Adaptive Multi-dimensional Taylor Network Control for Stochastic Nonlinear Systems with Full State Constraints and Dead-zone Input

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Abstract: In this paper, the stochastic nonlinear systems with full state constraints and dead-zone input are considered in the same framework for the first time, and a new adaptive controller based on MTN is designed. Firstly, for the dead-zone input constraint problem, the characteristic function is introduced to avoid the nonlinearity of the dead-zone. Secondly, in the process of adaptive control design, MTN technology is employed to approximate the nonlinear structure generated in backstepping, and a simple controller is designed. By constructing the barrier Lyapunov functions (BLFs) to deal with the full state constraint problem, the proposed scheme ensures that all states of the closed-loop system do not exceed the constraint limit. Then, Lyapunov stability theory is used to analyze the stability. Finally, two simulation examples are given to illustrate the effectiveness and applicability of the proposed scheme.

Key Words: Adaptive control, dead-zone, full state constraints, multi-dimensional Taylor network

1 INTRODUCTION

In recent years, with the continuous exploration of science and technology, the actual engineering systems need better control performance, and the random interference as the source of system instability has attracted more and more attention. This promotes the research of stochastic nonlinear systems [1-4]. Among many control methods, adaptive control method is widely used in the research of control problems of stochastic nonlinear systems because it is more suitable for the study of uncertain problems, and has become one of the important methods to study stochastic nonlinear systems [4, 5].

With the development of research, various intelligent control methods based on neural networks (NNs) and fuzzy logic systems (FLSs) control methods are proposed. Due to the advantages of adaptive control, combining the above methods with adaptive control method has produced a series of interesting results [6-9]. In recent years, a new network control method, multi-dimensional Taylor network (MTN) control technology was proposed in [10]. This method is approximated the uncertainty in the system in a simple polynomial combination, which can complete the approximation process in a short period of time, and carry lower computing costs. Once proposed, it is soon widely used in nonlinear systems [11-14], and even extended to stochastic nonlinear systems [15, 16]. However, the above results rarely consider the full state constraint problem.

In fact, due to physical restrictions, performance and security requirements, the actual control system is often affected by non-smooth nonlinear input constraints such as time delay, saturation and hysteresis [8, 12, 13, 17], which urges the study of constrained systems control. On the one hand, the existence of state constraints often leads to the decline of system control performance. In order to realize the state constrained control, the BLFs are usually constructed, and then the controller is designed based on the NNs or the FLSs [18-20]. Similarly, the MTN control technology is also used in the control research of full state constraint problems [21, 22]. Unfortunately, these achievements are difficult to take into account dead zone input constraints.

On the other hand, the emergence of dead-zone will reduce the system performance and make it difficult for the system to achieve stability, which has become a hot issue of widespread concern. In recent years, in order to solve the dead-zone problem, the method of constructing dead-zone inverse is mostly used for control research [23, 24]. So far, for systems with dead-zone nonlinearity at the input, a series of remarkable achievements have been made in NNs or FLSs [25-27]. Unfortunately, the control research on the combination of full-state constraint problem and dead zone input is not enough, and the control research based on MTN method is also blank.

Driven by the above discussion, this paper studies the adaptive MTN control for a class of stochastic nonlinear systems with full state constraints and dead-zone input. Firstly, the characteristic function is introduced to deal with the dead-zone nonlinearity, and BLFs are used to constrain states of the system. Then, the MTN technology is integrated into the backstepping process, and a new controller is proposed. Finally, the effectiveness of the proposed scheme is proved. The main innovations of this paper are as follows: 1) This is the first time to propose an adaptive control strategy based on MTN for stochastic nonlinear systems with full state constraints and dead-zone input; 2) Compared with the dead-zone inverse constructed by [24] , this paper introduces the characteristic function to

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deal with the dead-zone nonlinearity, the method is simple and the amount of calculation is reduced. In addition, this paper additionally considers the full state constraint problem, and uses BLFs to deal with this problem. The problems considered are more comprehensive, and the proposed scheme obtains satisfactory results; 3) Although authors in [19, 20, 26, 27] studied the full state constraint or dead-zone input problem, it don't consider the two kinds of constraint problems at the same time. Although the author considered both kinds of problems in [28], this paper considered the control problem of stochastic nonlinear systems on this basis, which is more complex than the research in this paper. Therefore, the research of this paper is of great significance.

2 PROBLEM PREPARATION

2.1 Problem description

Consider the following stochastic nonlinear systems with input dead-zone and full state constraints

$$\begin{cases} dx_i = \left(\psi_i\left(\boldsymbol{x}_i\right)+x_{i+1}\right)dt+g_i^{\mathrm{T}}\left(\boldsymbol{x}_i\right)d\omega \\ dx_n = \left(\psi_n\left(\boldsymbol{x}_n\right)+u\right)dt+g_n^{\mathrm{T}}\left(\boldsymbol{x}_n\right)d\omega \\ y = x_1 \end{cases} \tag{1}$$

where $\boldsymbol{x}_i = \left[x_1, x_2, ..., x_i\right]^{\mathrm{T}} \in \Re^i$, for $i=1,...,n-1$ are the state vectors of the system, x_{i+1} is the variable of the system. $y \in \Re$ denote the system output, and u represents the output of the dead-zone, ω is the r -dimensional standard Brownian motion defined on a complete probability space; $\psi_i\left(\boldsymbol{x}_i\right): \Re^i \to \Re$, and $g_i\left(\boldsymbol{x}_i\right): \Re^i \to \Re^r$ are the unknown smooth nonlinear functions. In addition, all states are constrained in set $\aleph_{x_i} = \left\{x_i \in \Re \| x_i \right| < h_{a,i} \}$, where $h_{a,i}$ are positive constants. In particular, the output of the dead-zone u can be expressed as follows:

$$u = T\left(\tau\right) = \begin{cases} \varphi_r\left(\tau\right), & \tau \geq s_r \\ 0, & s_l < \tau < s_r \\ \varphi_l\left(\tau\right), & \tau \leq s_l \end{cases} \tag{2}$$

where τ is the input of dead-zone and s_r and s_l are sign determined unknown bounded constants.

The control target: A suitable adaptive controller should be designed so that 1) the output of the system tracks the desired trajectory, and 2) all states do not exceed the limit.

Assumption 1 [18]**:** The desired signal y_d satisfies $|y_d| \leq \beta_0 \leq h_{a,1}$ and $|y_d^{(i)}| \leq \varpi_i$, where $\beta_0 > 0$ and $\varpi_i > 0$ are constants. y_d and $y_d^{(i)}$, $i=1,...,n$ are continuous and bounded.

According to [27]**,** the dead-zone can be rewritten as

$$u = T\left(\tau\right) = \Psi^{\mathrm{T}}\left(t\right)\hbar\left(t\right)\tau + D\left(\tau\right) \tag{3}$$

where $\hbar = \left[\phi_r, \phi_l\right]^{\mathrm{T}}$ with $\phi_r = \begin{cases} 1, & \tau > s_l \\ 0, & \tau \leq s_l \end{cases}$, and $\phi_l = \begin{cases} 1, & \tau < s_r \\ 0, & \tau \geq s_r \end{cases}$. $\left|D\left(\tau\right)\right| \leq \bar{d}$ with $\bar{d}$ is an unknown positive constant. And $\Psi = \left[\Psi_r, \Psi_l\right]^{\mathrm{T}}$ with $\Psi_r = \begin{cases} 0, & \tau \leq s_l \\ \varphi_r', s_l < \tau < +\infty \end{cases}$ and $\Psi_l = \begin{cases} \varphi_l', -\infty < \tau < s_r \\ 0, & \tau \geq s_r \end{cases}$. We can easily get $\eta_0 \leq \Psi^{\mathrm{T}}\left(t\right)\hbar\left(t\right) \leq \bar{d}$ with $\eta_0 > 0$ is a constant.

2.2 Multidimensional Taylor network

MTN is a new type of NN with special three-layer structure [15], according to **Weierstrass Theorem**, the following Lemma is true:

Lemma 1: For a compact set Λ, if $f\left(\boldsymbol{S}\right): \Re^n \to \Re$ is a continuous function, then for $\forall \varepsilon > 0$, there is always an MTN $\boldsymbol{\theta}^{\mathrm{T}} P_{m_n}\left(\boldsymbol{S}\right)$ that can be used for approximation $f\left(\boldsymbol{S}\right)$, such that

$$f\left(\boldsymbol{S}\right) = \boldsymbol{\theta}^{\mathrm{T}} P_{m_n}\left(\boldsymbol{S}\right) + \delta\left(\boldsymbol{S}\right) \tag{4}$$

where $\boldsymbol{S} = \left[s_1, ..., s_n\right]^{\mathrm{T}} \in \Re^n$, $\boldsymbol{\theta} = \left[\theta_1, ..., \theta_l\right]^{\mathrm{T}} \in \Re^l$, and $P_{m_n}\left(\boldsymbol{S}\right) = \left[s_1, ..., s_n, s_1^2, ..., s_1^m, ..., s_n^m\right]^{\mathrm{T}}$, n and m are the input number and highest power of MTN intermediate layer. $\delta\left(\boldsymbol{S}\right)$ is the error between $f\left(\boldsymbol{S}\right)$ and $\boldsymbol{\theta}^{\mathrm{T}} P_{m_n}\left(\boldsymbol{S}\right)$, and it satisfies $|\delta\left(\boldsymbol{S}\right)| \leq \varepsilon$.

Lemma 2 [18]**:** Exist a positive constant k_b, for $\forall z \in R$, if z satisfies $|z| \leq k_b$, then the following inequality holds:

$$\log \frac{k_b^{2p}}{k_b^{2p} - z^{2p}} < \frac{z^{2p}}{k_b^{2p} - z^{2p}} \tag{5}$$

where $\log\left(\bullet\right)$ is the logarithm of , and p is a positive constant.

3 MAIN RESULTS

3.1 Adaptive control based on MTN

First, make the following coordinate transformation

$$\begin{cases} z_1 = x_1 - y_d \\ z_i = x_i - \alpha_{i-1}, i = 1, ..., n \end{cases} \tag{6}$$

where α_{i-1} is virtual control signal will be designed later.

Step 1: The derivative of z_1 with respect to t can be calculated as follows

$$dz_1 = \left(\psi_1 + x_2 - \dot{y}_d\right)dt + g_1^{\mathrm{T}} d\omega \tag{7}$$

Selecting the first Lyapunov candidate function as follows

$$V_1 = \frac{1}{4}\log \frac{k_1^4}{k_1^4 - z_1^4} + \frac{1}{2}\tilde{\boldsymbol{\vartheta}}_1^{\mathrm{T}} \tilde{\boldsymbol{\vartheta}}_1 \tag{8}$$

where $\tilde{\boldsymbol{\vartheta}}_i = \boldsymbol{\vartheta}_i - \hat{\boldsymbol{\vartheta}}_i$, $i=1,2,...,n$ is parameter error. $k_1 = h_{a,1} - \beta_1$, and β_i, $i=1,2,...,n$ is positive constant.

The following formula can be calculated

$$LV_1 = \frac{z_1^3 \dot{z}_1}{\left(k_1^4 - z_1^4\right)} + \frac{z_1^2\left(3k_1^4 + z_1^4\right)}{2(k_1^4 - z_1^4)^2}\|g_1\|^2 - \tilde{\boldsymbol{\vartheta}}_1^{\mathrm{T}} \dot{\hat{\boldsymbol{\vartheta}}}_1 \tag{9}$$

Using Young's inequality, the following inequality is true

$$\begin{aligned} LV_1 \le & \frac{z_1^3}{k_1^4 - z_1^4}\left(x_2 + G_1\right) - \frac{3z_1^4}{4}\zeta_1^{\frac{4}{3}}\left(k_1^4 - z_1^4\right)^{-\frac{4}{3}} \\ & - \frac{3z_1^4}{4}\xi_1^{\frac{4}{3}}\left(k_1^4 - z_1^4\right)^{-\frac{4}{3}} + \frac{\varsigma_1^2}{4} - \tilde{\boldsymbol{\vartheta}}_1^{\mathrm{T}} \dot{\hat{\boldsymbol{\vartheta}}}_1 \end{aligned} \tag{10}$$

where

$$\begin{aligned} G_1 = & \psi_1 - \dot{y}_d + \frac{1}{4\varsigma_1^2}\frac{z_1\left(3k_1^4 + z_1^4\right)^2}{(k_1^4 - z_1^4)^3}\|g_1\|^4 \\ & + \frac{3}{4}z_1\left(\xi_1^{\frac{4}{3}} + \zeta_1^{\frac{4}{3}}\right)\left(k_1^4 - z_1^4\right)^{-\frac{1}{3}} \end{aligned}$$

with ς_1, ζ_1 and ξ_1 are positive constants.

It's easy to find that G_1 is an unknown nonlinear function, according to **Lemma 1**, for $\forall \varepsilon_1 > 0$, there must be an MTN structure $\boldsymbol{\vartheta}^{\mathrm{T}} P_{m_1}(\boldsymbol{S})$, such that

$$G_1 = \boldsymbol{\vartheta}_1^{\mathrm{T}} P_{m_1}\left(\boldsymbol{z}_1\right) + \delta_1\left(\boldsymbol{z}_1\right), \quad \left|\delta_1\left(\boldsymbol{z}_1\right)\right| \le \varepsilon_1 \tag{11}$$

where $\boldsymbol{z}_1 = \left[z_1\right]^{\mathrm{T}}$ is the input vector of MTN.

By employing **Young's inequality**, and designed the first virtual controller as follows

$$\alpha_1 = -\sigma_1 z_1 - \hat{\boldsymbol{\vartheta}}_1^{\mathrm{T}} P_{m_1}, \quad \sigma_1 > 0 \tag{12}$$

where $\sigma_i, i = 1,...,n$ is a positive design parameter. Then, equation (10) can be simplified as follows

$$\begin{aligned} LV_1 \le & -\sigma_1\frac{z_1^4}{k_1^4 - z_1^4} + \tilde{\boldsymbol{\vartheta}}_1^{\mathrm{T}}\left(P_{m_1}\frac{z_1^3}{k_1^4 - z_1^4} - \dot{\hat{\boldsymbol{\vartheta}}}_1\right) \\ & + \frac{1}{4\zeta_1^4}\varepsilon_1^4 + \frac{1}{4\xi_1^4}z_2^4 + \frac{\varsigma_1^2}{4} \end{aligned} \tag{13}$$

Step $2 \le i \le n-1$: According to $z_i = x_i - \alpha_{i-1}$, we can easily get the derivative of z_i as follows

$$dz_i = \left(x_{i+1} + \psi_i - \nabla\alpha_{i-1}\right)dt + \tilde{g}_i^{\mathrm{T}} d\omega \tag{14}$$

with $\nabla\alpha_{i-1}$ is $\dot{\alpha}_{i-1}\left(\boldsymbol{x}_{i-1}, y_d, ..., y_d^{(i-1)}, \hat{\boldsymbol{\vartheta}}_1, ..., \hat{\boldsymbol{\vartheta}}_{i-1}\right)$ and $\tilde{g}_i = g_i - \sum_{j=1}^{i-1}\left(\partial\alpha_{i-1}/\partial x_j\right)g_j$, $i = 2,3,...,n$.

Selecting the $i-$th Lyapunov candidate function as follows

$$V_i = \frac{1}{4}\log\frac{k_i^4}{k_i^4 - z_i^4} + \frac{1}{2}\tilde{\boldsymbol{\vartheta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\vartheta}}_i + V_{i-1} \tag{15}$$

where $k_i = h_{a,i} - \beta_{i-1}$, $\left|\alpha_{i-1}\right| \le \beta_{i-1}$.

Similar to **Step 1**, the following inequality can get

$$\begin{aligned} LV_i \le & -\sum_{j=1}^{i}\sigma_j\log\frac{z_j^4}{k_j^4 - z_j^4} + \frac{1}{4\xi_i^4}z_{i+1}^4 + \sum_{j=1}^{i}\frac{\varsigma_j^2}{4} \\ & + \sum_{j=1}^{i}\frac{1}{4\zeta_j^4}\varepsilon_j^4 + \sum_{j=1}^{i}\tilde{\boldsymbol{\vartheta}}_j^{\mathrm{T}}\left(\frac{z_j^3}{k_j^4 - z_j^4}P_{m_j} - \dot{\hat{\boldsymbol{\vartheta}}}_j\right) \end{aligned} \tag{16}$$

The virtual control signal α_i is selected as follows

$$\alpha_i = -\sigma_i z_i - \hat{\boldsymbol{\vartheta}}_i^{\mathrm{T}} P_{m_i}, \quad \sigma_i > 0 \tag{17}$$

Step n **:** It is easy to calculate the derivative of z_n as follows

$$dz_n = \left(\Psi^{\mathrm{T}}\hbar\tau + D + \psi_n - \nabla\alpha_{n-1}\right)dt + \tilde{g}_n^{\mathrm{T}} d\omega \tag{18}$$

Selecting the $n-$th Lyapunov candidate function as follows

$$V_n = \frac{1}{4}\log\frac{k_n^4}{k_n^4 - z_n^4} + \frac{1}{2}\tilde{\boldsymbol{\vartheta}}_n^{\mathrm{T}}\tilde{\boldsymbol{\vartheta}}_n + V_{n-1} \tag{19}$$

Then the following formula can be calculated

$$\begin{aligned} LV_n = & \frac{z_n^3\left(\Psi^{\mathrm{T}}\hbar\tau + D + G_n\right)}{k_n^4 - z_n^4} - \frac{z_n^4}{4\xi_{n-1}^4} - \tilde{\boldsymbol{\vartheta}}_n^{\mathrm{T}}\dot{\hat{\boldsymbol{\vartheta}}}_n \\ & - \frac{3z_n^4 + 3z_n^4\zeta_n^{\frac{4}{3}}}{4}\left(k_n^4 - z_n^4\right)^{-\frac{4}{3}} + \frac{\varsigma_n^2}{4} + LV_{n-1} \end{aligned} \tag{20}$$

According to **Lemma 1**, for $\forall \varepsilon_n > 0$, there must be an MTN structure $\boldsymbol{\vartheta}_n^{\mathrm{T}} P_{m_n}(\boldsymbol{S})$, such that

$$G_n = \boldsymbol{\vartheta}_n^{\mathrm{T}} P_{m_n}\left(\boldsymbol{z}_n\right) + \delta_n\left(\boldsymbol{z}_n\right), \quad \left|\delta_n\left(\boldsymbol{z}_n\right)\right| \le \varepsilon_n \tag{21}$$

where $\boldsymbol{z}_n = \left[z_1, z_2, ..., z_n\right]^{\mathrm{T}}$ is the input vector of MTN.

Using Young's inequality, and construct the actual control input signal τ and adaptive law $\dot{\hat{\boldsymbol{\vartheta}}}_i$ as follows

$$\tau = -\frac{1}{\eta_0}\left(\mu_n\left|z_n\right| + \left|\hat{\boldsymbol{\vartheta}}_n^{\mathrm{T}} P_{m_n}\right|\right)\mathrm{sgn}\left(z_n\right) \tag{22}$$

$$\dot{\hat{\boldsymbol{\vartheta}}}_i = -\mu_i\hat{\boldsymbol{\vartheta}}_i + \frac{z_i^3}{k_i^4 - z_i^4}P_{m_i}, \ i = 1,2,...,n \tag{23}$$

Substituting (22) and (23) into (20), according to **Lemma 2**, there is

$$\begin{aligned} LV_n = & -\sum_{i=1}^{n}\sigma_i\log\frac{k_i^4}{k_i^4 - z_i^4} + \sum_{i=1}^{n}\mu_i\tilde{\boldsymbol{\vartheta}}_i^{\mathrm{T}}\hat{\boldsymbol{\vartheta}}_i \\ & + \sum_{i=1}^{n}\frac{1}{4\zeta_i^4}\varepsilon_i^4 + \frac{1}{4}\bar{d}^4 + \sum_{i=1}^{n}\frac{\varsigma_i^2}{4} \end{aligned} \tag{24}$$

3.2 Stability analysis

Theorem 1: Consider nonlinear system (1), which is consist of intermediate virtual control signals (12) and (17), control signal (22) and adaptive law (23). It can ensure that all signals of closed-loop systems are bounded, the tracking error converges to a small neighborhood of the origin, and all states are constrained within the given limits.

Proof: Select the following Lyapunov function as follows

$$V = \frac{1}{4}\sum_{i=1}^{n}\log\frac{k_i^4}{k_i^4 - z_i^4} + \frac{1}{2}\sum_{i=1}^{n}\tilde{\boldsymbol{\vartheta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\vartheta}}_i \tag{25}$$

For term $\mu_i\tilde{\boldsymbol{\vartheta}}_i^{\mathrm{T}}\hat{\boldsymbol{\vartheta}}_i$ in inequality (24), it is pretty easy to obtain that $\mu_i\tilde{\boldsymbol{\vartheta}}_i^{\mathrm{T}}\hat{\boldsymbol{\vartheta}}_i \le -\frac{1}{2}\mu_i\tilde{\boldsymbol{\vartheta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\vartheta}}_i + \frac{1}{2}\mu_i\boldsymbol{\vartheta}_i^{\mathrm{T}}\boldsymbol{\vartheta}_i$.

Then, one has

$$\begin{aligned} LV \le & -\sum_{i=1}^{n}\sigma_i\log\frac{k_i^4}{k_i^4 - z_i^4} - \frac{1}{2}\sum_{i=1}^{n}\mu_i\tilde{\boldsymbol{\vartheta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\vartheta}}_i \\ & + \frac{1}{2}\sum_{i=1}^{n}\mu_i\boldsymbol{\vartheta}_i^{\mathrm{T}}\boldsymbol{\vartheta}_i + \sum_{i=1}^{n}\frac{1}{4\zeta_i^4}\varepsilon_i^4 + \sum_{i=1}^{n}\frac{\varsigma_i^2}{4} \end{aligned} \tag{26}$$

Let

$$B=\frac{1}{2}\sum_{i=1}^{n}\mu_i\vartheta_i^{\mathrm{T}}\vartheta_i+\sum_{i=1}^{n}\frac{1}{4\zeta_i^4}\varepsilon_i^4+\sum_{i=1}^{n}\frac{\varsigma_i^2}{4}$$

$$A=\min\{4\sigma_i,2\mu_i:i=1,\ldots,n\}$$

Then, we can get

$$LV\le -AV+B \tag{27}$$

Next, the conclusion of **Theorem 1** follows readily by using the similar analytical method in the work of [9].

4 SIMULATION RESULTS

Example 1 (Numerical example): Consider the following third-order stochastic nonlinear systems

$$\begin{cases} dx_1=x_2dt-x_1^3\cos x_1 d\omega \\ dx_2=\left(x_3+\sin\left(\frac{1}{4}(x_1+x_2)\right)\right)dt+x_2^2\cos x_1 d\omega \\ dx_3=(u+x_1x_2x_3)dt+\sin x_3 d\omega \\ y=x_1 \end{cases} \tag{28}$$

where the initial condition is $x_1(0)=x_2(0)=x_3(0)=0$, the reference signal $y_d=\sin t$, and the parameters of dead-zone $T(\tau)$ are designed as $\varphi_r(\tau)=1.5(\tau-2.5)$, $\varphi_l(\tau)=\tau-0.5$, $s_r=2$, and $s_l=-0.8$.

Based on **Theorem 1**, the intermediate virtual control signal, actual control law and adaptive law are designed as formula (17), (22) and (23). The design parameters are selected as $\mu_1=1$, $\mu_2=2$, $\mu_3=5$, $\sigma_1=6$, $\sigma_2=6$, $\sigma_3=15$, $\eta_0=1$. The state constraints are selected as $|x_1|\le 1.03$, $|x_2|\le 1.5$, and $|x_3|\le 4.5$. The simulation results are shown in Figs 1-5.

Fig 1 shows a trajectory of the output y tracking desired signal y_d. It can be seen that the tracking effect is satisfactory according to Fig 3. Fig 2 shows the curve of system states x_2 and x_3, and it can be seen from the figure that all states are limited within a given constraint range. Figs 4 and 5 show the trajectories of dead-zone input u and control input τ. It can be seen from Figs 1-5 that all signals of the closed-loop system are bounded.

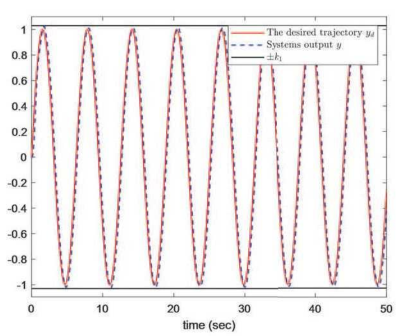

Fig 1. The output y tracks the curve of the desired trajectory y_d

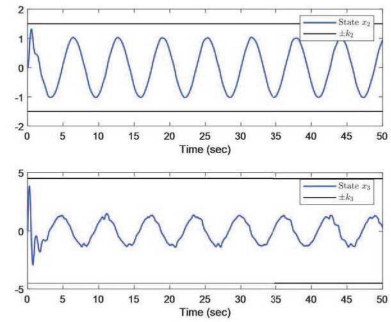

Fig 2. The trajectories of states x_2 and x_3

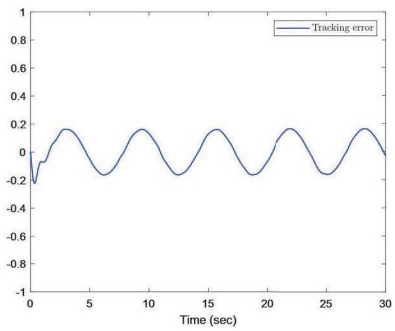

Fig 3. Trajectory curve of tracking error

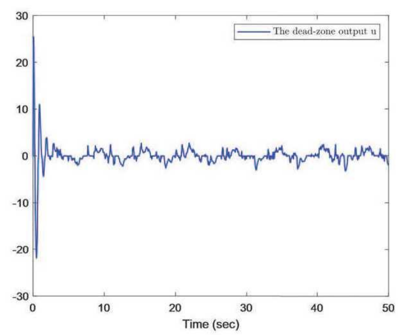

Fig 4. Output of system dead-zone input u

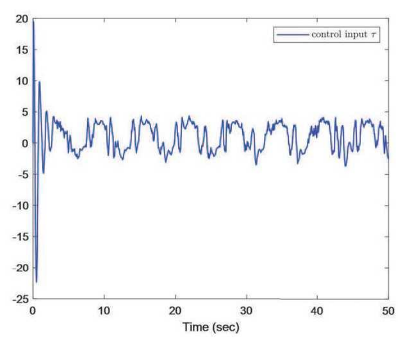

Fig 5. The trajectory of τ in the example

Example 2 (Practical example): As shown in [29], continuous stirred tank reactor is considered, and the system is as follows

$$\begin{cases} dx_1 = (x_2 + 0.1x_1)dt + 0.1x_1 d\omega \\ dx_2 = udt \\ y = x_1 \end{cases} \tag{29}$$

with the initial condition $x_1(0) = x_2(0) = 0$, the reference signal is defined as $y_d = 0.5\sin t$, where the parameters of dead-zone $T(\tau)$ are designed as $\varphi_r(\tau) = 0.1(\tau - 2.5)^2 + (\tau - 2.5)$, $\varphi_l(\tau) = \tau + 1.5$, $s_r = 2.5$, and $s_l = -1.5$.

Based on **Theorem 1**, the intermediate virtual control signal, actual control law and adaptive law are designed as formula (20), (25) and (26). The design parameters are selected as $\mu_1 = 0.1$, $\mu_2 = 0.1$, $\sigma_1 = 20$, $\sigma_2 = 10$, $\eta_0 = 2$. All state constraints are $|x_1| \le 0.7$ and $|x_2| \le 0.7$. The simulation results are shown in Figs 6-10.

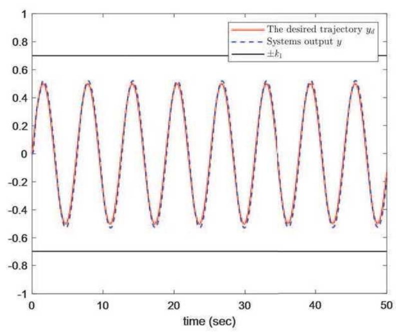

Fig 6. The output y tracks the curve of the desired trajectory y_d

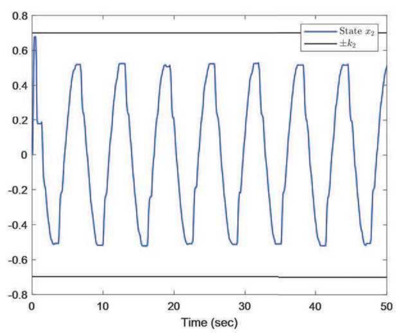

Fig 7. The trajectories of states x_2

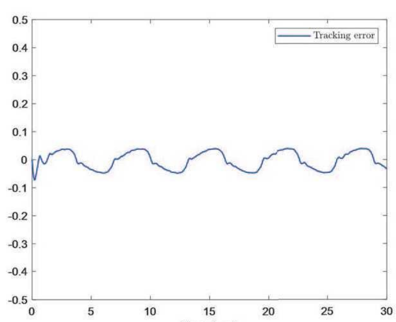

Fig 8. Trajectory curve of tracking error

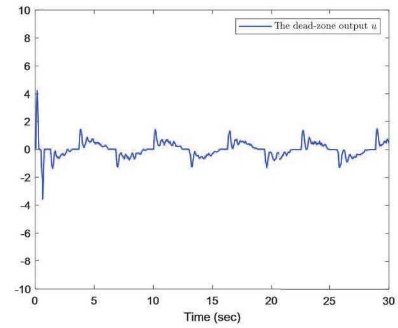

Fig 9. Output of system dead-zone input u

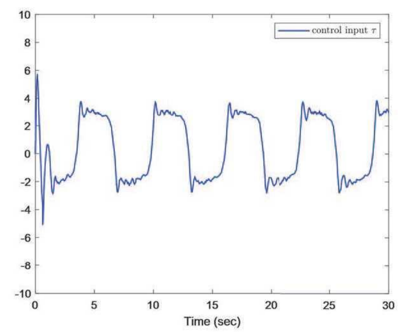

Fig 10. The trajectory of τ in the example

It can be seen from Figs 6-10 that the simulation effect we obtained is satisfactory, which shows the applicability of the proposed scheme.

5 CONCLUSION

In this paper, the problem of adaptive control for a class of stochastic nonlinear systems with full state constraints and dead-zone input is studied. Firstly, the dead-zone nonlinearity is cleverly avoided by introducing the characteristic function. Secondly, a new adaptive controller based on backstepping design technology is proposed by combining MTN control technology with BLFs in the process of control design. The stability of the closed-loop system is proved by using Lyapunov stability theorem. All signals are bounded, and all states can't violate the given constraints. Finally, the effectiveness of the designed controller is verified by two simulation examples.

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