilde{\theta}_2, \tag{28}$$

where $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$ denotes the parameter error vector, $\Gamma_2 = \Gamma_2^{\mathrm{T}} > 0$ denotes a constant matrix.

Then, one has the derivative of V_2 with respect to time t as follows:

$$\dot{V}_2 \le \dot{V}_1 + z_2(x_3 + f_2 + \Delta_2 - \dot{\alpha}_1) - \tilde{\theta}_2^{\mathrm{T}}\Gamma_2^{-1}\dot{\hat{\theta}}_2. \tag{29}$$

Denote $\bar{\Delta}_2 = \Delta_2 - \nabla\alpha_{x_1}^{[1]}\Delta_1$, according to Assumption 3 and Lemma 2, one has

$$\begin{aligned}\left|z_2\bar{\Delta}_2\right| \le & |z_2|\left(\varphi_{2,1}(|\bar{x}_2|) + \left|\nabla\alpha_{x_1}^{[1]}\right|\varphi_{1,1}(|\bar{x}_1|)\right) \\ & + |z_2|\left(\varphi_{2,2}(|\xi|) + \left|\nabla\alpha_{x_1}^{[1]}\right|\varphi_{1,2}(|\xi|)\right),\end{aligned} \tag{30}$$

where $\varphi_{2,1}$ and $\varphi_{2,2}$ are non-negative increasing smooth functions.

According to Assumption 3, Lemma 3 and Lemma 4, for any given $\varepsilon_{2,1} > 0$, the following inequality holds

$$|z_2|\left(\varphi_{2,1}(|\bar{x}_2|) + \left|\nabla\alpha_{x_1}^{[1]}\right|\varphi_{1,1}(|\bar{x}_1|)\right) \le z_2\tilde{\varphi}_{2,1} + \varepsilon_{2,1}, \tag{31}$$

where $\tilde{\varphi}_{2,1}(\cdot)$ is a smooth function satisfies $\tilde{\varphi}_{2,1}(0) = 0$.

Using the same argument in [28], for any given $\varepsilon_{1,2} > 0$, the following inequality can be obtained

$$|z_2|\left(\varphi_{2,2}(|\xi|) + \left|\nabla\alpha_{x_1}^{[1]}\right|\varphi_{1,2}(|\xi|)\right) \le z_2\tilde{\varphi}_{2,2} + \frac{1}{4}z_2^2\Pi_1 + 2\cdot 2\varepsilon_{2,2} + h_2(t), \tag{32}$$

where $\Pi_1 = 1 + \sum_{j=1}^{1}\left(\nabla\alpha_{x_j}^{[1]}\right)^2$, $\tilde{\varphi}_{2,2}$ is a smooth function of variables x_1, x_2, ζ and $\hat{\theta}_1$, $h_2(t) = \sum_{j=1}^{2}\left(\varphi_{j,2}\circ\kappa_1^{-1}(2H(t))\right)^2$ satisfies $h_2(t) \ge 0$ for all $t \ge 0$ and when $t \ge T_0$, $h_2(t) = 0$.

Substituting (30), (31) and (32) into (29) gives

$$\dot{V}_2 \le \dot{V}_1 + z_2\left(x_3 + \bar{f}_2\right) - z_2^2 + \varepsilon_{2,1} + 2\cdot 2\varepsilon_{2,2} + h_2(t) - \tilde{\theta}_2^{\mathrm{T}}\Gamma_2^{-1}\dot{\hat{\theta}}_2, \tag{33}$$

where $\Theta_1 = \nabla\alpha_{x_1}^{[1]}(x_2 + f_1) + \nabla\alpha_{\hat{\theta}_1}^{[1]}\dot{\hat{\theta}}_1 + \sum_{j=0}^{1}\nabla\alpha_{y_d^{(j)}}^{[1]}y_d^{(j+1)} + \nabla\alpha_{\zeta}^{[1]}\dot{\zeta}$ and $\bar{f}_2 = f_2 + \tilde{\varphi}_{2,1} + \tilde{\varphi}_{2,2} + \frac{1}{4}z_2\Pi_1 - \Theta_1 + z_2$.

Similarly, unknown functions $\bar{f}_2$ cannot be used directly for controller design, according to Lemma 5, for any given constant $\varsigma_2 > 0$, there exists a MTN $\theta_2^{\mathrm{T}}P_{m_2}(\mathbf{z}_2)$ such that

$$\bar{f}_2 = \theta_2^{\mathrm{T}}P_{m_2}(\mathbf{z}_2) + \delta_2(\mathbf{z}_2), \quad |\delta_2(\mathbf{z}_2)| \le \varsigma_2, \tag{34}$$

where $\mathbf{z}_2 = [z_1, z_2]^{\mathrm{T}}$, and $\delta_2(\mathbf{z}_2)$ denotes the estimate error.

Considering that $x_3 = z_3 + \alpha_2$, based on (33) and (34), by Young's inequality, we have

$$\dot{V}_2 \le \dot{V}_1 + z_2\alpha_2 + \frac{1}{2}z_3^2 + z_2\theta_2^{\mathrm{T}}P_{m_2} + \frac{1}{2}\varsigma_2^2 + \varepsilon_{2,1} + 2\cdot 2\varepsilon_{2,2} + h_2(t) - \tilde{\theta}_2^{\mathrm{T}}\Gamma_2^{-1}\dot{\hat{\theta}}_2. \tag{35}$$

Choosing the virtual control law α_2 as follows

$$\alpha_2 = -r_2z_2 - \hat{\theta}_2^{\mathrm{T}}P_{m_2}, \tag{36}$$

and $r_2 > 0$ denotes a design constant.

Combining (27) with (35) and (36), one has

$$\dot{V}_2 \le -\sum_{j=1}^{2}r_jz_j^2 + \frac{1}{2}\sum_{j=1}^{2}z_{j+1}^2 - \frac{\bar{c}}{\lambda_0}\zeta + \sum_{j=1}^{2}h_j(t) + \sum_{j=1}^{2}\tilde{\theta}_j^{\mathrm{T}}\left(z_jP_{m_j} - \Gamma_j^{-1}\dot{\hat{\theta}}_j\right) + \Xi_2, \tag{37}$$

where $\Xi_2 = \sum_{j=1}^{2}\varepsilon_{j,1} + 2\sum_{j=1}^{2}j\varepsilon_{j,2} + \frac{1}{2}\sum_{j=1}^{2}\varsigma_j^2 + \frac{1}{\lambda_0}d_0$.

Step $3 \le i \le n-1$: Based on Step 1 and Step 2, using the recursion method, the following Lemma can be drawn.

Lemma 6: For each $i = 3, \cdots, n-1$, choose a Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i, \tag{38}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ denotes the parameter error vector, $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ denotes a constant matrix.

Choosing the ith virtual control law α_i as follows:

$$\alpha_i = -r_iz_i - \hat{\theta}_i^{\mathrm{T}}P_{m_i}, \tag{39}$$

then we have

$$\dot{V}_i \le -\sum_{j=1}^{i}r_jz_j^2 + \frac{1}{2}\sum_{j=1}^{i}z_{j+1}^2 - \frac{\bar{c}}{\lambda_0}\zeta + \sum_{j=1}^{i}h_j(t) + \sum_{j=1}^{i}\tilde{\theta}_j^{\mathrm{T}}\left(z_jP_{m_j} - \Gamma_j^{-1}\dot{\hat{\theta}}_j\right) + \Xi_i, \tag{40}$$

where $r_i > 0$ are the design constants and $\Xi_i = \sum_{j=1}^{i}\varepsilon_{j,1} + 2\sum_{j=1}^{i}j\varepsilon_{j,2} + \frac{1}{2}\sum_{j=1}^{i}\varsigma_j^2 + \frac{1}{\lambda_0}d_0$.

Step n: According to (17), we have

$$\dot{z}_n = u + f_n + \Delta_n - \dot{\alpha}_{n-1}, \tag{41}$$

where $\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1}\nabla\alpha_{x_j}^{[n-1]}(x_{j+1} + f_j + \Delta_j) + \sum_{j=1}^{n-1}\nabla\alpha_{\hat{\theta}_j}^{[n-1]}\dot{\hat{\theta}}_j + \sum_{j=0}^{n-1}\nabla\alpha_{y_d^{(j)}}^{[n-1]}y_d^{(j+1)} + \nabla\alpha_{\zeta}^{[n-1]}\dot{\zeta}$ and $\nabla\alpha_{x_j}^{[n-1]} = \frac{\partial\alpha_{n-1}}{\partial x_j}$, $\nabla\alpha_{\hat{\theta}_j}^{[n-1]} = \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}$, $\nabla\alpha_{y_d^{(j)}}^{[n-1]} = \frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}$ for $i = 3, \cdots, n-1$.

Select the Lyapunov function candidate as follows:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{\theta}_n^{\mathrm{T}}\Gamma_n^{-1}\tilde{\theta}_n, \tag{42}$$

where $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$ denotes the parameter error vector, $\Gamma_n = \Gamma_n^{\mathrm{T}} > 0$ denotes a constant matrix.

Then, one can get the derivative of V_n with respect to time t as follows:

$$\dot{V}_n \le \dot{V}_{n-1} + z_n\left(u + f_n + \bar{\Delta}_n - \Theta_{n-1}\right) - \tilde{\theta}_n^{\mathrm{T}}\Gamma_n^{-1}\dot{\hat{\theta}}_n, \tag{43}$$

where $\Theta_{n-1} = \sum_{j=1}^{n-1}\nabla\alpha_{x_j}^{[n-1]}(x_{j+1} + f_j) + \sum_{j=1}^{n-1}\nabla\alpha_{\hat{\theta}_j}^{[n-1]}\dot{\hat{\theta}}_j + \sum_{j=0}^{n-1}\nabla\alpha_{y_d^{(j)}}^{[n-1]}y_d^{(j+1)} + \nabla\alpha_{\zeta}^{[n-1]}\dot{\zeta}$ and $\bar{\Delta}_n = \Delta_n - \sum_{j=1}^{n-1}\nabla\alpha_{x_j}^{[n-1]}\Delta_j$.

Repeating the procedure (30), (31) and (32) in Step 2, we have

$$\left|z_n\bar{\Delta}_n\right| \le |z_n|\left(\varphi_{n,1}(|\bar{x}_n|) + \sum_{j=1}^{n-1}\left|\nabla\alpha_{x_j}^{[n-1]}\right|\varphi_{j,1}\right) + |z_n|\left(\varphi_{n,2}(|\xi|) + \sum_{j=1}^{n-1}\left|\nabla\alpha_{x_j}^{[n-1]}\right|\varphi_{j,2}\right), \tag{44}$$

$$|z_n|\left(\varphi_{n,1}(|\bar{x}_n|) + \sum_{j=1}^{n-1}\left|\nabla\alpha_{x_j}^{n-1}\right|\varphi_{j,1}\right) \le z_n\tilde{\varphi}_{n,1} + \varepsilon_{n,1}, \tag{45}$$

$$
\begin{aligned}
&|z_n|\left(\varphi_{n,2}(|\xi|)+\left|\nabla\alpha_{x_j}^{n-1}\right|\varphi_{j,2}\right)\\
&\leq z_n\tilde{\varphi}_{n,2}+\frac{1}{4}z_n^2\Pi_{n-1}+2n\varepsilon_{n,2}+h_n(t),
\end{aligned}\tag{46}
$$

where $\Pi_{n-1}=1+\sum_{j=1}^{n-1}\left(\nabla\alpha_{x_j}^{[n-1]}\right)^2$, $\tilde{\varphi}_{n,2}$ is a smooth function of variables $x_1,\cdots,x_n$, ζ, and $\hat{\theta}_1,\cdots,\hat{\theta}_{n-1}$, $\varphi_{j,1}=\varphi_{j,1}(|\bar{x}_j|)$, $\varphi_{j,2}=\varphi_{j,2}(|\xi|)$. $h_n(t)=\sum_{j=1}^{n}\left(\varphi_{j,2}\circ\kappa_1^{-1}(2H(t))\right)^2$ satisfies $h_n(t)\geq 0$ for all $t\geq 0$ and when $t\geq T_0$, $h_n(t)=0$.

Substituting (44), (45) and (46) into (43), we have

$$
\begin{aligned}
\dot{V}_n\leq&\dot{V}_{n-1}+z_n\left(u+\bar{f}_n\right)-\frac{1}{2}z_n^2+\varepsilon_{n,1}\\
&+2n\varepsilon_{n,2}+h_n(t)-\tilde{\theta}_n^{\mathrm{T}}\Gamma_n^{-1}\dot{\hat{\theta}}_n,
\end{aligned}\tag{47}
$$

where $\bar{f}_n=f_n-\Theta_{n-1}+\tilde{\varphi}_{n,1}+\tilde{\varphi}_{n,2}+\frac{1}{4}z_n\Pi_{n-1}+\frac{1}{2}z_n$.

Similarly, unknown functions $\bar{f}_n$ cannot be used directly for controller design, according to Lemma 5, for any given constant $\varsigma_n>0$, there exists a MTN $\theta_n^{\mathrm{T}}P_{m_n}(\mathbf{z}_n)$ such that

$$
\bar{f}_n=\theta_n^{\mathrm{T}}P_{m_n}(\mathbf{z}_n)+\delta_n(\mathbf{z}_n),\quad|\delta_n(\mathbf{z}_n)|\leq\varsigma_n,\tag{48}
$$

where $\mathbf{z}_n=[z_1,z_2,\cdots,z_n]^{\mathrm{T}}$, and $\delta_n(\mathbf{z}_n)$ denotes the estimate error.

By Young's inequality, the following inequality holds

$$
z_n\bar{f}_n(z_n)\leq z_n\theta_n^{\mathrm{T}}P_{m_n}+\frac{1}{2}z_n^2+\frac{1}{2}\varsigma_n^2.\tag{49}
$$

Design the actual control input u as follows:

$$
u=-r_nz_n-\hat{\theta}_n^{\mathrm{T}}P_{m_n},\tag{50}
$$

where $r_n>0$ denotes a design constant.

Combining (40), (47) with (49) and (50), one has

$$
\begin{aligned}
\dot{V}_n\leq&-\sum_{j=1}^{n}r_jz_j^2+\frac{1}{2}\sum_{j=1}^{n-1}z_{j+1}^2-\frac{\bar{c}}{\lambda_0}\zeta+\sum_{j=1}^{n}h_j(t)\\
&+\sum_{j=1}^{n}\tilde{\theta}_j^{\mathrm{T}}\left(z_jP_{m_j}-\Gamma_j^{-1}\dot{\hat{\theta}}_j\right)+\Xi_n,
\end{aligned}\tag{51}
$$

where $\Xi_n=\sum_{j=1}^{n}\varepsilon_{j,1}+2\sum_{j=1}^{n}j\varepsilon_{j,2}+\frac{1}{2}\sum_{j=1}^{n}\varsigma_j^2+\frac{1}{\lambda_0}d_0$.

3.2. Stability analysis

Theorem 1: Under Assumptions 1-5, for the nonlinear system with input-delay (1), if a control law is chosen as (50) with the intermediate virtual control signals described as (26), (36) and (39), the adaptive laws defined as

$$
\dot{\hat{\theta}}_i=z_i\Gamma_iP_{m_i}-\eta_i\Gamma_i\hat{\theta}_i,\ i=1,\cdots,n,\tag{52}
$$

where $\eta_i>0$ are designed parameters. Then, for bounded initial conditions, we can draw the conclusion that the system output y follows a given reference signal y_d, and all the signals in the closed-loop are bounded as well.

Proof: According to the design process of the controller, the Lyapunov function is chosen as follows:

$$
V_n=\frac{1}{2}\sum_{i=1}^{n}z_i^2+\frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i+\frac{1}{\lambda_0}\zeta.\tag{53}
$$

According to (51), we have

$$
\begin{aligned}
\dot{V}_n\leq&-\sum_{j=1}^{n}r_jz_j^2+\frac{1}{2}\sum_{j=1}^{n-1}z_{j+1}^2-\frac{\bar{c}}{\lambda_0}\zeta+\sum_{j=1}^{n}h_j(t)\\
&+\sum_{j=1}^{n}\tilde{\theta}_j^{\mathrm{T}}\left(z_jP_{m_j}-\Gamma_j^{-1}\dot{\hat{\theta}}_j\right)+\Xi_n.
\end{aligned}\tag{54}
$$

According to (52), the following inequality holds:

$$
\begin{aligned}
&\sum_{j=1}^{n}\tilde{\theta}_j^{\mathrm{T}}\left(z_jP_{m_j}-\Gamma_j^{-1}\dot{\hat{\theta}}_j\right)\\
&\leq-\sum_{j=1}^{n}\bar{\eta}_j\tilde{\theta}_j^{\mathrm{T}}\Gamma_j^{-1}\tilde{\theta}_j+\frac{1}{2}\sum_{j=1}^{n}\eta_j\|\theta_j\|^2,
\end{aligned}\tag{55}
$$

where $\bar{\eta}_j=\frac{\eta_j}{2\lambda_{\max}\left(\Gamma_j^{-1}\right)}$.

Substituting (55) into (54), the following holds:

$$
\begin{aligned}
\dot{V}_n\leq&-\sum_{j=1}^{n}c_jz_j^2-\frac{\bar{c}}{\lambda_0}\zeta+\sum_{j=1}^{n}h_j(t)\\
&-\sum_{j=1}^{n}\bar{\eta}_j\tilde{\theta}_j^{\mathrm{T}}\Gamma_j^{-1}\tilde{\theta}_j+\frac{1}{2}\sum_{j=1}^{n}\eta_j\|\theta_j\|^2+\Xi_n,
\end{aligned}\tag{56}
$$

where $c_j=\begin{cases}r_1, & j=1,\\ r_j-\frac{1}{2}, & j=2,\cdots,n.\end{cases}$.

Define $a_0=\min\left\{2c_j,\ \bar{\eta}_j,\ \bar{c}\text{: } j=1,\cdots,n\right\}$ and $b_0=\frac{1}{2}\sum_{j=1}^{n}\eta_j\|\theta_j\|^2+\Xi_n$. Then, the inequality (56) can be reduced as follows:

$$
\dot{V}_n\leq-a_0V_n+b_0+\sum_{j=1}^{n}h_j(t).\tag{57}
$$

Noticing that $\sum_{j=1}^{n}h_j(t)\geq 0$, $t\geq 0$ and $\sum_{j=1}^{n}h_j(t)=0$, $t\geq T_0$, the following holds:

$$
\int_0^{\infty}\sum_{j=1}^{n}h_j(t)dt<+\infty.\tag{58}
$$

Therefore, combing (57) and (58), for $\forall t\geq 0$, we have

$$
\begin{aligned}
0\leq V_n(t)\leq&\left(V_n(0)-\frac{b_0}{a_0}\right)e^{-a_0t}\\
&+\frac{b_0}{a_0}+\int_0^{\infty}\sum_{j=1}^{n}h_j(t)dt.
\end{aligned}\tag{59}
$$

Using similar arguments in [28], it follows that the system output y follows a given reference signal y_d, and all the signals in the closed-loop are bounded as well. □

Remark 6: It should be noted that the computation burden of the control strategy designed in this paper is significantly decreased through the following aspects. By using the backstepping technique, makes the controller design simpler and easier. Meanwhile, thanks to the simple structure of MTN, the proposed MTN-based controller (50) is of less calculation.

4. SIMULATION RESULTS

In this section, the following three examples are given to validate the effectiveness of the proposed control approach of this paper, including one numerical example, one practical example and one comparative experiment.

Example 1: Consider the following nonlinear system with dynamic uncertainties and input delay:

$$\begin{cases} \dot{\xi} = -\xi + 0.5x_1^2 + 0.5, \\ \dot{x}_1 = x_2 + 0.5x_1x_2^2 + 0.5\xi x_1 \sin x_1, \\ \dot{x}_2 = u(t-\tau) + 2x_2^2 e^{-0.5x_1} + x_1x_2\xi, \\ y = x_1. \end{cases} \tag{60}$$

According to Theorem 1, the virtual control law α_1 and control law u are designed as $\alpha_1 = -r_1z_1 - \hat{\theta}_1^{\mathrm{T}} P_{m_1}(\mathbf{z}_1)$ and $u = -r_2z_2 - \hat{\theta}_2^{\mathrm{T}} P_{m_2}(\mathbf{z}_2)$, where $\mathbf{z}_1 = [z_1]^{\mathrm{T}}$, $\mathbf{z}_2 = [z_1, z_2]^{\mathrm{T}}$, and $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$.

The adaptive laws are designed as $\dot{\hat{\theta}}_i = z_i \Gamma_i P_{m_i} - \Gamma_i \eta_i \hat{\theta}_i$, $i = 1, 2$.

In the simulation, the reference signal is chosen as $y_d = 0.5(\sin(t) + \sin(0.5t))$, which is investigated commonly in the work, see, for example [16,43,44]. The design parameters are chosen as follows: $r_1 = 10$, $r_2 = 10$, $\eta_1 = 5$, $\eta_2 = 10$. The simulation running with the initial states $\xi(0) = 0$, $x_1(0) = 0$, $x_2(0) = 0$ and $\tau = 0.04$. The simulation results are shown in Figs. 1-2.

Fig. 1 shows the trajectories of output y and the reference signal y_d, so it is clear that a good tracking performance has been achieved. Figs. 2-4 depict the trajectories of control input u, state x_2 and unmodelled dynamics ξ, respectively. Fig. 5 depicts the trajectory of tracking error, it can be seen that the tracking error can converge to a small region of the origin. Figs. 1-5 indicate that all the signals of the closed-loop system are bounded.

Example 2: To further validate the effectiveness of the proposed control approach of this paper, consider the following case-hard disk drive by VCM actuator with input-delay, according to [45], the nonlinear system can be expressed as follows:

$$\begin{cases} \dot{\xi} = x_1 - \dfrac{\sigma_0 \xi}{f_c + (f_s - f_c)\, e^{-(x_2/\dot{x}_s)}}, \\ \dot{x}_1 = x_2, \\ \dot{x}_2 = u(t-\tau) - \sigma_0\xi - \sigma_1\dot{\xi} - \sigma_2 x_2 - d(t), \\ y = x_1. \end{cases} \tag{61}$$

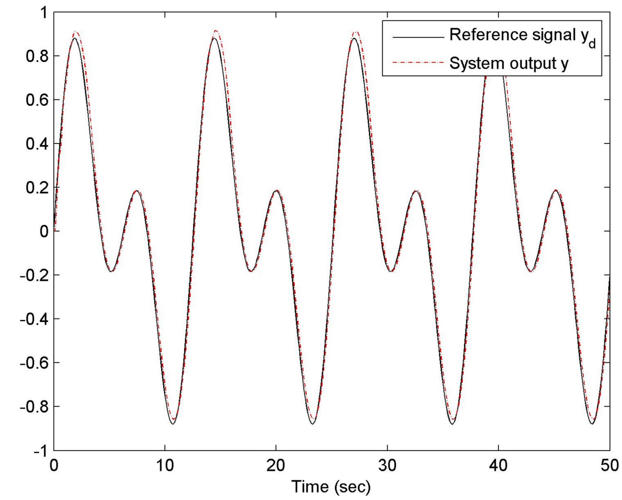

Fig. 1. The trajectories of output y and the reference signal y_d of Example 1.

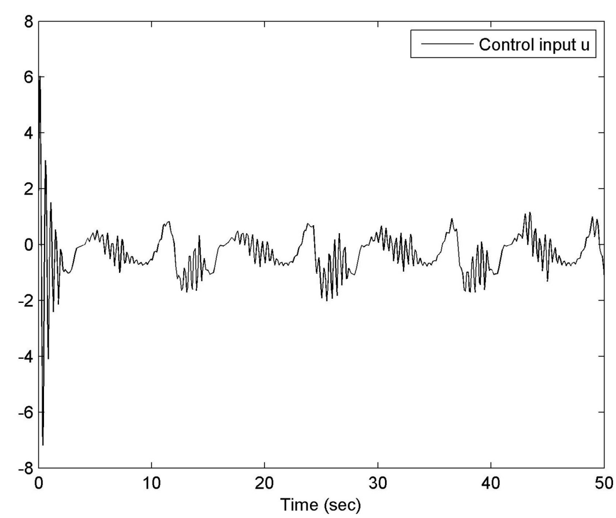

Fig. 2. The trajectory of control input u of Example 1.

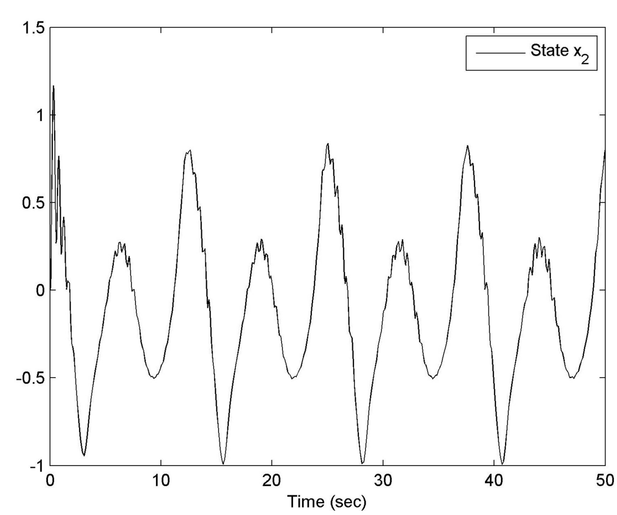

Fig. 3. The trajectory of state x_2 of Example 1.

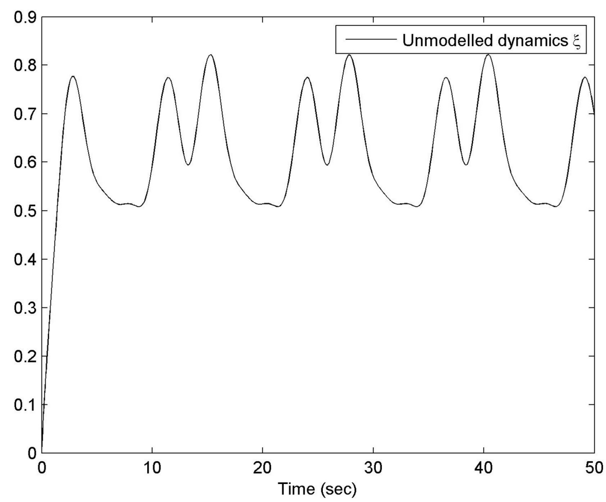

Fig. 4. The trajectory of unmodelled dynamics ξ of Example 1.

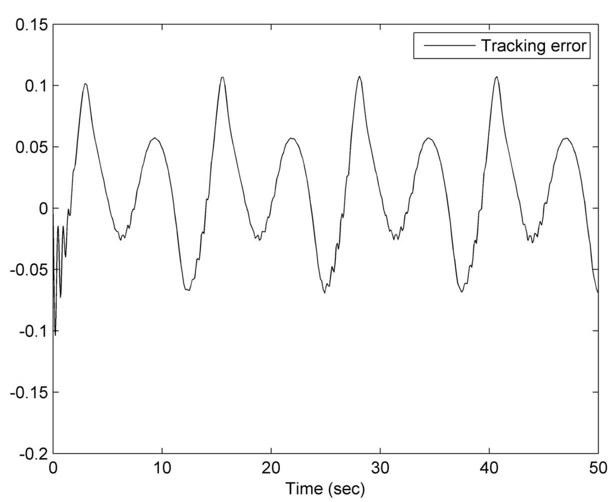

Fig. 5. The trajectory of tracking error of Example 1.

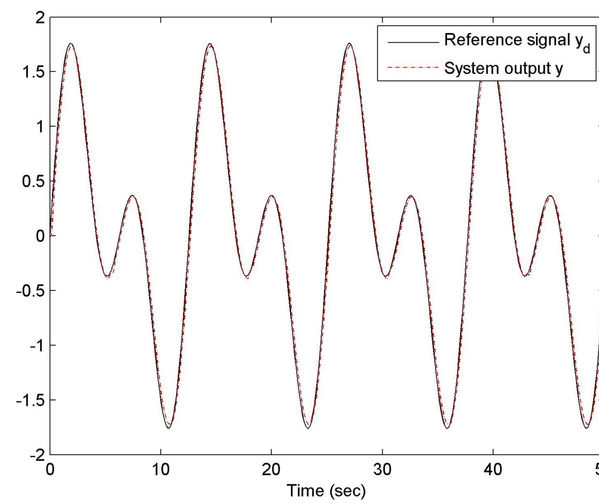

Fig. 6. The trajectories of output y and the reference signal y_d of Example 2.

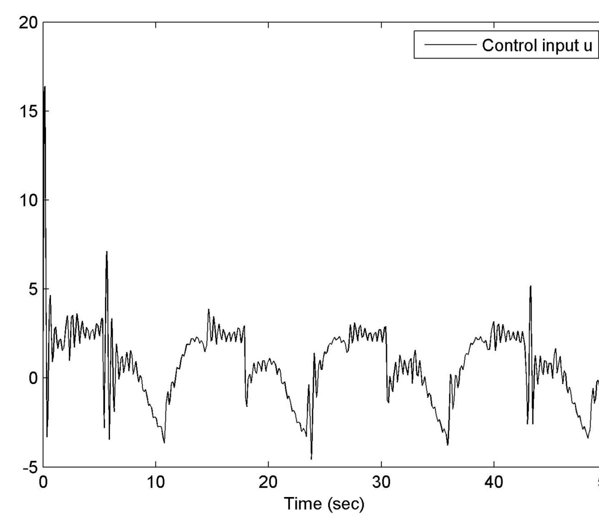

Fig. 7. The trajectory of control input u of Example 2.

Parameters of system (61) are chosen as: $\sigma_0 = \sigma_1 = \sigma_2 = 1$, $f_s = 2$, $f_c = 1$, $\dot{x}_s = 0.001$ and $d(t) = \sin t$, and the reference signal is $y_d = \sin t + \sin(0.5t)$.

In the simulation, the parameters of controller are the same as those in Example 1, and the simulation run with the initial states $\xi(0) = 0$, $x_1(0) = 0$, $x_2(0) = 0$ and $\tau = 0.04$. The simulation results are illustrated in Figs. 3-4, respectively.

Fig. 6 shows that a good tracking performance has been achieved. From Figs. 6-10, it can be seen that all signals of the closed-loop system are semi-globally bounded. Fig. 10 depicts that the tracking error converges to a small neighborhood around the origin. These simulation results further verify the feasibility of the proposed control scheme.

Remark 7: The results of Examples 1 and 2 show that the control method proposed in this paper can obtain satisfactory control effect with low computational cost, and have good convergence and stability.

Example 3: As stated in [25,27], MTN and radial basis function (RBF) neural network are very similar in form. Therefore, the following comparison is carried out. For the system (60), we use RBF neural networks instead of MTNs to handle the unknown nonlinearities of system. In simulation, the RBF neural network for α_1 contains 5 nodes with centres spaced evenly in the interval $[-2,2]$ and widths being equal to 10, and the RBF neural network for u contains 9 nodes with centres spaced evenly in the interval $[-4,4]$ and widths being equal to 10. Other design parameters are the same as Example 1. The simulation results are shown in Fig. 11.

Fig. 11 shows that both MTN-based approach and RBF neural network-based approach can achieve a good tracking performance, and the former is slightly better than the latter when the signal changes.

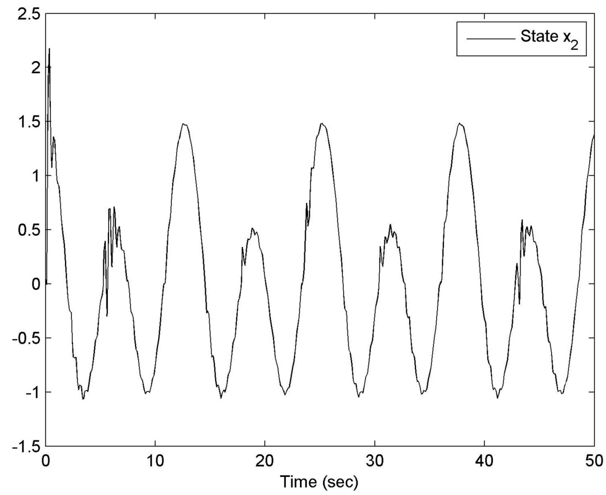

Fig. 8. The trajectory of state x_2 of Example 2.

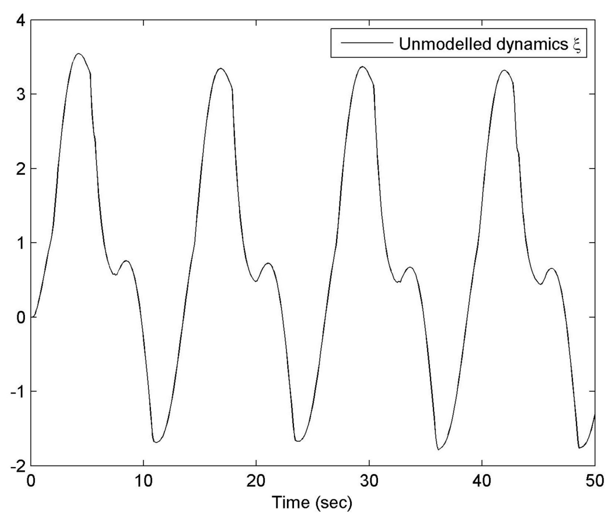

Fig. 9. The trajectory of unmodelled dynamics ξ of Example 2.

The computational complexity of the algorithm can be measured by the number of flops [46] and a flop is floating point add, subtract, multiply, or divide. Since the hidden layer of RBF neural network contains exponential functions, and the middle layer of MTN consists of an array of polynomials, which contain only addition and multiplication. Obviously, the number of flops of the control method proposed in this paper much less than the RBF neural network-based method. Therefore, we may conclude that the proposed method can get satisfactory tracking results with low computational cost.

5. CONCLUSION

In this paper, we extend the MTN control approach to a class of nonlinear systems with dynamic uncertainties and input delay. Specifically, Padé approximation with Laplace transformation is used to deal with the input delay and MTNs are used to handle the unknown nonlinearities of system, and then proposed a novel MTN-based adaptive tracking control scheme by using backstepping technique, adaptive control and Padé approximation. Meanwhile, the proposed controller has the advantages of simple structure and good real-time performance.

In recent years, finite-time control has become one of the hotspot for its finite time convergence. Therefore, our future work will be focus on extending the proposed MTN-based control algorithm to finite-time tracking control for nonlinear systems with input delay.

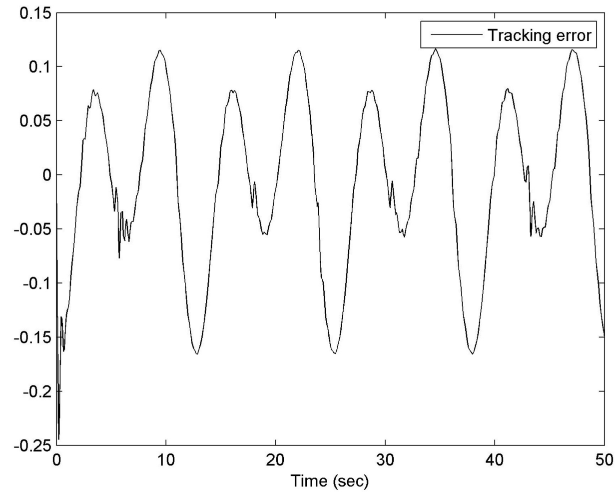

Fig. 10. The trajectory of tracking error of Example 2.

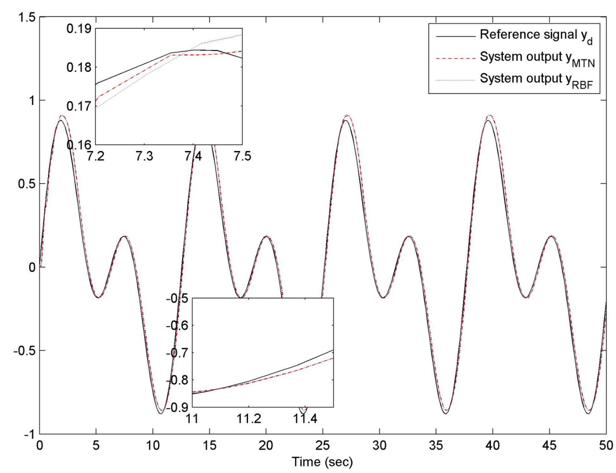

Fig. 11. The trajectories of reference signal y_d and output y under two approaches.

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