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Novel adaptive controller design for a class of switched nonlinear systems subject to input delay using multi-dimensional Taylor network

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Abstract

For the tracking control problem of a class of switched nonlinear systems with input delay, this article proposes an adaptive tracking control scheme based on multi-dimensional Taylor network (MTN) approach. First, Padé approximation and Laplace transform are employed to overcome the problem of input delay, and a new variable is introduced. Second, the MTNs are used to approximate the uncertain nonlinear structures in the process of controller design via backstepping. The results show that the controller designed in this article can keep all the signals in the closed-loop system are bounded, meanwhile, the output tracking error can be converged to arbitrarily small. It should be pointed out that MTN-based approach is applied to switched nonlinear systems subject to input delay for the first time. Finally, three examples are presented to verify the effectiveness of the proposed control strategy.

KEYWORDS

adaptive control, multi-dimensional Taylor network, input delay, switched nonlinear systems

1 | INTRODUCTION

As is known to all, with the rapid development of automatic control in recent years, more and more complex problems have been encountered in practical applications. Consequently, many control issues cannot be solved by a single continuous system or discrete systems control method. Therefore, as a mathematical model for many important applications, the research on hybrid systems has received extensive attention.[1–4] Among them, the switched systems, one of the important branches of hybrid systems, has a wide engineering background and acquired a lot of achievements.[5–10] It is important to note that backstepping technique has been widely used in switched nonlinear systems.[11–13] The basic idea of this technique is to design the virtual control variable and construct the Lyapunov function for each subsystem of the nonlinear systems, and finally get the control law of the whole system. Since the interaction between system parameters and subsystems are unknown, adaptive backstepping control technique is usually used to deal with the control problem of switched nonlinear systems.[14–16] However, the control problem of complex systems with uncertain nonlinear structures is difficult to achieve only based on adaptive backstepping control technology.

In response to the above problems, many control methods based on neural networks (NNs)[17–19] and fuzzy logic systems (FLSs)[20–24] have been developed to approximate the unknown nonlinear structures of systems. By combining above approximation method and adaptive backstepping control technology, a lot of research results have been achieved for nonlinear systems[25–31] and stochastic nonlinear systems.[32–35] In addition, a lot of meaningful research results have been acquired for switched nonlinear systems.[36–38] It is worth noting that, as a NN with special structure, multi-dimensional

Taylor network (MTN) has become one of design tools for nonlinear systems in recent years, and its underlying idea is to approximate unknown functions by linear combination of polynomials. The results of References 39–44 showed that MTN has good approximation ability, and it is especially suitable for the control of nonlinear systems. However, the controllers of the above-mentioned MTN are mostly concentrated on nonlinear systems,[39,43] large-scale systems,[44] and stochastic systems,[41,42] and there are few results on switched nonlinear systems. Recently, for a class of switched nonlinear systems with input nonlinearity, Zhu et al.[45] studied the tracking control problem, and proposed a MTN-based adaptive strategy. Unfortunately, the problem of input delay is not considered.

On the other hand, the phenomenon of time delay is inevitable in the actual systems, and their existences reduce the performance of the system or even induce the systems instability. Especially, the problem of input delay has been widely concerned for linear time-invariant systems[46] and nonlinear systems.[47,48] In particular, by combining Padé approximation method and Laplace transform technique, a new variable is introduced to eliminate the influence of input delay, which can better improve the system performance. And then, this design idea was extended to several different cases, and good results have been obtained, such as stochastic nonlinear systems.[49] Therefore, it is of great significance to extend the above method to the switched nonlinear systems with input delay, which has important theoretic significance and great practical value.

On the basis of investigation, for a class of switched nonlinear systems with input delay, this article tries to develop an adaptive tracking control strategy via MTN method. The input delay problem is processed by using Padé approximation method and Laplace transform, and then, an adaptive controller based on MTN is designed by using backstepping control technique. The results show that the control method proposed in this article is feasible, that is, the tracking effect of the system output is satisfactory and all the signals of the closed-loop system are bounded. The main contributions of this article are as follows.

1. For the first time, input delay, MTN technology and switched nonlinear systems appear in the same framework. With consideration of the tracking control problem of switched nonlinear systems subject to input time delay, an adaptive control strategy is proposed by combining backstepping method, MTN technology and adaptive control, and a common Lyapunov function is constructed to reduce the complexity of the switching law structure.
2. To tackle the problem of input delay, we extended the method in Reference 50 for general nonlinear systems to switched nonlinear systems. More specifically, by introducing a new variable, the switched system with input delay is transformed into a new system without input delay. Therefore, the design process of the controller is greatly simplified.
3. MTN technology is applied to the switched systems. As a novel neural network, MTN can greatly reduce the computational complexity. As far as our current knowledge is concerned, the MTN-based control strategy for switched systems only appears in Reference 45. Unfortunately, this strategy cannot handle the problem caused by input delay. Although the authors of Reference 51 studied the tracking control problem subject to input delay for switched nonlinear systems, a new NN controller is proposed in this article, which has a simpler controller structure than Reference 51.

The remainder of this article is organized as follows. Problem formulation and preliminaries are given in Section 2. An adaptive MTN controller is proposed and the stability of the system is proved in Section 3. Section 4 gives a numerical example, a practical example and a comparative example. Section 5 concludes the work.

2 | SYSTEM DESCRIPTION AND PRELIMINARIES

2.1 | Problem description

Consider the following switched nonlinear systems with input delay

$$\begin{cases} \dot{x}_i = x_{i+1} + f_{i,\sigma(t)}\left(\overline{\boldsymbol{x}}_i\right) + d_{i,\sigma(t)}, \quad i = 1, 2, \dots, n-1, \\ \dot{x}_n = u(t-\tau) + f_{n,\sigma(t)}\left(\overline{\boldsymbol{x}}_n\right) + d_{n,\sigma(t)}, \\ y = x_1, \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \dots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector with $\overline{\boldsymbol{x}}_i = [x_1, x_2, \dots, x_i]^{\mathrm{T}} \in \mathbb{R}^i$, $i = 1, 2, \dots, n$. $y \in \mathbb{R}$ is the system output, $u \in \mathbb{R}$ is the control input, and τ is the input delay, $\tau > 0$. $\sigma(t) : \mathbb{R}_+ \to M = \{1, 2, \dots, m\}$ denotes the switching signal with m is the number of subsystem. $f_{i,k}(\cdot), k \in M, i = 1, 2, \dots, n$ are unknown smooth nonlinear functions and $d_{i,k}(\cdot), i = 1, 2, \dots, n, k \in M$ are unknown external disturbance.

Remark 1. It is important to note that external disturbance $d_{i,\sigma(t)}$ in system (1) depend on the switching signal $\sigma(t)$. Compared with References 29 and 47, the external disturbance considered in this article is more general. In other words, the subsystems of different switched nonlinear systems have different external disturbances.

Control objectives: An adaptive MTN controller is designed for the closed-loop system, so that all signals in the closed-loop system remain bounded and the output y of the system can keep up with the reference signal y_d.

Assumption 1. For $i = 1, 2, \dots, n, k \in M$, external disturbance $d_{i,k}(\cdot)$ satisfy $|d_{i,k}| \le \overline{d}_{i,k}$, with constants $\overline{d}_{i,k}$ denote the upper bound on $d_{i,k}$.

Assumption 2. For $\forall t \ge 0$ and $\underline{Y}_0, \overline{Y}_0, Y_1, \dots, Y_n > 0$ are constants, the reference signal y_d and up to the nth order time derivative satisfy the inequality as follows: $-\underline{Y}_0 \le y_d(t) \le \overline{Y}_0, |\dot{y}_d(t)| < Y_1, \dots, |y_d^{(n)}(t)| < Y_n$.

2.2 | Input delay transformation

Similar to the work of Reference 50, the problem of input delay for switched nonlinear systems can be divided into the following four steps.

Step 1: Let s is the Laplace variable, according to the Laplace transform technique, the following equation holds

$$\psi\{u(t-\tau)\} = e^{-\tau s}\psi\{u(t)\} = e^{-\frac{\tau s}{2}}/e^{\frac{\tau s}{2}}\psi\{u(t)\}, \tag{2}$$

where $\psi\{\cdot\}$ is defined as the Laplace transform.

Step 2: Using Padé approximation method to approximate term $e^{-\frac{\tau s}{2}}/e^{\frac{\tau s}{2}}$, one has

$$e^{-\frac{\tau s}{2}}/e^{\frac{\tau s}{2}} \approx \left(1 - \frac{\tau s}{2}\right) \Big/ \left(1 + \frac{\tau s}{2}\right). \tag{3}$$

Step 3: Define a new variable x_{n+1}, combine (2) and (3) to get the following formula

$$\psi\{u(t-\tau)\} = \left(1 - \frac{\tau s}{2}\right) \Big/ \left(1 + \frac{\tau s}{2}\right)\psi\{u(t)\} = \psi\{x_{n+1}(t)\} - \psi\{u(t)\}. \tag{4}$$

Step 4: Define $\chi = \frac{2}{\tau}$, one has

$$\begin{aligned} u(t-\tau) &= x_{n+1}(t) - u(t), \\ \dot{x}_{n+1} &= -\chi x_{n+1} + 2\chi u. \end{aligned} \tag{5}$$

Substituting formula (5) into system (1), one has

$$\begin{cases} \dot{x}_i = x_{i+1} + f_{i,\sigma(t)}\left(\overline{\boldsymbol{x}}_i\right) + d_{i,\sigma(t)}, \quad i = 1, 2, \dots, n-1, \\ \dot{x}_n = x_{n+1} - u + f_{n,\sigma(t)}\left(\overline{\boldsymbol{x}}_n\right) + d_{n,\sigma(t)}, \\ \dot{x}_{n+1} = -\chi x_{n+1} + 2\chi u, \\ y = x_1. \end{cases} \tag{6}$$

Remark 2. Due to the defects of Padé approximation in dealing with large delay, the above method is suitable for small delay case.[50]

2.3 | Multi-dimensional Taylor network

Lemma 1. [41] *On a compact set Ω, for a continuous function $f(S) : \mathbb{R}^n \to \mathbb{R}$, and $\forall \varepsilon > 0$, there exists a MTN $\boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(S)$, as follow*

$$f(S) = \boldsymbol{\theta}^{*\mathrm{T}} P_{m_n}(S) + \delta(S), \tag{7}$$

where $P_{m_n}(S) \triangleq \left[s_1, \dots, s_n, s_1^2, \dots, s_n^2, \dots, s_1^m, \dots, s_n^m\right]^{\mathrm{T}} \in \mathbb{R}^l$ *is the middle layer vector of MTN.* $S \triangleq [s_1, s_2, \dots, s_n]^{\mathrm{T}} \in \mathbb{R}^n$ *is the input vector of MTN.* $\delta(S)$ *is the approximate error between* $f(S)$ *and* $\theta^{\mathrm{T}} P_{m_n}(S)$*, and* $|\delta(S)| < \varepsilon$*.* $\boldsymbol{\theta}$ *is the weight vector of MTN, and* $\boldsymbol{\theta}^* := \arg\min_{\theta \in \mathbb{R}^l} \left\{ \sup_{z\in\Omega} \left| f(s) - \boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(S) \right| \right\} \in \mathbb{R}^l$.

Remark 3. One thing should be pointed out is that the structure of MTN is similar to that of radial basis function neural network (RBFNN). However, their middle layers are essentially different. Say concretely, RBFNN realizes the nonlinear transformation of non-adjustable parameters by introducing basis functions in the middle layer, while MTN approximates the nonlinearity by polynomials.

3 | MAIN RESULTS

First of all, define unknown constants $\theta_i, i = 1, 2, \dots, n, \theta_i = \max\left\{ \|\boldsymbol{\theta}_{i,k}\|^2 : k \in M \right\}$, where $\boldsymbol{\theta}_{i,k}$ are the weight vectors of MTN, and its value will be given later. $\hat{\theta}_i$ are the estimated value of θ_i and satisfy $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

3.1 | Design of adaptive MTN controller

Before design controller via backstepping, define the following coordinate transformation.

$$\begin{cases} z_1 = x_1 - y_d, \\ z_i = x_i - \alpha_{i-1}, \quad i = 2, 3, \dots, n-1, \\ z_n = x_n - \alpha_{n-1} + \frac{1}{\chi} x_{n+1}, \end{cases} \tag{8}$$

where y_d is the reference signal. $\alpha_i, \ i = 1, 2, \dots, n-1$ are the virtual control signals, the specific design will be given below.

Step 1: Consider the candidate Lyapunov function as follows

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}_1^2. \tag{9}$$

By taking the derivative of (9), the following equation holds

$$\dot{V}_1 = z_1 \left(x_2 + \tilde{f}_{1,k}\left(\bar{\boldsymbol{x}}_1\right) + d_{1,k} \right) - \frac{3}{2} z_1^2 - \tilde{\theta}_1 \dot{\hat{\theta}}_1, \tag{10}$$

where $\tilde{f}_{1,k}\left(\bar{\boldsymbol{x}}_1\right) = f_{1,k}\left(\bar{\boldsymbol{x}}_1\right) - \dot{y}_d + \frac{3}{2} z_1$ is an unknown function.

According to Lemma 1, for $\forall \varepsilon_{1,k} > 0$, exist a MTN that can be used to approximate $\tilde{f}_{1,k}$, such as

$$\tilde{f}_{1,k}\left(\bar{\boldsymbol{x}}_1\right) = \boldsymbol{\theta}_{1,k}^{\mathrm{T}} P_{m_1}\left(\boldsymbol{z}_1\right) + \delta_{1,k}\left(\boldsymbol{z}_1\right), \quad \left|\delta_{1,k}\left(\boldsymbol{z}_1\right)\right| \le \varepsilon_{1,k}, \tag{11}$$

where $\boldsymbol{z}_1 = [z_1]^{\mathrm{T}}$ is the input vector, $\delta_{1,k}\left(\boldsymbol{z}_1\right)$ is the approximation error.

From (11), Assumption 1 and Young's inequality, the following inequalities hold

$$\begin{aligned} z_1 \tilde{f}_{1,k} &\le \frac{1}{2}\ell_1^2 + \frac{1}{2\ell_1^2} z_1^2 \|\boldsymbol{\theta}_{1,k}\|^2 P_{m_1}^{\mathrm{T}} P_{m_1} + \frac{1}{2} z_1^2 + \frac{1}{2}\varepsilon_{1,k}^2 \\ &\le \frac{1}{2}\ell_1^2 + \frac{1}{2\ell_1^2} z_1^2 \theta_1 P_{m_1}^{\mathrm{T}} P_{m_1} + \frac{1}{2} z_1^2 + \frac{1}{2}\varepsilon_{1,k}^2, \end{aligned} \tag{12}$$

$$z_1 d_{1,k} \le \frac{1}{2} z_1^2 + \frac{1}{2} \bar{d}_{1,k}^{2}, \tag{13}$$

where $\ell_1 > 0$ is a constant.

According to the coordinate transformation Equation (8) and Young's inequality, one has

$$z_1x_2 = z_1(z_2+\alpha_1) \le \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + z_1\alpha_1. \tag{14}$$

Substituting (12)–(14) into (10), the following inequality holds

$$\dot{V}_1 \le \frac{1}{2}z_2^2 + z_1\alpha_1 + \frac{1}{2}\ell_1^2 + \frac{1}{2\ell_1^2}z_1^2\theta_1 P_{m_1}^{\mathrm{T}}P_{m_1} + \frac{1}{2}\varepsilon_{1,k}^2 + \frac{1}{2}\bar{d}_{1,k}^2 - \widetilde{\theta}_1\dot{\widehat{\theta}}_1. \tag{15}$$

Based on (15), the virtual control signal α_1 is designed as follows

$$\alpha_1 = -r_1z_1 - \frac{1}{2\ell_1^2}z_1\widehat{\theta}_1P_{m_1}^TP_{m_1}, \tag{16}$$

where $r_1 > 0$ is a constant.

Substituting (16) into (15), the following inequality holds

$$\dot{V}_1 \le -r_1z_1^2 + \frac{1}{2}z_2^2 + \widetilde{\theta}_1\left(\frac{1}{2\ell_1^2}z_1^2P_{m_1}^{\mathrm{T}}P_{m_1} - \dot{\widehat{\theta}}_1\right) + \frac{1}{2}\ell_1^2 + \frac{1}{2}\left(\varepsilon_{1,k}^2 + \bar{d}_{1,k}^2\right). \tag{17}$$

Step 2: Consider the candidate Lyapunov function as follows

$$V_2 = \frac{1}{2}z_2^2 + \frac{1}{2}\widetilde{\theta}_2^2 + V_1. \tag{18}$$

By taking the derivative of (18), the following equation holds

$$\dot{V}_2 = z_2\left(x_3 + \widetilde{f}_{2,k} + d_{2,k}\right) - 2z_2^2 - \widetilde{\theta}_2\dot{\widehat{\theta}}_2 + \dot{V}_1, \tag{19}$$

where $\dot{\alpha}_1 = \frac{\partial\alpha_1}{\partial x_1}\left(x_2 + f_{1,k} + d_{1,k}\right) + \frac{\partial\alpha_1}{\partial\widehat{\theta}_1}\dot{\widehat{\theta}}_1 + \sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(j)}}y_d^{(j+1)}$ is the derivative of virtual control signal. $\widetilde{f}_{2,k} = f_{2,k} - \dot{\alpha}_1 + 2z_2$ is an unknown function.

According to Lemma 1, for $\forall\varepsilon_{2,k} > 0$, exist a MTN that can be used to approximate $\widetilde{f}_{2,k}$, one has

$$\widetilde{f}_{2,k} = \boldsymbol{\theta}_{2,k}^{\mathrm{T}}P_{m_2}(\mathbf{z}_2) + \delta_{2,k}(\mathbf{z}_2), \quad \left|\delta_{2,k}(\mathbf{z}_2)\right| \le \varepsilon_{2,k}, \tag{20}$$

where $\mathbf{z}_2 = [z_1, z_2]^{\mathrm{T}}$ is the input vector, $\delta_{2,k}(\boldsymbol{z}_2)$ is the approximation error.

From (20), Assumption 1, and Young's inequality, the following inequalities hold

$$\begin{aligned} z_2\widetilde{f}_{2,k} &\le \frac{1}{2}\ell_2^2 + \frac{1}{2\ell_2^2}z_2^2\left\|\boldsymbol{\theta}_{2,k}\right\|^2P_{m_2}^{\mathrm{T}}P_{m_2} + \frac{1}{2}z_2^2 + \frac{1}{2}\varepsilon_{2,k}^2 \\ &\le \frac{1}{2}\ell_2^2 + \frac{1}{2\ell_2^2}z_2^2\theta_2P_{m_2}^{\mathrm{T}}P_{m_2} + \frac{1}{2}z_2^2 + \frac{1}{2}\varepsilon_{2,k}^2, \end{aligned} \tag{21}$$

$$z_2d_{2,k} \le \frac{1}{2}z_2^2 + \frac{1}{2}\bar{d}_{2,k}^2, \tag{22}$$

where $\ell_2 > 0$ is a constant.

According to the coordinate transformation equation (8) and Young's inequality, one has

$$z_2x_3 = z_2(z_3+\alpha_2) \le \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 + z_2\alpha_2. \tag{23}$$

Substituting (17) and (21)–(23) into (19), the following inequality holds

$$\dot{V}_2 \leq \frac{1}{2}z_3^2 + z_2\alpha_2 + \frac{1}{2\ell_2^2}z_2^2\theta_2 P_{m_2}^{\mathrm{T}} P_{m_2} - \widetilde{\theta}_2\dot{\widehat{\theta}}_2 - r_1 z_1^2$$
$$+ \widetilde{\theta}_1\left(\frac{1}{2\ell_1^2}z_1^2 P_{m_1}^{\mathrm{T}} P_{m_1} - \dot{\widehat{\theta}}_1\right) + \frac{1}{2}\sum_{j=1}^{2}\ell_j^2 + \frac{1}{2}\sum_{j=1}^{2}\left(\varepsilon_{j,k}^2 + \bar{d}_{j,k}^2\right). \tag{24}$$

Based on (24), the virtual control signal α_2 is designed as follows

$$\alpha_2 = -r_2 z_2 - \frac{1}{2\ell_2^2}z_2\widehat{\theta}_2 P_{m_2}^{\mathrm{T}} P_{m_2}, \tag{25}$$

where $r_2 > 0$ is a constant.

Substituting (25) into (24), the following inequality holds

$$\dot{V}_2 \leq -\sum_{j=1}^{2} r_j z_j^2 + \sum_{j=1}^{2}\widetilde{\theta}_j\left(\frac{1}{2\ell_j^2}z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\widehat{\theta}}_j\right) + \frac{1}{2}\sum_{j=1}^{2}\ell_j^2 + \frac{1}{2}\sum_{j=1}^{2}\left(\varepsilon_{j,k}^2 + \bar{d}_{j,k}^2\right) + \frac{1}{2}z_3^2. \tag{26}$$

Step i$(3 \leq i \leq n-2)$: Consider the candidate Lyapunov functions as follows

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\widetilde{\theta}_i^2. \tag{27}$$

By taking the derivative of (27), the following equation holds

$$\dot{V}_i = \dot{V}_{i-1} + z_i\left(x_{i+1} + \widetilde{f}_{i,k} + d_{i,k}\right) - 2z_i^2 - \widetilde{\theta}_i\dot{\widehat{\theta}}_i, \tag{28}$$

where $\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(x_{j+1} + f_{j,k} + d_{j,k}\right) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\widehat{\theta}_j}\dot{\widehat{\theta}}_j + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)}$ are the derivatives of virtual control signal. $\widetilde{f}_{i,k} = f_{i,k} - \dot{\alpha}_{i-1} + 2z_i$ are unknown functions.

According to Lemma 1, for $\forall\varepsilon_{i,k} > 0$, exist a MTN that can be used to approximate $\widetilde{f}_{i,k}$, one has

$$\widetilde{f}_{i,k} = \boldsymbol{\theta}_{i,k}^{\mathrm{T}} P_{m_i}(\boldsymbol{z}_i) + \delta_{i,k}(\boldsymbol{z}_i), \quad \left|\delta_{i,k}(\boldsymbol{z}_i)\right| \leq \varepsilon_{i,k}, \tag{29}$$

where $\boldsymbol{z}_i = [z_1, z_2, \ldots, z_i]^{\mathrm{T}}$ are the input vectors, $\delta_{i,k}(\boldsymbol{z}_i)$ are the approximation error.

From (29), Assumption 1, and Young's inequality, the following inequalities hold

$$\begin{aligned} z_i\widetilde{f}_{i,k} &= z_i\boldsymbol{\theta}_{i,k}^{\mathrm{T}} P_{m_i} + z_i\delta_{i,k} \\ &\leq \frac{1}{2}\ell_i^2 + \frac{1}{2\ell_i^2}z_i^2\left\|\boldsymbol{\theta}_{i,k}\right\|^2 P_{m_i}^{\mathrm{T}} P_{m_i} + \frac{1}{2}z_i^2 + \frac{1}{2}\varepsilon_{i,k}^2 \\ &\leq \frac{1}{2}\ell_i^2 + \frac{1}{2\ell_i^2}z_i^2\theta_i P_{m_i}^{\mathrm{T}} P_{m_i} + \frac{1}{2}z_i^2 + \frac{1}{2}\varepsilon_{i,k}^2, \end{aligned} \tag{30}$$

$$z_i d_{i,k} \leq \frac{1}{2}z_i^2 + \frac{1}{2}\bar{d}_{i,k}^2, \tag{31}$$

where $\ell_i > 0$ are constants.

According to the coordinate transformation equation (8) and Young's inequality, one has

$$z_i x_{i+1} = z_i\left(z_{i+1} + \alpha_i\right) \leq \frac{1}{2}z_i^2 + \frac{1}{2}z_{i+1}^2 + z_i\alpha_i. \tag{32}$$

Substituting (30)–(32) into (28), the following inequality holds

$$\dot{V}_i \leq \dot{V}_{i-1} + \frac{1}{2}z_{i+1}^2 + z_i\alpha_i + \frac{1}{2}\ell_i^2 + \frac{1}{2\ell_i^2}z_i^2\theta_i P_{m_i}^{\mathrm{T}} P_{m_i} + \frac{1}{2}\varepsilon_{i,k}^2 + \frac{1}{2}\bar{d}_{i,k}^2 - \widetilde{\theta}_i\dot{\widehat{\theta}}_i - \frac{1}{2}z_i^2. \tag{33}$$

Based on (33), the virtual control signals α_i are designed as follows

$$\alpha_i = -r_i z_i - \frac{1}{2\ell_i^2} z_i \hat{\theta}_i P_{m_i}^{\mathrm{T}} P_{m_i}, \tag{34}$$

where $r_i > 0$ are constants.

Substituting (34) into (33) and repeating Step 2, the following inequality holds

$$\dot{V}_i \leq -\sum_{j=1}^{i} r_j z_j^2 + \sum_{j=1}^{i} \tilde{\theta}_j \left(\frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\hat{\theta}}_j \right) + \frac{1}{2}\sum_{j=1}^{i} \ell_j^2 + \frac{1}{2}\sum_{j=1}^{i} \left(\varepsilon_{j,k}^2 + \bar{d}_{j,k}^2 \right) + \frac{1}{2} z_{i+1}^2. \tag{35}$$

Step $n-1$: Consider the candidate Lyapunov function as follows

$$V_{n-1} = V_{n-2} + \frac{1}{2} z_{n-1}^2 + \frac{1}{2} \tilde{\theta}_{n-1}^2. \tag{36}$$

By taking the derivative of (36), the following equation holds

$$\dot{V}_{n-1} = \dot{V}_{n-2} + z_{n-1} \left(x_n + \tilde{f}_{n-1,k} + d_{n-1,k} \right) - 2 z_{n-1}^2 - \tilde{\theta}_{n-1} \dot{\hat{\theta}}_{n-1}, \tag{37}$$

where $\dot{\alpha}_{n-2} = \sum_{j=1}^{n-2} \frac{\partial \alpha_{n-2}}{\partial x_j} \left(x_{j+1} + f_{j,k} + d_{j,k} \right) + \sum_{j=1}^{n-2} \frac{\partial \alpha_{n-2}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j + \sum_{j=0}^{n-2} \frac{\partial \alpha_{n-2}}{\partial y_d^{(j)}} y_d^{(j+1)}$ is the derivative of virtual control signal. $\tilde{f}_{n-1,k} = f_{n-1,k} - \dot{\alpha}_{n-2} + 2 z_{n-1}$ is an unknown function.

According to Lemma 1, for $\forall \varepsilon_{n-1,k} > 0$, exist a MTN that can be used to approximate $\tilde{f}_{n-1,k}$, such as

$$\tilde{f}_{n-1,k} = \boldsymbol{\theta}_{n-1,k}^{\mathrm{T}} P_{m_{n-1}} (\boldsymbol{z}_{n-1}) + \delta_{n-1,k} (\boldsymbol{z}_{n-1}), \quad \left| \delta_{n-1,k} (\boldsymbol{z}_{n-1}) \right| \leq \varepsilon_{n-1,k}, \tag{38}$$

where $\boldsymbol{z}_{n-1} = [z_1, z_2, \ldots, z_{n-1}]^{\mathrm{T}}$ is the input vector, $\delta_{n-1,k}(\boldsymbol{z}_{n-1})$ is the approximation error.

From (38), Assumption 1, and Young's inequality, the following inequalities hold

$$\begin{aligned} z_{n-1} \tilde{f}_{n-1,k} &\leq \frac{1}{2} \ell_{n-1}^2 + \frac{1}{2\ell_{n-1}^2} z_{n-1}^2 \left\| \boldsymbol{\theta}_{n-1,k} \right\|^2 P_{m_{n-1}}^{\mathrm{T}} P_{m_{n-1}} + \frac{1}{2} z_{n-1}^2 + \frac{1}{2} \varepsilon_{n-1,k}^2 \\ &\leq \frac{1}{2} \ell_{n-1}^2 + \frac{1}{2\ell_{n-1}^2} z_{n-1}^2 \theta_{n-1} P_{m_{n-1}}^{\mathrm{T}} P_{m_{n-1}} + \frac{1}{2} z_{n-1}^2 + \frac{1}{2} \varepsilon_{n-1,k}^2, \end{aligned} \tag{39}$$

$$z_{n-1} d_{n-1,k} \leq \frac{1}{2} z_{n-1}^2 + \frac{1}{2} \bar{d}_{n-1,k}^2, \tag{40}$$

where $\ell_{n-1} > 0$ is a constant.

According to the coordinate transformation equation (8) and Young's inequality, one has

$$z_{n-1} x_n = z_{n-1} \left(z_n + \alpha_{n-1} - \frac{1}{\chi} x_{n+1} \right) \leq z_{n-1} z_n + z_{n-1} \alpha_{n-1} \leq \frac{1}{2} z_{n-1}^2 + \frac{1}{2} z_n^2 + z_{n-1} \alpha_{n-1}. \tag{41}$$

Substituting (39)–(41) into (37), the following inequality holds

$$\begin{aligned} \dot{V}_{n-1} \leq\; & \dot{V}_{n-2} + \frac{1}{2} z_n^2 + z_{n-1} \alpha_{n-1} + \frac{1}{2} \ell_{n-1}^2 + \frac{1}{2\ell_{n-1}^2} z_{n-1}^2 \theta_{n-1} P_{m_{n-1}}^{\mathrm{T}} P_{m_{n-1}} \\ & + \frac{1}{2} \varepsilon_{n-1,k}^2 + \frac{1}{2} \bar{d}_{n-1,k}^2 - \tilde{\theta}_{n-1} \dot{\hat{\theta}}_{n-1} - \frac{1}{2} z_{n-1}^2. \end{aligned} \tag{42}$$

Based on (42), the virtual control signal α_{n-1} is designed as follows

$$\alpha_{n-1} = -r_{n-1} z_{n-1} - \frac{1}{2\ell_{n-1}^2} z_{n-1} \hat{\theta}_{n-1} P_{m_{n-1}}^{\mathrm{T}} P_{m_{n-1}}, \tag{43}$$

where $r_{n-1} > 0$ is a constant.

Substituting (43) into (42) and combining with (35), the following inequality holds

$$\dot{V}_{n-1} \leq -\sum_{j=1}^{n-1} r_j z_j^2 + \sum_{j=1}^{n-1} \widetilde{\theta}_j \left(\frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\hat{\theta}}_j \right) + \frac{1}{2}\sum_{j=1}^{n-1} \ell_j^2 + \frac{1}{2}\sum_{j=1}^{n-1} \left(\varepsilon_{j,k}^2 + \overline{d}_{j,k}^2 \right) + \frac{1}{2} z_n^2. \tag{44}$$

Remark 4. Similarly, $\frac{1}{2}z_{i+1}^2$ in Step i ($i = 1, 2, \dots, n-1$) will be eliminated in Step $i+1$.

Step n: Consider the candidate Lyapunov function as follows

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\widetilde{\theta}_n^2. \tag{45}$$

By taking the derivative of (45), the following equation holds

$$\begin{aligned} \dot{V}_n &= \dot{V}_{n-1} + z_n \left(x_{n+1} - u + f_{n,k} + d_{n,k} - \dot{\alpha}_{n-1} + \frac{1}{\chi}\left(-\chi x_{n+1} + 2\chi u\right) \right) - \widetilde{\theta}_n \dot{\hat{\theta}}_n \\ &= \dot{V}_{n-1} + z_n \left(u + \widetilde{f}_{n,k} + d_{n,k} \right) - \frac{3}{2} z_n^2 - \widetilde{\theta}_n \dot{\hat{\theta}}_n, \end{aligned} \tag{46}$$

where $\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\left(x_{j+1} + f_{j,k} + d_{j,k}\right) + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j}\dot{\hat{\theta}}_j + \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)}$ is the derivative of virtual control signal. $\widetilde{f}_{n,k} = f_{n,k} - \dot{\alpha}_{n-1} + \frac{3}{2}z_n$ is an unknown function.

According to Lemma 1, for $\forall \varepsilon_{n,k} > 0$, exist a MTN that can be used to approximate $\widetilde{f}_{n,k}$, one has

$$\widetilde{f}_{n,k} = \boldsymbol{\theta}_{n,k}^{\mathrm{T}} P_{m_n}(\boldsymbol{z}_n) + \delta_{n,k}(\boldsymbol{z}_n), \quad \left|\delta_{n,k}(\boldsymbol{z}_n)\right| \leq \varepsilon_{n,k}, \tag{47}$$

where $\boldsymbol{z}_n = [z_1, z_2, \dots, z_n]^{\mathrm{T}}$ is the input vector, $\delta_{n,k}(\boldsymbol{z}_n)$ is the approximation error.

From (47), Assumption 1, and Young's inequality, the following inequalities hold

$$\begin{aligned} z_n \widetilde{f}_{n,k} &= z_n \boldsymbol{\theta}_{n,k}^{\mathrm{T}} P_{m_n} + z_n \delta_{n,k} \\ &\leq \frac{1}{2}\ell_n^2 + \frac{1}{2\ell_n^2} z_n^2 \left\|\boldsymbol{\theta}_{n,k}\right\|^2 P_{m_n}^{\mathrm{T}} P_{m_n} + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_{n,k}^2 \\ &\leq \frac{1}{2}\ell_n^2 + \frac{1}{2\ell_n^2} z_n^2 \theta_n P_{m_n}^{\mathrm{T}} P_{m_n} + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_{n,k}^2, \end{aligned} \tag{48}$$

$$z_n d_{n,k} \leq \frac{1}{2}z_n^2 + \frac{1}{2}\overline{d}_{n,k}^2, \tag{49}$$

where $\ell_n > 0$ is a constant.

Substituting (48) and (49) into (46), the following inequality holds

$$\dot{V}_n \leq \dot{V}_{n-1} + z_n u + \frac{1}{2}\ell_n^2 + \frac{1}{2\ell_n^2} z_n^2 \theta_n P_{m_n}^{\mathrm{T}} P_{m_n} + \frac{1}{2}\varepsilon_{n,k}^2 + \frac{1}{2}\overline{d}_{n,k}^2 - \frac{1}{2}z_n^2 - \widetilde{\theta}_n \dot{\hat{\theta}}_n. \tag{50}$$

Based on (50), the control input u is designed as follows

$$u = -r_n z_n - \frac{1}{2\ell_n^2} z_n \hat{\theta}_n P_{m_n}^{\mathrm{T}} P_{m_n}, \tag{51}$$

where $r_n > 0$ is a constant.

Substituting (51) into (50) and combining with (44), the following inequality holds

$$\dot{V}_n \leq -\sum_{j=1}^{n-1} r_j z_j^2 + \sum_{j=1}^{n-1} \widetilde{\theta}_j \left(\frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\hat{\theta}}_j \right) + \frac{1}{2}\sum_{j=1}^{n-1} \ell_j^2 + \frac{1}{2}\sum_{j=1}^{n-1} \left(\varepsilon_{j,k}^2 + \overline{d}_{j,k}^2 \right) + \frac{1}{2} z_n^2$$

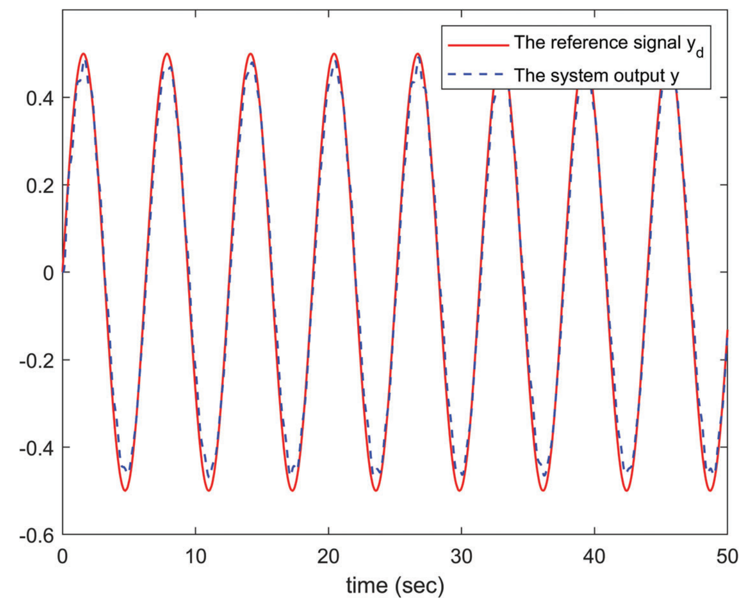

FIGURE 1 System output y and reference signal y_d of Example 1

$$- r_n z_n^2 - \frac{1}{2\ell_n^2} z_n^2 \hat{\theta}_n P_{m_n}^{\mathrm{T}} P_{m_n} + \frac{1}{2}\ell_n^2 + \frac{1}{2\ell_n^2} z_n^2 \theta_n P_{m_n}^{\mathrm{T}} P_{m_n} + \frac{1}{2}\varepsilon_{n,k}^2 + \frac{1}{2}\overline{d}_{n,k}^2 - \frac{1}{2}z_n^2 - \widetilde{\theta}_n \dot{\hat{\theta}}_n$$

$$= -\sum_{j=1}^{n} r_j z_j^2 + \sum_{j=1}^{n} \widetilde{\theta}_j \left(\frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\hat{\theta}}_j \right) + \frac{1}{2}\sum_{j=1}^{n} \ell_j^2 + \frac{1}{2}\sum_{j=1}^{n} \left(\varepsilon_{j,k}^2 + \overline{d}_{j,k}^2 \right). \quad (52)$$

Remark 5. It can be seen from the controller design process that only one MTN is used to copy with the combination of nonlinear functions in each step of backstepping. Although FLSs and NNs can also achieve this goal, the computation complexity of controller of this article is significantly reduced in the view of the following reasons. (i) Compared with the control methods based on NNs or FLSs[51,52] for switched nonlinear systems, the proposed MTN-based controller (51) with fewer numbers of middle layer and simpler structure. (ii) Thanks to the simple structure and small calculation of MTN, no lengthy calculation is involved.

3.2 | Stability analysis

Theorem 1. *Consider a class of switched nonlinear systems* (1) *with input delay, if the virtual control signals and control law are designed as* (16), (25), (34), (43), *and* (51), *the adaptive control laws are designed as*

$$\dot{\hat{\theta}}_i = -\eta_i \hat{\theta}_i + \frac{1}{2\ell_i^2} z_i^2 P_{m_i}^{\mathrm{T}} P_{m_i}, \quad (53)$$

where $\eta_i > 0, \ell_i > 0$. *For any bounded initial conditions, one has*

1. *All signals in a closed loop system are bounded.*
2. *The tracking error will converge to a small region of the origin.*

Proof. Let the Lyapunov function be

$$V = V_n = \frac{1}{2}\sum_{j=1}^{n} z_j^2 + \frac{1}{2}\sum_{j=1}^{n} \widetilde{\theta}_j^2. \quad (54)$$

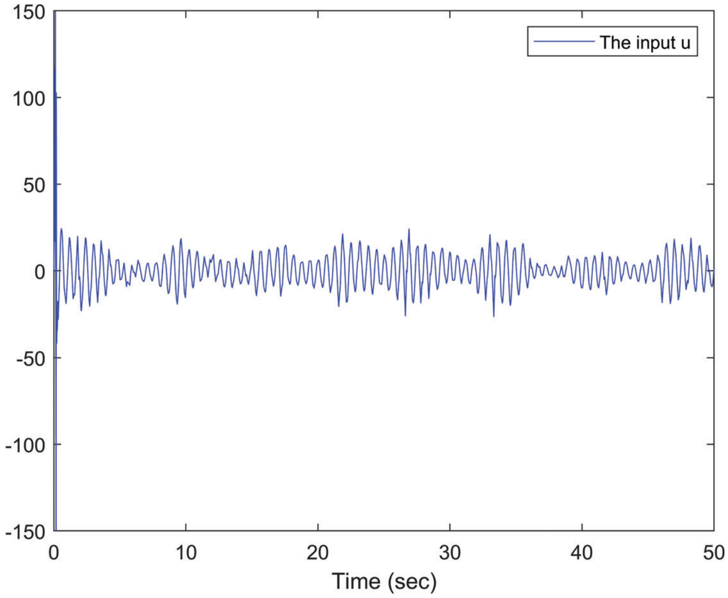

FIGURE 2 System control input u of Example 1

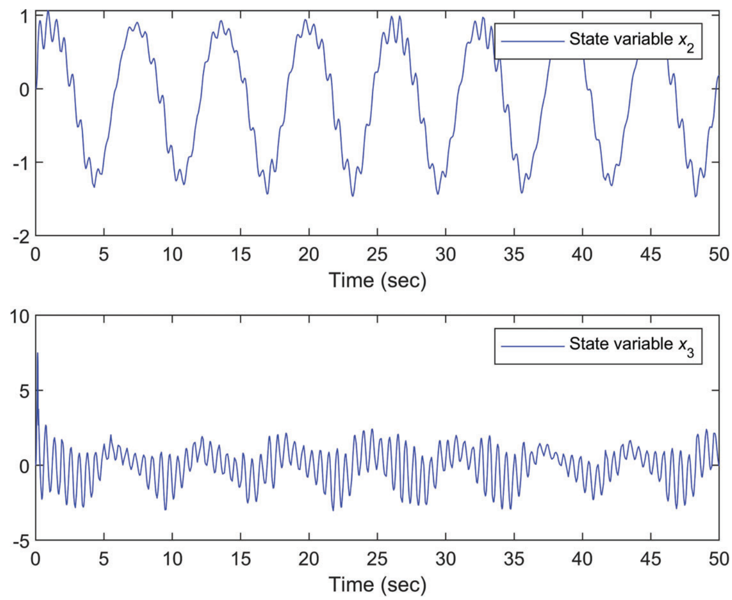

FIGURE 3 State variables x_2, x_3 of Example 1

According to Equation (52), the following inequality holds

$$\dot{V} \leq -\sum_{j=1}^{n} r_j z_j^2 + \sum_{j=1}^{n} \widetilde{\theta}_j \left(\frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\hat{\theta}}_j \right) + \frac{1}{2} \sum_{j=1}^{n} \ell_j^2 + \frac{1}{2} \sum_{j=1}^{n} \left(\varepsilon_{j,k}^2 + \overline{d}_{j,k}^2 \right), \tag{55}$$

Substituting (53) into term $\sum_{j=1}^{n} \widetilde{\theta}_j \left(\frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\hat{\theta}}_j \right)$, the following inequality holds

$$\sum_{j=1}^{n} \widetilde{\theta}_j \left(\frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\hat{\theta}}_j \right) = \sum_{j=1}^{n} \widetilde{\theta}_j \left(\frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} + \eta_j \hat{\theta}_j - \frac{1}{2\ell_j^2} z_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} \right)$$

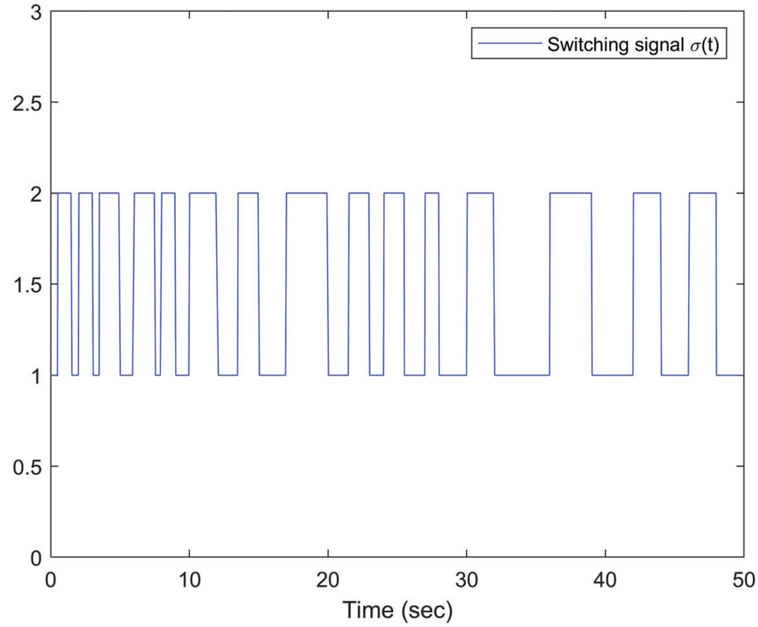

FIGURE 4 Switching signal $\sigma(t)$ of Example 1

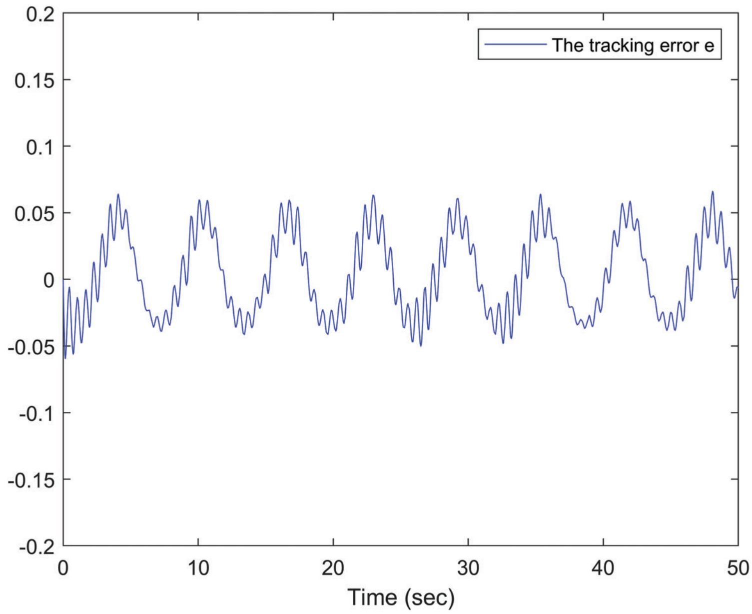

FIGURE 5 The tracking error of Example 1

$$
\begin{aligned}
&= \sum_{j=1}^{n} \eta_j \tilde{\theta}_j \hat{\theta}_j = \sum_{j=1}^{n} \eta_j \tilde{\theta}_j \left(\theta_j - \tilde{\theta}_j \right) \\
&\leq \frac{1}{2} \sum_{j=1}^{n} \eta_j \tilde{\theta}_j^2 + \frac{1}{2} \sum_{j=1}^{n} \eta_j \theta_j^2 - \sum_{j=1}^{n} \eta_j \tilde{\theta}_j^2 \\
&\leq -\frac{1}{2} \overline{\eta}_j \sum_{j=1}^{n} \tilde{\theta}_j^2 + \frac{1}{2} \sum_{j=1}^{n} \eta_j \theta_j^2,
\end{aligned} \tag{56}
$$

where $\overline{\eta}_j = \min \left\{ \eta_j | j = 1, 2, \ldots, n \right\}$.

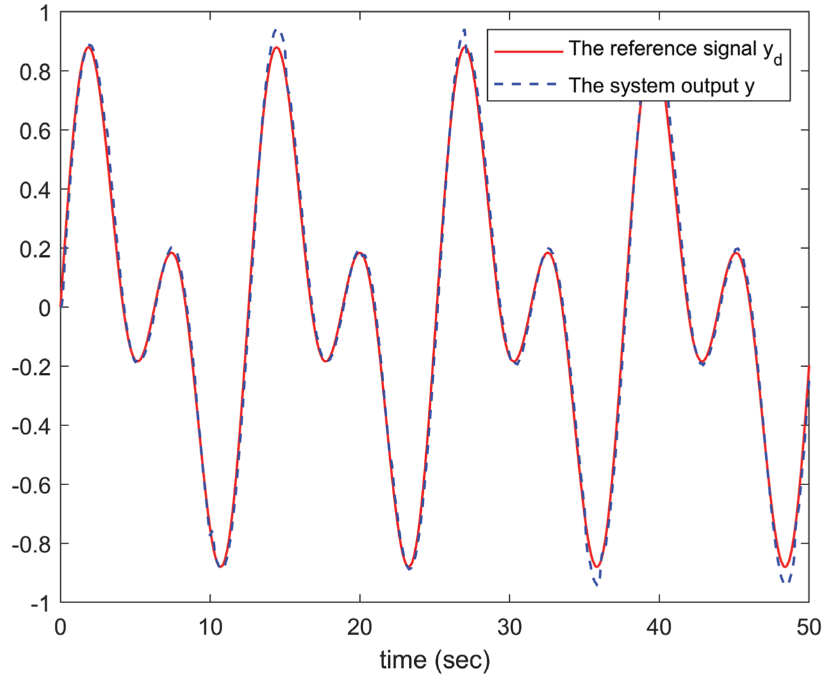

FIGURE 6 System output y and reference signal y_d of Example 2

Substituting (56) into (55), the following inequality holds

$$\begin{aligned}\dot{V} &\leq -\sum_{j=1}^{n} r_j z_j^2 - \frac{1}{2}\overline{\eta}_j \sum_{j=1}^{n} \widetilde{\theta}_j^2 + \frac{1}{2}\sum_{j=1}^{n} \eta_j \theta_j^2 + \frac{1}{2}\sum_{j=1}^{n} \ell_j^2 + \frac{1}{2}\sum_{j=1}^{n}\left(\varepsilon_{j,k}^2 + \overline{d}_{j,k}^2\right) \\ &\leq -\sum_{j=1}^{n} r_j z_j^2 - \frac{1}{2}\overline{\eta}_j \sum_{j=1}^{n} \widetilde{\theta}_j^2 + \frac{1}{2}\sum_{j=1}^{n} \eta_j \theta_j^2 + \frac{1}{2}\sum_{j=1}^{n}\left(\ell_j^2 + \varepsilon_{j,\max}^2 + \overline{d}_{j,\max}^2\right) \\ &\leq -a_0 V + b_0, \end{aligned} \tag{57}$$

where $a_0 = \min\left\{2r_j, \overline{\eta}_j | j = 1, 2, \dots, n\right\}$, $b_0 = \frac{1}{2}\sum_{j=1}^{n} \eta_j \theta_j^2 + \frac{1}{2}\sum_{j=1}^{n}\left(\ell_j^2 + \varepsilon_{j,\max}^2 + \overline{d}_{j,\max}^2\right)$.

Integrating (57), for $\forall t \geq 0$, the following inequality holds

$$0 \leq V \leq \left[V(0) - \frac{b_0}{a_0}\right] e^{-a_0 t} + \frac{b_0}{a_0}. \tag{58}$$

Based on inequality (58), using the method in References 36 and 53, it can be concluded that all signals in the closed-loop system are bounded, and the output tracking error can be converged to a small region of the origin by designing appropriate parameters.

That completes the proof of Theorem 1. ■

Remark 6. It can be concluded from the above process that the construct of Lyapunov functions and the design of controller have the characteristics of systematized and structured, which will make it easier for the reader to follow and in-depth research.

4 | SIMULATION RESULT

In order to verify the effectiveness of the control strategy proposed in this article, three examples are presented in this section.

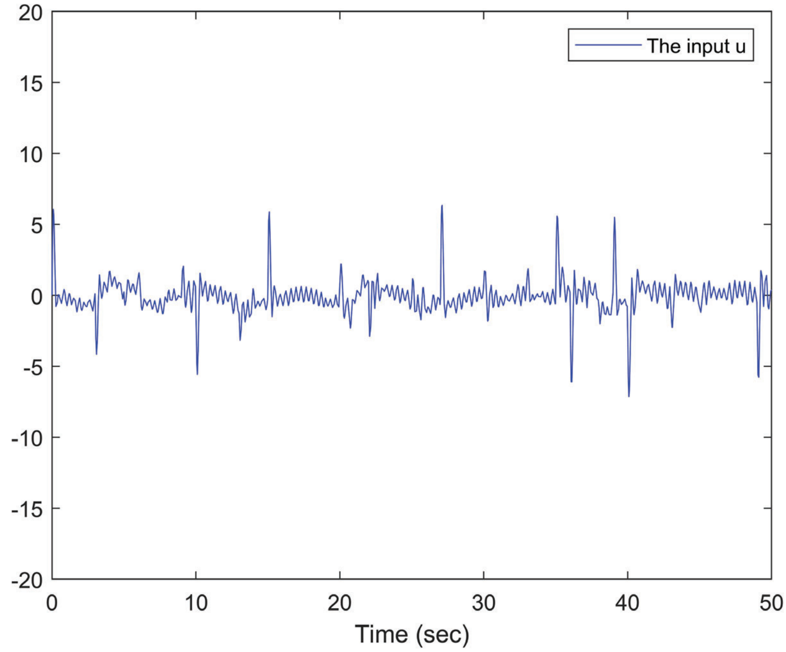

FIGURE 7 System control input u of Example 2

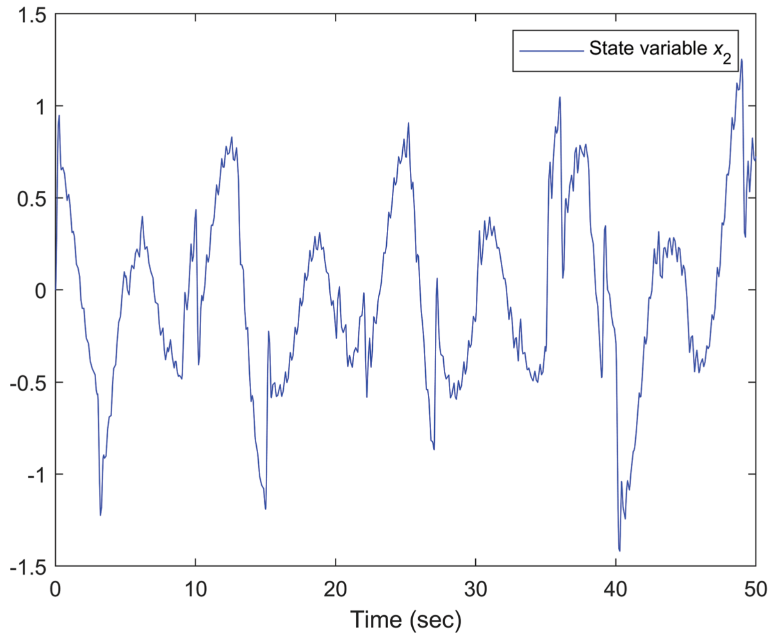

FIGURE 8 State variable x_2 of Example 2

Example 1. In order to further verify the effectiveness of the proposed controller designed in this article, the following third-order nonlinear switched systems with input delay are considered.

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma(t)}\left(\overline{\boldsymbol{x}}_1\right) + d_{1,\sigma(t)}, \\ \dot{x}_2 = x_3 + f_{2,\sigma(t)}\left(\overline{\boldsymbol{x}}_2\right) + d_{2,\sigma(t)}, \\ \dot{x}_3 = u(t-\tau) + f_{3,\sigma(t)}\left(\overline{\boldsymbol{x}}_3\right) + d_{3,\sigma(t)}, \\ y = x_1, \end{cases} \tag{59}$$

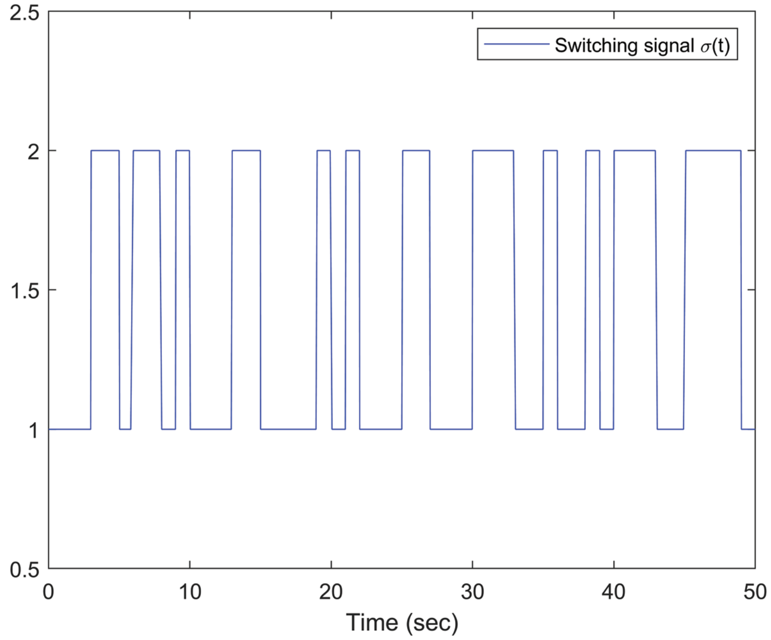

FIGURE 9 Switching signal $\sigma(t)$ of Example 2

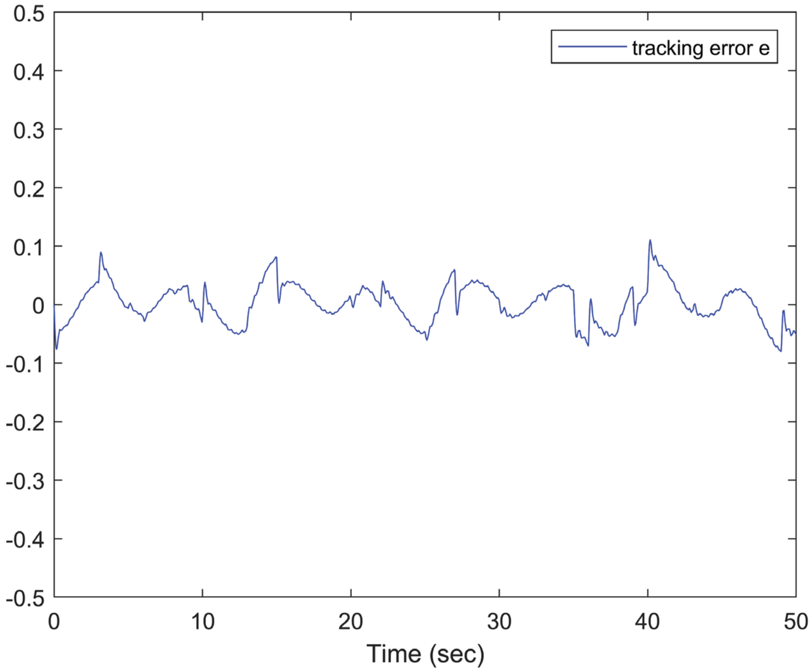

FIGURE 10 The tracking error of Example 2

where $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0$, $\sigma(t) : [0, \infty] \to M = \{1, 2\}$, $f_{1,1} = -2x_1 e^{-0.5x_1}$, $f_{1,2} = -2\sin(x_1) e^{-0.5x_1}$, $f_{2,1} = -x_1 \cos(x_2^2)$, $f_{2,2} = -x_1 \sin(x_2^2)$, $f_{3,1} = x_2 x_3$, $f_{3,2} = x_1 x_2 x_3$, $d_{1,1} = 0.2 \sin t \cos t$, $d_{1,2} = 0.1 \sin t \cos t$, $d_{2,1} = 0.2 \sin t$, $d_{2,2} = 0.1 \cos t$, $d_{3,1} = 0.2 \cos t$, $d_{3,2} = 0.1 \sin t$. The reference signal is chosen as $y_d = 0.5 \sin t$. Input delay is set to $\tau = 0.01$.

According to Theorem 1, the control structure of system (59) can be designed as follows

$$\alpha_i = -r_i z_i - \frac{1}{2\ell_i^2} z_i \hat{\theta}_i P_{m_i}^{\mathrm{T}} P_{m_i}, \quad i = 1, 2,$$

$$u = -r_3 z_3 - \frac{1}{2\ell_3^2} z_3 \hat{\theta}_3 P_{m_3}^{\mathrm{T}} P_{m_3},$$
$$\dot{\hat{\theta}}_i = -\eta_i \hat{\theta}_i + \frac{1}{2\ell_i^2} z_i^2 P_{m_i}^{\mathrm{T}} P_{m_i}, \quad i = 1, 2, 3, \tag{60}$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $z_3 = x_3 - \alpha_2$, $\boldsymbol{z}_1 = [z_1]^{\mathrm{T}}$, $\boldsymbol{z}_2 = [z_1, z_2]^{\mathrm{T}}$, and $\boldsymbol{z}_3 = [z_1, z_2, z_3]^{\mathrm{T}}$. The parameters of the design controller are as follows $r_1 = 18.5$, $r_2 = 3.5$, $r_3 = 16.5$, $\eta_1 = 6$, $\eta_2 = 1$, $\eta_3 = 0.2$, $\ell_1 = 1$, $\ell_2 = 1$, $\ell_3 = 1$. The simulation results are shown in Figures 1–5.

Figure 1 depicts the tracking performance. It can be seen from Figure 1 that the proposed MTN-based controller has achieved satisfactory tracking control effect. Figures 2–4 describe the trajectories of the control input $u(t)$, state variables x_2, x_3, and switching signal $\sigma(t)$, respectively. Figure 5 displays the trajectory of the tracking error. A small region where the tracking error converges to the origin can be obtained. Through the simulation results in Figures 1–5, it is proved that all signals in the controlled system are bounded, and the control strategy proposed in this article is effective.

Example 2. The continuous stirred tank reactor (CSTR) with two modes feed stream is used to verify the effectiveness of the proposed controller, which can be modeled as switched nonlinear system[54] as follows.

$$\begin{cases} \dot{x}_1 = f_{1,\sigma(t)} + x_2, \\ \dot{x}_2 = g_{\sigma(t)} u, \\ y = x_1, \end{cases} \tag{61}$$

where $x_1(0) = x_2(0) = 0$, $\sigma(t) : [0, \infty] \rightarrow M = \{1, 2\}$, $f_{1,1}(x_1) = 0.1x_1$, $f_{1,2}(x_1) = x_1$, $g_1 = g_2 = [1, 1]$. The reference signal is designed as $y_d = 0.5(\sin t + \sin(0.5t))$. Input delay is set to $\tau = 0.01$. The parameters of the design controller are as follows $r_1 = 15$, $r_2 = 10$, $\eta_1 = 0.4$, $\eta_2 = 0.1$, $\ell_1 = \ell_2 = 1$. The simulation results are shown in Figures 6–10.

The control strategy presented in this article still has satisfactory results for practical systems, which further proves the effectiveness of the controller proposed in this article.

Example 3. (Comparative example): In order to further verify the effectiveness of the control strategy proposed in this article, a comparative experiment between the adaptive MTN and RBFNN control methods is gave on the basis of Example 1. The simulation comparison result is shown in Figure 11.

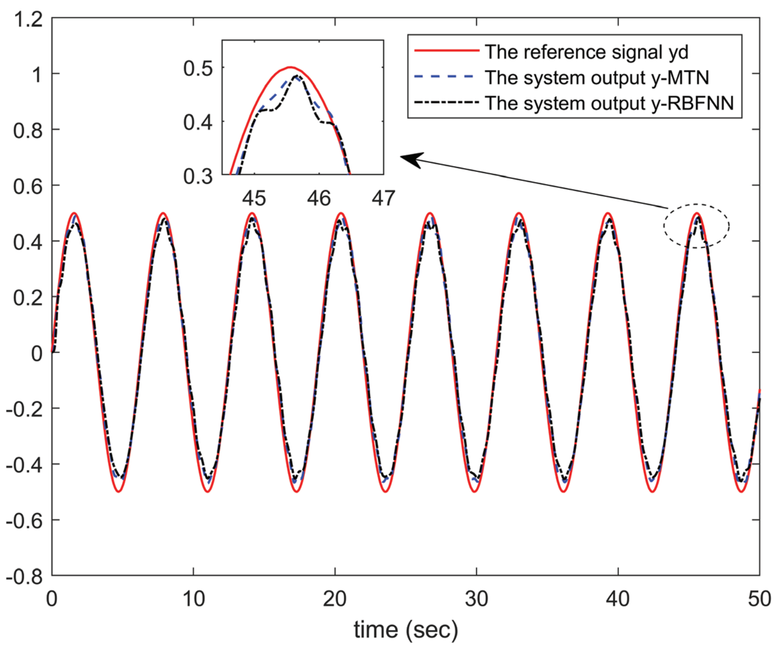

FIGURE 11 The tracking performance comparison of MTN and RBFNN

The simulation results show that the controller based on MTN and the controller based on RBFNN can achieve satisfactory tracking control effect. However, the proposed control strategy can obtain similar results to RBFNN with lower computational cost.

5 CONCLUSION

In this article, the tracking control problem for a class of switched nonlinear systems subject to input delay is studied. First, Padé approximation method and Laplace transform are used to copy with the input delay problem in the system, and a new variable is introduced to eliminate the effect of input delay. Furthermore, an adaptive MTN tracking controller is constructed by combining the approximating ability of MTN and backstepping technology. In addition, the signals in the closed-loop system are bounded. It is worth noting that MTN technology is applied for the first time to switched nonlinear systems subject to input delay. In the end, three examples are presented to verify the effectiveness of the proposed control strategy.

Future research will be focused on adaptive MTN controller design for switched nonlinear systems with output constraint based on the developed results of this article.

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