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Adaptive multi-dimensional Taylor network funnel control of a class of nonlinear systems with asymmetric input saturation

Yu-Qun Han[1,2,3] | Na Li[1,2] | Wen-Jing He[1,2] | Shan-Liang Zhu[1,2]

[1]School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao, China

[2]Research Institute for Mathematics and Interdisciplinary Sciences, Qingdao University of Science and Technology, Qingdao, China

[3]Key Laboratory of Measurement and Control of Complex Systems of Engineering, Ministry of Education, Nanjing, China

Correspondence
Shan-Liang Zhu, School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao 266061, China.
Email: zhushanliang@qust.edu.cn

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Summary

In view of the result and performance of control are affected by the existence of input constraints and requirements, adaptive multi-dimensional Taylor network (MTN) funnel control problem is studied for a class of nonlinear systems with asymmetric input saturation in this paper. Firstly, the effect of asymmetric input saturation can overcome by introducing the Gaussian error function, namely, the asymmetric saturation model is represented as a simple linear model with a bounded disturbance. Secondly, MTNs are employed to approximate the unknown functions in the controller design. Then, an adaptive MTN tracking controller is developed by blends the idea of funnel control into backstepping, which can guarantee that the tracking error always meets the given prescribed performance regarding the transient and steady state responses as well as the output of system tracks the give continuous reference signal. Finally, the effectiveness of the proposed control is demonstrated using two examples.

KEYWORDS

asymmetric input saturation, funnel control, multi-dimensional Taylor network, nonlinear systems

1 | INTRODUCTION

The analysis and synthesis of nonlinear systems, as an important part of cybernetics, have been paid attention in recent years. Meanwhile, in view of nonlinearity and uncertainty exists widely in any real system, adaptive control showed its great advantages and potential, and emerged as an important design approach, and many great success have achieved.[1-7] This technique has enabled a lot of advanced intelligent control method, such as fuzzy control,[8] neural network (NN) control,[9] backstepping technique,[10] can be used to control strategies design for nonlinear systems. As a favorable approach, approximation-based control supplied an effective solution to the control of complex nonlinear systems, meanwhile, a series of meaningful and fruitful achievements centered on fuzzy control and NN-based control are published. For example, adaptive fuzzy-based or NN-based control approach have been introduced to solve the control problem of nonlinear systems in References 11-13, Markov jump chaotic systems,[14] switched complex dynamical networks model,[15] stochastic nonlinear systems in References 16,17, multi-inputs and multi-outputs (MIMO) nonlinear systems in References 18-20, MIMO stochastic nonlinear systems in References 21,22, and switched stochastic nonlinear systems in References 23,24. However, the open problem of slow convergence and partial minimum of NN still remain to be solved. Similarly, fuzzy control also exist some drawbacks, such as the selection of fuzzy rules is more subjective, control performance is vulnerable to the impact of parameter changes. Under this background, as a new discovery of the application study on the nonlinear systems, the multi-dimensional Taylor network (MTN) comes into being.

The underlying idea of MTN is to approximate unknown functions by the linear combination of the polynomial. Once the theory of MTN is put forward, MTN has become one of the important design tools in solving nonlinear control problems because of its good approaching capability, and many significant results have been achieved. For example, authors in References 25,26 investigated some MTN tracking control schemes for nonlinear systems. Authors in References 27-31 investigated some adaptive MTN tracking control schemes for stochastic nonlinear systems. Authors in Reference 32 proposed an adaptive MTN control approach for MIMO nonlinear systems with time-varying and noises. Authors in Reference 33 proposed a MTN-based control scheme for a class of non-affine discrete MIMO nonlinear systems with time-varying to achieve the real-time tracking control. However, the above-mentioned MTN control focus mostly on system without actuator constraints. Thus, these control methods cannot directly used to control nonlinear systems with input constraints.

Recently, new research aspect has been increasingly transferred to nonlinear systems with input constraints, and there have been many research achievements in this field, including input saturation constraint[34-39] and quantized nonlinearity input.[40] More recently, MTN-based control method also has been generalized to nonlinear systems with input constraints, for example, by employing MTN, authors in Reference 41 studied the problem of dynamic regulation problem for nonlinear systems with actuator saturation and time varying delay. Authors in Reference 42 developed a MTN-based control scheme for a class of stochastic non-linear systems with input saturation constraint. For a class of nonlinear switched systems with input sector nonlinearity, authors in Reference 43 proposed an adaptive MTN tracking control scheme. Moreover, in order to achieve tracking control with prescribed transient behavior, funnel control was proposed in Reference 44. Since then, many scholars have done a great deal of research on funnel control.[45-48] Up to now, however, the problem of funnel control for nonlinear systems with asymmetric input saturation based on MTN approach has achieved no concrete results.

Based on the above analysis, this paper tried to propose a novel adaptive MTN funnel control algorithm for a class of nonlinear system with asymmetric input saturation. The asymmetric input saturation is firstly represented as a simple linear model with a bounded disturbance by introducing the Gaussian error function. Secondly, the unknown functions are handled by employing the approximation of MTN. Then, an adaptive MTN tracking controller is constructed by blends the idea of funnel control into backstepping. Finally, the tracking error always meets a given prescribed performance regarding the transient and steady state responses as well as the output of system tracks the give continuous reference signal. The main contribution of our work is as following:

1. A new MTN-based adaptive funnel controller design approach is firstly expanded in this paper for nonlinear systems with asymmetric input saturation. On the premise of the stability of the closed-loop system, in order to achieve tracking control with prescribed transient behavior, a new adaptive MTN controller is developed via backstepping.
2. Although the approximation-based control schemes for nonlinear systems has been investigated in References 25,36,45, these cannot be directly applied to the system studied in this paper due to the presence of prescribed performance and asymmetric input saturation simultaneously. This paper first combines MTN method and funnel control together realizes the tracking control for nonlinear system with asymmetric input saturation. On the one hand, a new variable with given prescribed performance is introduced to achieve funnel control. On the other hand, by introducing the Gaussian error function, the asymmetric input saturation can be expressed as a simple linear model with a bounded disturbance. Based on the work of above, a stable adaptive control method is designed using the approximation of MTN.

2 | PRELIMINARIES AND FORMULATION

2.1 | Problem explanation

In this paper, the following nonlinear system with asymmetric input saturation is considered

$$
\begin{cases}
\dot{x}_1 = x_2 + f_1(\bar{\boldsymbol{x}}_1) \\
\vdots \\
\dot{x}_{n-1} = x_n + f_{n-1}(\bar{\boldsymbol{x}}_{n-1}) \\
\dot{x}_n = u(v) + f_n(\bar{\boldsymbol{x}}_n) \\
y = x_1
\end{cases}
\tag{1}
$$

with the systems input $u(v)$ satisfies the following asymmetric constraints

$$
u(v) = \begin{cases}
u_m, & v < u_m \\
v, & u_m \leq v \leq u_M \\
u_M, & v > u_M
\end{cases}
\tag{2}
$$

where $x_1, x_2, \cdots, x_n$ are the states of system and $\bar{\boldsymbol{x}}_i = [x_1, \cdots, x_i]^{\mathrm{T}} \in R^i$, $i = 1, \cdots, n$, $y \in R$ is the output of system. In addition, for any $i = 1, \cdots, n$, $f_i(\bar{\boldsymbol{x}}_i) : R^i \rightarrow R$ is an unknown function and satisfies $f_i(\mathbf{0}) = 0$.

Remark 1. Different from the results in References 45,49,50, this paper solves the input saturation and the tracking error satisfies the prescribed performance problems simultaneously. To the best of our knowledge, there is no MTN-based control result has been reported to address this paper.

The purpose of this paper is to design a controller u, and can obtain the following two properties:

1. Under the premise of ensuring all signals of the closed-loop system are bounded, system output y tracks the give continuous reference signal y_r.
2. The tracking error satisfies the prescribed performance regarding the transient and steady state responses. Namely, the tracking error $e(t) = y(t) - y_r(t)$ remain fall into a given prescribed performance funnel, that is, $\{(t, e) \in R^+ \times R \,||e| < F_\chi(t)\}$, where $F_\chi(t)$ is the boundary of the prescribed performance funnel.

Remark 2. Similar to Reference 45 in this paper, $F_\chi(t)$ is defined as follows

$$
F_\chi(t) = (\chi_0 - \chi_\infty)e^{-\beta t} + \chi_\infty
\tag{3}
$$

where χ_0, χ_∞, and β are positive constants.

2.2 | Preliminary knowledge

As stated in Reference 36 the following smooth model is introduced to approximate the asymmetric saturation nonlinearity (2):

$$
u(v) = \bar{u} \cdot G_e(\sqrt{\pi} v / 2\bar{u})
\tag{4}
$$

where $\bar{u} = (u_M + u_m/2) + (u_M - u_m/2) sign(v)$, and $G_e(\cdot)$ is a Gaussian error function define as $G_e(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2} ds$. Then, define a function as

$$
\Xi(v) = u - cv
\tag{5}
$$

where $c > 0$ is a constant.

Combining (1) with (5), one has

$$
\begin{cases}
\dot{x}_1 = x_2 + f_1(\bar{\boldsymbol{x}}_1) \\
\vdots \\
\dot{x}_{n-1} = x_n + f_{n-1}(\bar{\boldsymbol{x}}_{n-1}) \\
\dot{x}_n = (\Xi(v) + cv) + f_n(\bar{\boldsymbol{x}}_n)
\end{cases}
\tag{6}
$$

For simplified controller design purposes, the following two Assumptions are necessary.

Assumption 1. The reference signal y_r and its i-order ($i = 1, \cdots, n$) derivative $y_r^{(i)}$ are continuous and bounded.

Assumption 2. (see Reference 36)**:** For $\forall c \in [c, \overline{c}]$, $\Xi(v)$ satisfies $\Xi(v) \leq e_u$, where $e_u > 0$, $c > 0$ and $\overline{c} > 0$ are constants.

2.3 | Multi-dimensional Taylor network

In this paper, the design of the controller makes full use of the approximating to nonlinear functions capability of MTN. The theory of MTN has been introduced in detail in our recent work,[25,30,42] only a useful Lemma is included here.

Lemma 1. (see Reference 30)**:** Suppose that $\varphi(\hbar_1, \cdots, \hbar_n) : R^n \to R$ is a function defined on a closed interval $\Omega \subset R^n$, for any given constant $\tau > 0$, there exists a MTN with approximation error $\gamma(\boldsymbol{\hbar})$ satisfies

$$\varphi(\boldsymbol{\hbar}) = \boldsymbol{\theta}^{*\mathrm{T}} P_{m_n}(\boldsymbol{\hbar}) + \gamma(\boldsymbol{\hbar}), \quad |\gamma(\boldsymbol{\hbar})| \leq \tau \tag{7}$$

with $\boldsymbol{\hbar} = [\hbar_1, \cdots, \hbar_n]^{\mathrm{T}}$, $\boldsymbol{\theta}^* = [\theta_1^*, \cdots, \theta_l^*]^{\mathrm{T}} \in R^l$ is the ideal constant weights, and $P_{m_n}(\boldsymbol{\hbar}) = [\hbar_1, \cdots, \hbar_n, \hbar_1^2, \hbar_1\hbar_2, \cdots, \hbar_1^2, \cdots, \hbar_n^m] \in R^l$.

Remark 3. MTN is a feedforward network, including three layers, that is, input layer, middle layer and output layer. The middle layer is constructed by polynomials formed by input, and its main function is information processing. In addition, MTN has such advantages as simple structure, low-calculation and easy realization, and is especially suitable for controlling of nonlinear systems. More information of MTN can be found in our recent work.[28,30]

3 | MAIN RESULTS

This section includes two aspects: controller design and stability analysis.

3.1 | MTN-based adaptive controller design

Before controller design, the following coordinate transformation is introduced

$$\begin{cases} z_1 = x_1 - y_r \\ z_i = x_i - \alpha_{i-1} \end{cases} \tag{8}$$

where $i = 1, \cdots, n-1$, and α_{i-1} will be designed in the following steps of backstepping.

Step 1: According to (6) and (8) with $i = 1$, one can get the derivative of z_1 as follows

$$\dot{z}_1 = x_2 + f_1 - \dot{y}_r \tag{9}$$

Then, choose the Lyapunov function candidate as

$$V_1 = \frac{1}{4}\xi^2 + \frac{1}{2}\widetilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\widetilde{\boldsymbol{\theta}}_1 \tag{10}$$

where $\xi = z_1^2/(F_\chi^2 - z_1^2)$, and $\widetilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \widehat{\boldsymbol{\theta}}_1$ denotes the parameter estimation error.

According to (10), one can get the derivative of V_1 as follows

$$\begin{aligned} \dot{V}_1 &= \xi \Gamma_\chi (x_2 + f_1 - \dot{y}_r - z_1 \dot{F}_\chi / F_\chi) - \widetilde{\boldsymbol{\theta}}_1^{\mathrm{T}} \dot{\widehat{\boldsymbol{\theta}}}_1 \\ &= \xi(\Gamma_\chi x_2 + \overline{f}_1) - \frac{1}{2}\xi^2 - \widetilde{\boldsymbol{\theta}}_1^{\mathrm{T}} \dot{\widehat{\boldsymbol{\theta}}}_1 \end{aligned} \tag{11}$$

where $\overline{f}_1 = \Gamma_\chi f_1 - \Gamma_\chi \dot{y}_r - \Gamma_\chi z_1 \dot{F}_\chi / F_\chi + \frac{1}{2}\xi$ is an unknown function with $\Gamma_\chi = z_1 F_\chi^2 / (F_\chi^2 - z_1^2)^2$.

Since unknown function $\bar{f}_1$ is not available to design the virtual control, by using Lemma 1, $\bar{f}_1$ can be approximate by a MTN with any given approximation accuracy $\varepsilon_1 > 0$. Namely, one has the following expression

$$\bar{f}_1 = \theta_1^{\mathrm{T}} P_{m_1}(Z_1) + \gamma_1(Z_1), \quad |\gamma_1(Z_1)| \le \varepsilon_1 \tag{12}$$

where $Z_1 = [z_1]^{\mathrm{T}}$.

Based on (12), according to Young's inequality, the following inequality holds

$$\begin{aligned} \xi \bar{f}_1 &= \xi(\theta_1^{\mathrm{T}} P_{m_1}(Z_1) + \gamma_1(Z_1)) \\ &\le \xi \theta_1^{\mathrm{T}} P_{m_1} + \frac{1}{2}\xi^2 + \frac{1}{2}\varepsilon_1^2 \end{aligned} \tag{13}$$

Substituting (13) into (11), and taking $x_2 = z_2 + \alpha_1$ into account, one has

$$\dot{V}_1 \le \xi \Gamma_\chi z_2 + \xi \Gamma_\chi \alpha_1 + \xi \theta_1^{\mathrm{T}} P_{m_1} - \tilde{\theta}_1^{\mathrm{T}} \dot{\hat{\theta}}_1 + \frac{1}{2}\varepsilon_1^2 \tag{14}$$

According to (14), design the intermediate control signal α_1 as follows

$$\alpha_1 = -\frac{1}{\Gamma_\chi}(k_1 \xi + \hat{\theta}_1^{\mathrm{T}} P_{m_1}) \tag{15}$$

where $k_1 > 0$ is a constant.

Then, substituting (15) into (14), one has

$$\dot{V}_1 \le -k_1 \xi^2 + \xi \Gamma_\chi z_2 + \tilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\hat{\theta}}_1) + \frac{1}{2}\varepsilon_1^2 \tag{16}$$

Step 2: According to (6) and (8) with $i = 2$, one can get the derivative of z_2 as follows

$$\dot{z}_2 = x_3 + f_2 - \dot{\alpha}_1 \tag{17}$$

with $\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1}(x_2 + f_1) + \frac{\partial \alpha_1}{\partial \hat{\theta}_1}\dot{\hat{\theta}}_1 + \sum_{i=0}^{1} \frac{\partial \alpha_1}{\partial y_r^{(i)}} y_r^{(i+1)}$.

Then, choose the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\theta}_2^{\mathrm{T}}\tilde{\theta}_2 \tag{18}$$

where $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$ denotes the parameter error.

According to (18), one can get the derivative of V_2 as follows

$$\dot{V}_2 \le -k_1 \xi^2 + z_2(x_3 + \bar{f}_2) - z_2^2 + \frac{1}{2}\varepsilon_1^2 + \tilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\hat{\theta}}_1) - \tilde{\theta}_2^{\mathrm{T}}\dot{\hat{\theta}}_2 \tag{19}$$

where $\tilde{f}_2 = \xi \Gamma_\chi + f_2 - \frac{\partial \alpha_1}{\partial x_1}(x_2 + f_1) - \frac{\partial \alpha_1}{\partial \hat{\theta}_1}\dot{\hat{\theta}}_1 - \sum_{i=0}^{1} \frac{\partial \alpha_1}{\partial y_d^{(i)}} y_d^{(i+1)} + z_2$ is an unknown function.

Since unknown function $\bar{f}_2$ is not available to design the virtual control, by using Lemma 1, $\bar{f}_2$ can be approximate by a MTN with any given approximation accuracy $\varepsilon_2 > 0$. Namely, one has the following expression

$$\bar{f}_2 = \theta_2^{\mathrm{T}} P_{m_2}(Z_2) + \gamma_2(Z_2), |\gamma_2(Z_2)| \le \varepsilon_2 \tag{20}$$

with $Z_2 = [z_1, z_2]^{\mathrm{T}}$ is the input vector.

Then, the following inequality holds

$$z_2 \bar{f}_2 \le z_2 \theta_2^{\mathrm{T}} P_{m_2} + \frac{1}{2}z_2^2 + \frac{1}{2}\varepsilon_2^2 \tag{21}$$

Substituting (21) into (19), and taking $x_3 = z_3 + \alpha_2$ into account, one has

$$\dot{V}_2 \le -k_1\xi^2 + z_2z_3 + z_2\alpha_2 + z_2\theta_2^{\mathrm{T}}P_{m_2} - \frac{1}{2}z_2^2 + \frac{1}{2}\sum_{i=1}^{2}\varepsilon_i^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) - \widetilde{\theta}_2^{\mathrm{T}}\dot{\widehat{\theta}}_2 \tag{22}$$

According to (22), design the intermediate control signal α_2 as follows

$$\alpha_2 = -k_2z_2 - \widehat{\theta}_2^{\mathrm{T}}P_{m_2} \tag{23}$$

where $k_2 > 0$ is a constant.

Substituting (23) into (22), one has

$$\begin{aligned}\dot{V}_2 &\le -k_1\xi^2 + z_2z_3 - k_2z_2^2 - z_2\widehat{\theta}_2^{\mathrm{T}}P_{m_2} + z_2\theta_2^{\mathrm{T}}P_{m_2} - \frac{1}{2}z_2^2 + \frac{1}{2}\sum_{i=1}^{2}\varepsilon_i^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) - \widetilde{\theta}_2^{\mathrm{T}}\dot{\widehat{\theta}}_2\\ &\le -k_1\xi^2 - k_2z_2^2 + \frac{1}{2}z_3^2 + \frac{1}{2}\sum_{i=1}^{2}\varepsilon_i^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) + \widetilde{\theta}_2^{\mathrm{T}}(z_2P_{m_2} - \dot{\widehat{\theta}}_2)\end{aligned} \tag{24}$$

Step $i(3 \le i \le n-1)$: According to (6) and (8), one can get the derivative of z_i as follows

$$\dot{z}_i = \dot{x}_i - \dot{\alpha}_{i-1} = x_{i+1} + f_i - \dot{\alpha}_{i-1} \tag{25}$$

where $\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1} + f_j) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\widehat{\theta}_j}\dot{\widehat{\theta}}_j + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j)}}y_r^{(j+1)}$.

Then, choose the Lyapunov function as follows

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\widetilde{\theta}_i^{\mathrm{T}}\widetilde{\theta}_i \tag{26}$$

where $\widetilde{\theta}_i = \theta_i - \widehat{\theta}_i$ is the parameter error.

Following the similarly procedure of (19)–(24) in Step 2, the following inequality holds

$$\dot{V}_i \le -k_1\xi^2 - \sum_{j=2}^{i}k_jz_j^2 + \frac{1}{2}z_{i+1}^2 + \frac{1}{2}\sum_{j=1}^{i}\varepsilon_j^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) + \sum_{j=2}^{i}\widetilde{\theta}_j^{\mathrm{T}}(z_jP_{m_j} - \dot{\widehat{\theta}}_j) \tag{27}$$

with the intermediate control signal α_i as follows

$$\alpha_i = -k_iz_i - \widehat{\theta}_i^{\mathrm{T}}P_{m_i} \tag{28}$$

where $k_i > 0$ are design constants.

Step n: According to (6) and (8) with $i = n$, one can get the derivative of z_n as follows

$$\dot{z}_n = \Xi(v) + cv + f_n - \dot{\alpha}_{n-1} \tag{29}$$

where $\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(x_{j+1} + f_j) + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\widehat{\theta}_j}\dot{\widehat{\theta}}_j + \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(j)}}y_r^{(j+1)}$.

Then, choose the Lyapunov function as follows

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\widetilde{\theta}_n^{\mathrm{T}}\widetilde{\theta}_n \tag{30}$$

where $\widetilde{\theta}_i = \theta_i - \widehat{\theta}_i$ is the parameter error.

According to (30), one can get the derivative of V_n as follows

$$\dot{V}_n \leq -k_1\xi^2 - \sum_{j=2}^{n-1} k_j z_j^2 + \frac{1}{2}z_n^2 + \frac{1}{2}\sum_{j=1}^{n-1}\varepsilon_j^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) + \sum_{j=2}^{n-1}\widetilde{\theta}_j^{\mathrm{T}}(z_j P_{m_j} - \dot{\widehat{\theta}}_j) + z_n(cv + \Xi(v) + f_n - \dot{\alpha}_{n-1}) - \widetilde{\theta}_n^{\mathrm{T}}\dot{\widehat{\theta}}_n$$
$$\leq -k_1\xi^2 - \sum_{j=2}^{n-1} k_j z_j^2 + \frac{1}{2}z_n^2 + \frac{1}{2}\sum_{j=1}^{n-1}\varepsilon_j^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) + \sum_{j=2}^{n-1}\widetilde{\theta}_j^{\mathrm{T}}(z_j P_{m_j} - \dot{\widehat{\theta}}_j) + z_n(cv + \bar{f}_n) - \frac{3}{2}z_n^2 + z_n\Xi(v) - \widetilde{\theta}_n^{\mathrm{T}}\dot{\widehat{\theta}}_n \tag{31}$$

where $\bar{f}_n = f_n - \dot{\alpha}_{n-1} + \frac{3}{2}z_n$ is an unknown function.

Similarly, $\bar{f}_n$ also can be approximate by a MTN with any given approximation accuracy $\varepsilon_n > 0$. Namely, one has the following expression

$$\bar{f}_n = \theta_n^{\mathrm{T}} P_{m_n}(Z_n) + \gamma_n(Z_n), |\gamma_n(Z_n)| \leq \varepsilon_n \tag{32}$$

with $Z_n = [z_1, \cdots, z_n]^{\mathrm{T}}$ is the input vector.

Then, the following inequality holds

$$z_n\bar{f}_n \leq z_n\theta_n^{\mathrm{T}} P_{m_n}(Z_n) + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_n^2 \tag{33}$$

Substituting (33) into (31), one has

$$\dot{V}_n \leq -k_1\xi^2 - \sum_{j=2}^{n-1} k_j z_j^2 + \frac{1}{2}\sum_{j=1}^{n}\varepsilon_j^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) + \sum_{j=2}^{n-1}\widetilde{\theta}_j^{\mathrm{T}}(z_j P_{m_j} - \dot{\widehat{\theta}}_j) + cvz_n + z_n\theta_n^{\mathrm{T}} P_{m_n}(Z_n) - \frac{1}{2}z_n^2 + z_n\Xi(v) - \widetilde{\theta}_n^{\mathrm{T}}\dot{\widehat{\theta}}_n \tag{34}$$

According to (34), design the control input v as follows

$$v = -\frac{1}{c}(k_n|z_n| + |\widehat{\theta}_n^{\mathrm{T}} P_{m_n}|)\mathrm{sgn}(z_n) \tag{35}$$

Then, the following inequalities hold

$$z_n cv \leq -k_n z_n^2 - |z_n\widehat{\theta}_n^{\mathrm{T}} P_{m_n}| \tag{36}$$

$$z_n\Xi(v) \leq \frac{1}{2}z_n^2 + \frac{1}{2}e_u^2 \tag{37}$$

Furthermore, substituting (36) and (37) into (34), one has

$$\dot{V}_n \leq -k_1\xi^2 - \sum_{j=2}^{n} k_j z_j^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) + \sum_{j=2}^{n}\widetilde{\theta}_j^{\mathrm{T}}(z_j P_{m_j} - \dot{\widehat{\theta}}_j) + \frac{1}{2}\sum_{j=1}^{n}\varepsilon_j^2 + \frac{1}{2}e_u^2 \tag{38}$$

Here, the controller design process has been completed. And the following work will be transferred to the stability of the closed-loop system.

3.2 | Stability analysis of the closed-loop system

Theorem 1. Under Assumptions 1 and 2, consider the closed-loop system composes with nonlinear system (1), asymmetric input saturation (2), intermediate control signals (15), (23), (28), the control input (35), and adaptive laws $\dot{\widehat{\theta}}_1 = -\xi P_{m_1} + \kappa_1\widehat{\theta}_1$, $\dot{\widehat{\theta}}_i = -z_i P_{m_i} + \kappa_i\widehat{\theta}_i$, where $\kappa_l > 0$, $l = 1, \cdots, n$ Then, under any bounded initial conditions, one has:

1. Under the premise of all signals of the closed-loop system are bounded, system output y tracks the give continuous reference signal y_r.
2. The tracking error satisfies the prescribed performance regarding the transient and steady state responses.

Proof. For the closed-loop system, choose the Lyapunov function follows

$$V = \frac{1}{4}\xi^2 + \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}\sum_{i=1}^{n}\widetilde{\theta}_i^{\mathrm{T}}\widetilde{\theta}_i \tag{39}$$

■

According to (38), the following result holds:

$$\dot{V} \leq -k_1\xi^2 - \sum_{i=2}^{n} k_i z_i^2 + \widetilde{\theta}_1^{\mathrm{T}}(\xi P_{m_1} - \dot{\widehat{\theta}}_1) + \sum_{i=2}^{n}\widetilde{\theta}_i^{\mathrm{T}}(z_i P_{m_i} - \dot{\widehat{\theta}}_i) + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \frac{1}{2}e_u^2 \tag{40}$$

Substituting $\dot{\widehat{\theta}}_1 = -\kappa_1\widehat{\theta}_1 + \xi P_{m_1}$ and $\dot{\widehat{\theta}}_i = -\kappa_i\widehat{\theta}_i + z_i P_{m_i}$ into (40), one has

$$\dot{V} \leq -k_1\xi^2 - \sum_{i=2}^{n} k_i z_i^2 + \sum_{i=1}^{n}\kappa_i\widetilde{\theta}_i^{\mathrm{T}}\widehat{\theta}_i + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \frac{1}{2}e_u^2 \tag{41}$$

Meanwhile, by $\kappa_i\widetilde{\theta}_i^{\mathrm{T}}\widehat{\theta}_i = \kappa_i\widetilde{\theta}_i^{\mathrm{T}}(\theta_i - \widetilde{\theta}_i) \leq -\frac{\kappa_i}{2}\widetilde{\theta}_i^{\mathrm{T}}\widetilde{\theta}_i + \frac{\kappa_i}{2}\|\theta_i\|^2$, one has

$$\begin{aligned}\dot{V} &\leq -k_1\xi^2 - \sum_{i=2}^{n} k_i z_i^2 - \frac{1}{2}\sum_{i=1}^{n}\kappa_i\widetilde{\theta}_i^{\mathrm{T}}\widetilde{\theta}_i + \frac{1}{2}\sum_{i=1}^{n}\kappa_i\|\theta_i\|^2 + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \frac{1}{2}e_u^2\\ &\leq -a_0V + b_0\end{aligned} \tag{42}$$

where $a_0 = \min\{4k_1, \kappa_1, 2k_i, \kappa_i | i = 2, \cdots, n\}$ and $b_0 = \frac{1}{2}\sum_{i=1}^{n}\kappa_i\|\theta_i\|^2 + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \frac{1}{2}e_u^2$.

On the one hand, from (42), one has

$$0 \leq V(t) \leq V(0) + \frac{b_0}{a_0} \tag{43}$$

According to (43), recalling (39), all signals of the closed-loop system are bounded.

On the other hand, using the same discussion method in Reference 45 one can draw the conclusion that the tracking error satisfies the prescribed performance regarding the transient and steady state responses as well as y tracks the give continuous reference signal y_r.

Remark 4. It is well known that the funnel control has gradually become a frontier research topic, and many important results have been reported for different systems under different assumptions, such as nonlinear systems[40,42] and stochastic nonlinear systems.[16,51] However, in this paper, the adaptive tracking control problem has been considered for a class of nonlinear systems by combining MTN approach and backstepping technique. Compared with the existed literatures, there are two main differences: (i) Different from References 13,48, this paper solve the input saturation and the tracking error satisfies the prescribed performance problems simultaneously; (ii) In order to deal with the unknown nonlinearity in the system, the NN-based or fuzzy logic systems -based method (see References 34-36,38) has been replaced by MTN-based method to develop the adaptive control scheme, therefore, the computation burden is greatly reduced.

4 | SIMULATION RESULTS

In this paper, the effectiveness of the proposed control method is tested and verified through the following two examples.

Example 1. (numerical example): Consider the following nonlinear system with asymmetric input saturation

$$\begin{cases} \dot{x}_1 = x_2 + x_1 \\ \dot{x}_2 = x_3 + x_1 x_2^2 \\ \dot{x}_3 = u(v) + x_1 x_2 x_3 \\ y = x_1 \end{cases} \tag{44}$$

with the systems input $u(v)$ satisfies the following constraints

$$u(v) = \begin{cases} -5, & v < -5 \\ v, & -5 \le v \le 6 \\ 6, & v > 6 \end{cases} \tag{45}$$

According to Theorem 1, the control structure of system (44) is as follows:

$$\begin{aligned} \alpha_1 &= -\frac{1}{F_\chi}(k_1\xi + \hat{\theta}_1^{\mathrm{T}} P_{m_1}(Z_1)) \\ \alpha_2 &= -k_2 z_2 - \hat{\theta}_2^{\mathrm{T}} P_{m_2}(Z_2) \\ v &= -\frac{1}{c}(k_3|z_3| + |\hat{\theta}_3^{\mathrm{T}} P_{m_3}(Z_3)|)\mathrm{sgn}(z_3) \end{aligned}$$

with the adaptive laws as $\dot{\hat{\theta}}_1 = -\xi P_{m_1} + \kappa_1 \hat{\theta}_1$, $\dot{\hat{\theta}}_i = -z_i P_{m_i} + \kappa_i \hat{\theta}_i$, $i = 2, 3$, where $Z_1 = [z_1]^{\mathrm{T}}$, $Z_2 = [z_1, z_2]^{\mathrm{T}}$ and $Z_3 = [z_1, z_2, z_3]^{\mathrm{T}}$.

In simulation, the reference signal is chosen as $y_r = 0.5\sin t$ and the design parameters are chosen as $\kappa_1 = \kappa_2 = \kappa_3 = 0.1$, $k_1 = 6.5$, $k_2 = 3$, $k_3 = 2$. In addition, $F_\chi(t)$ is defined as $F_\chi(t) = (3 - 0.2)e^{-t} + 0.2$.

Figures 1-4 show the simulation results. Figure 1 indicates that the MTN-based controller implements the tracking goals and has a good control performance. Figure 2 shows the trajectories of u and v. Figure 3 displays the trajectories of x_2 and x_3. Figure 4 illustrates that the tracking error $e = y - y_r$ fall into the given prescribed performance funnel. Figures 1-4 demonstrate that all signals of the closed-loop system, including y, x_1, x_2, x_3, u, and v, are bounded.

Example 2. (practical example): Consider a class of uncertain Duffing-Holmes chaotic system with asymmetric input saturation, according to Reference 52, the system can be described as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u(v) + f_2(x_1, x_2) \\ y = x_1 \end{cases} \tag{46}$$

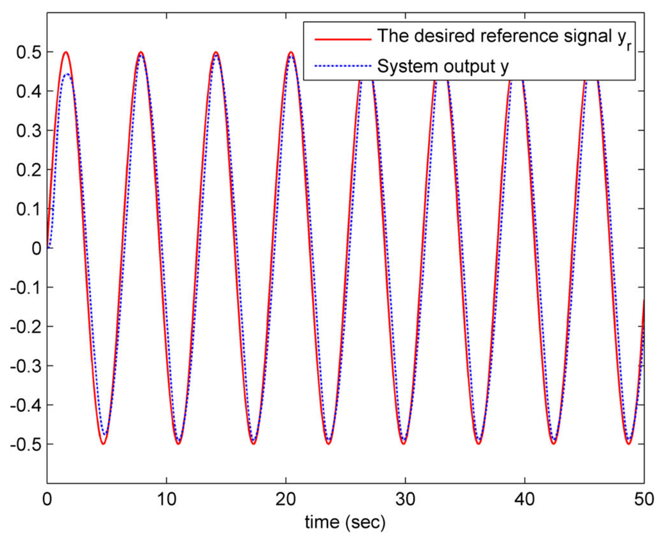

FIGURE 1 The trajectories of y and y_r of example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

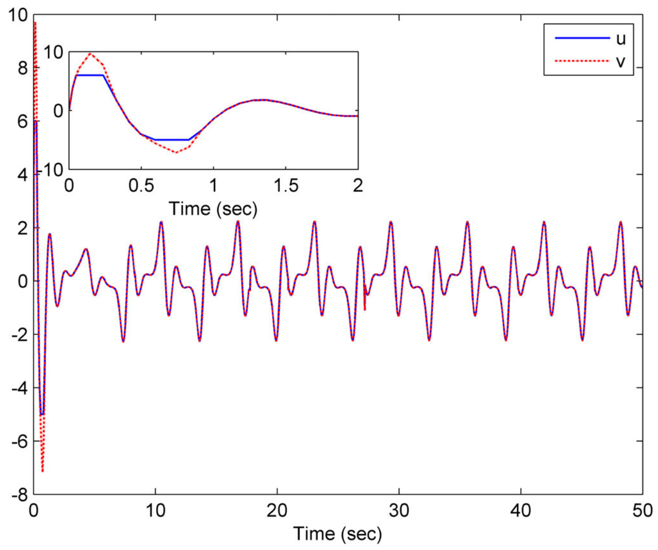

FIGURE 2 The trajectories of u and v of example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

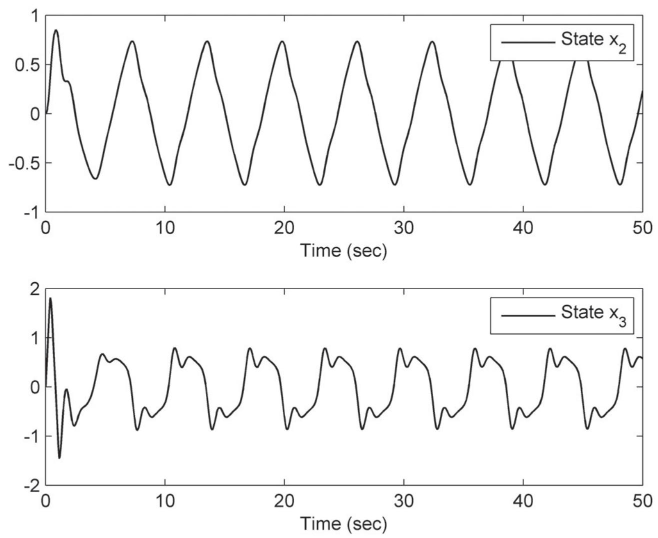

FIGURE 3 The trajectories of x_2 and x_3 of example 1

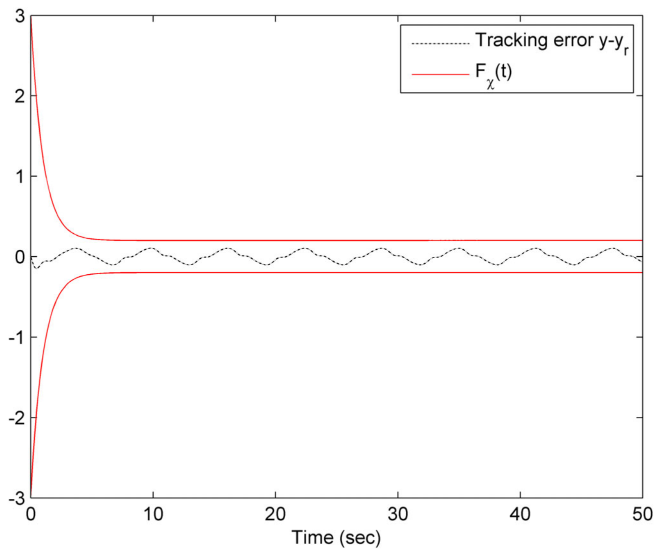

FIGURE 4 The trajectories of $y - y_r$ and $F_\chi(t)$ of example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

where $f_2(x_1, x_2) = x_1 - x_2 - x_1^3 + 0.5\cos(0.5t)$ and the systems input $u(v)$ satisfies the following constraints

$$u(v) = \begin{cases} -3, & v < -3 \\ v, -3 \le v \le 4 \\ 4, & v > 4 \end{cases} \tag{47}$$

According to Theorem 1, the control structure of system (46) is as follows:

$$\alpha_1 = -\frac{1}{\Gamma_\chi}(k_1\xi + \hat{\theta}_1^{\mathrm{T}} P_{m_1}(Z_1))$$
$$v = -\frac{1}{c}(k_2|z_2| + |\hat{\theta}_2^{\mathrm{T}} P_{m_2}(Z_2)|)\mathrm{sgn}(z_2)$$

with the adaptive laws as $\dot{\hat{\theta}}_1 = -\xi P_{m_1} + \kappa_1\hat{\theta}_1$, $\dot{\hat{\theta}}_2 = -z_2 P_{m_2} + \kappa_2\hat{\theta}_2$, where $Z_1 = [z_1]^{\mathrm{T}}$ and $Z_2 = [z_1, z_2]^{\mathrm{T}}$.

In simulation, the reference signal is chosen as $y_r = 0.5\sin t$ and the design parameters are chosen as $\kappa_1 = \kappa_2 = 0.1$, $k_1 = 20$, $k_2 = 2$. In addition, $F_\chi(t)$ is defined as $F_\chi(t) = (3 - 0.1)e^{-t} + 0.1$. The simulation results are shown in Figures 5-8, which further demonstrate the effectiveness of the proposed control method.

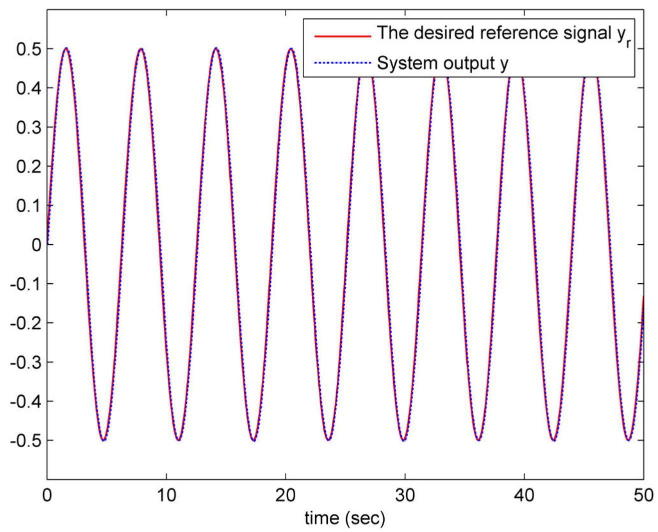

FIGURE 5 The trajectories of y and y_r of example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

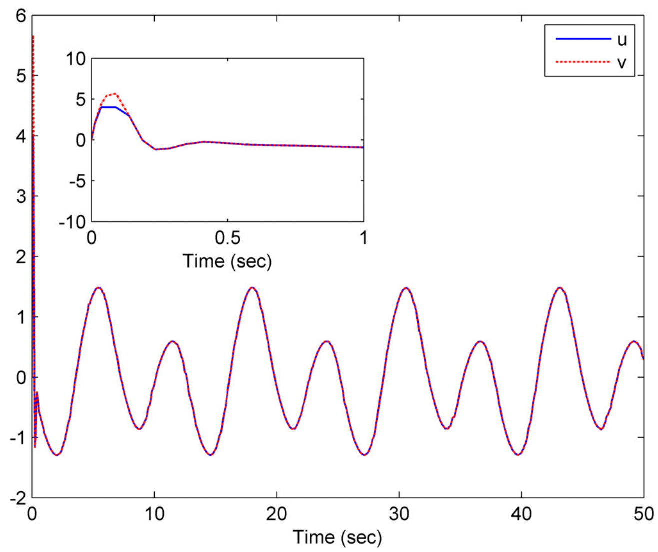

FIGURE 6 The trajectories of u and v of example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

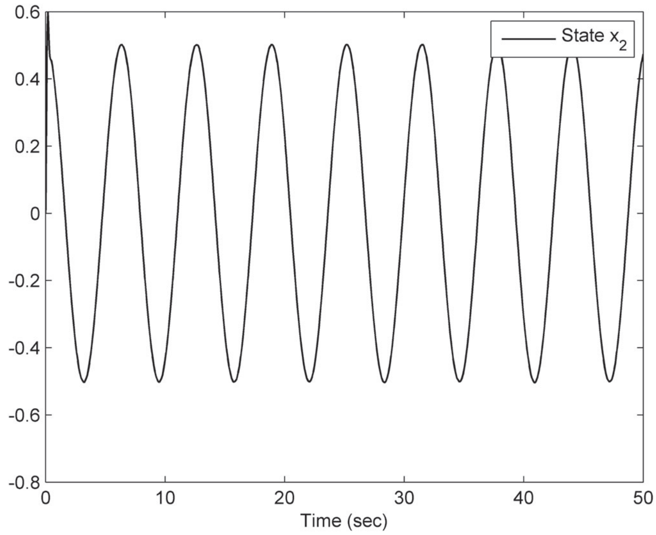

FIGURE 7 The trajectory of x_2 of example 2

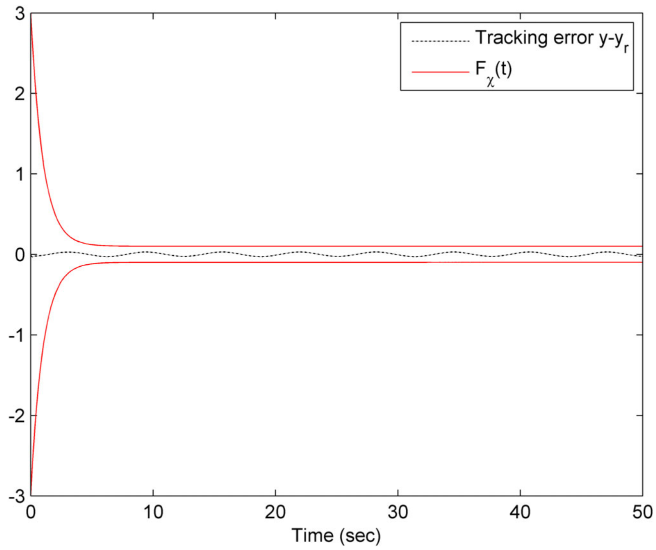

FIGURE 8 The trajectories of $y - y_r$ and $F_\chi(t)$ of example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

5 | CONCLUSION

An adaptive MTN funnel control approach is studied for nonlinear systems with asymmetric input saturation in this paper. And a MTN-based adaptive funnel control approach is firstly proposed for nonlinear systems with asymmetric input saturation. With the control approach developed in this paper, satisfactory results have been obtained, that is, the tracking error satisfies the prescribed performance regarding the transient and steady state responses as well as system output tracks the give continuous reference signal.

It is important to point out that the control approach proposed in this paper needs all of the state information, however, it is difficult to obtain the state information of many practical systems. Therefore, based on the control approach of this paper, design an effective observer-based adaptive control strategy for nonlinear system (1), which will be considered in our future work.

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DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

ORCID

Yu-Qun Han https://orcid.org/0000-0002-9055-2954
Na Li https://orcid.org/0000-0002-7911-8903
Wen-Jing He https://orcid.org/0000-0001-6370-592X

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