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Control Design for Stochastic Nonlinear Systems with Full-state Constraints and Input Delay: A New Adaptive Approximation Method

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Abstract: In this paper, the full state constraints and input delay of stochastic nonlinear systems are studied. A new adaptive control algorithm is proposed using backstepping approach and multi-dimensional Taylor network (MTN) method. Firstly, the input delay problem is dealt with by introducing a new variable using the Padé approximation with Laplace transform. Secondly, MTNs are employed to approximate unknown nonlinear functions, and the barrier Lyapunov functions (BLFs) are constructed to deal with the state constraints. Based on this, a new approximation-based adaptive controller is proposed. Thirdly, it is proved that the proposed control method can ensure that all signals in the closed-loop system are semi-global ultimately uniformly bounded (SGUUB) in probability and the tracking error converges to a small neighborhood of the origin. Finally, two simulation examples are given to illustrate the effectiveness of the proposed design method.

Keywords: Adaptive control, full-state constraints, input delay, multi-dimensional Taylor networks, stochastic nonlinear systems.

1. INTRODUCTION

As is known to all, the controlled systems of practical application are intrinsically nonlinear and uncertain, and often affected by external disturbances. Therefore, the research on the control theory of stochastic nonlinear systems will contribute values to the cybernetics development theoretically and practically, which has gradually been regarded by more and more researchers [1,2]. So far, many control methods of general nonlinear systems have been extended to stochastic cases, such as adaptive control [3–5], backstepping methods [6,7], Lyapunov stability theory [8], sliding mode control [9], fault-tolerant control [10,11], LaSalle invariance principle [12] and so on. Among them, adaptive control is of great significance for the study of uncertain systems since this method is not required to linearize the nonlinear system. Consequently, it also has become an attractive technique for stochastic nonlinear systems [4,5]. However, adaptive control is no longer fitted for the systems with uncertain structure.

In order to solve the above problems, many methods based on neural networks (NNs) or fuzzy logic systems (FLSs) have been proposed and widely used for nonlinear systems [13–15], switched systems [16–20], multiple-input multiple-output (MIMO) nonlinear systems [21,22], nonlinear time-varying delay systems [23,24] and so on. Meantime, the above methods have also obtained a lot of significant research results in stochastic nonlinear systems [25–27]. Nevertheless, the above control structures still inherit the drawbacks of NNs or FLSs. Fortunately, a novel network control method, multi-dimensional Taylor network (MTN), with the simple structure was conceived in [28]. The core idea of MTN is to copy with the unknown nonlinear functions on closed interval by the linear combination of polynomials, which improves the approximation performance and reduces the computational complexity. The MTN-based control approach has drawn the extensive attention, and many noteworthy achievements have been obtained for MIMO nonlinear systems [29], single-input single-output (SISO) nonlinear systems [30], stochastic nonlinear systems [31], large-scale stochastic nonlinear systems [32], nonlinear systems with input saturation [33–35], MIMO discrete-time nonlinear systems [36], large-scale nonlinear systems [37] and so on. However, the above control algorithms can not be directly available to nonlinear systems with input constraints. In particular, there are few studies on considering both input delay and state constraints simultaneously.

In fact, input delays can not be avoided in the application of industry. In particular, the existence of input delay seriously affects the control performance of the system. Therefore, it is extremely important to study the

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control problems for the systems with input delay phenomenon [38,39]. With further research, the adaptive control method also has been generalized to stochastic nonlinear systems with input delay [40]. On the other hand, whether in the practical engineering system or in the theoretical control system, the full state constraints is a problem that needs to be considered [23,24]. The significant reason is that the constraints are violated, and the control performance of the systems will be degraded, unstable or even damaged. Hence, more and more scholars are interested in these kinds of systems and many achievements have been gotten [41,42]. Unfortunately, there are few results in resolving the problem of input delay and full-state constraints for stochastic nonlinear systems under a unified framework, which prompts the research of this paper.

Based on the above discussion, this paper concerns with the full-state constraints and the input delay problem for a class of stochastic nonlinear systems, and a MTN-based adaptive tracking control method is proposed. The results show that the proposed control scheme is feasible and can ensure that the tracking error falls within the given range when all signals of the closed-loop system are restricted. The main innovations of this paper are as follows:

1) A new adaptive control algorithm using MTN-based method is first proposed for the stochastic nonlinear systems with full-state constraints and input delay. With the help of MTN, although the authors in [40] studied the problem of stochastic nonlinear systems with input delay, they failed to consider the issue of full-state constraints. In [23,41], the authors addressed the problem of full-state constraints, however, the controlled plants were restricted to nonlinear systems rather than stochastic nonlinear systems with input delay. Moreover, the authors in [42] focused on the stochastic systems with full state constraints, but not considering the existence of input delay. Thus, the problem in our study is undoubtedly considered as representation.
2) Although MTN-based control algorithms have been proposed for stochastic nonlinear systems in [31–33], they can not be directly used to deal with the control problem considered in this paper. That is to say, the research in this paper broadens the development of MTN. In addition, by constructing the BLFs in each step of backstepping, and further design control laws and adaptive laws, the state constraints can be successfully realized.
3) The problem of input delay is handled by introducing a new variable, thus simplifying the process of controller design. In addition, owing to the simple structure of MTN, the controller design in this study has the advantages of simple structure and less calculation. These reasons render the control algorithm proposed in this paper implementation much easier.

2. PROBLEM EXPRESSION AND PREMISE

2.1. Problem description

In this paper, we consider a class of stochastic nonlinear system with input delay as follows:

$$\begin{cases} dx_i = \left(x_{i+1} + f_i\left(\bar{\boldsymbol{x}}_i\right)\right)dt + g_i^{\mathrm{T}}\left(\bar{\boldsymbol{x}}_i\right)d\omega, \\ \qquad i = 1,\ \ldots,\ n-1, \\ dx_n = \left(u\left(t-\tau\right) + f_n\left(\bar{\boldsymbol{x}}_n\right)\right)dt + g_n^{\mathrm{T}}\left(\bar{\boldsymbol{x}}_n\right)d\omega, \\ y = x_1, \end{cases} \tag{1}$$

where $\bar{\boldsymbol{x}}_n = [x_1, x_2, ..., x_n]^{\mathrm{T}} \in \mathbb{R}^n$ represents the state vector of the system, with $\bar{\boldsymbol{x}}_i = [x_1, x_2, ..., x_i]^{\mathrm{T}} \in \mathbb{R}^i$. $y \in \mathbb{R}$ is the output of system, u stands for the system input, ω is a r-dimensional standard Wiener process defined in complete probability space, $\tau > 0$ is the input delay of the system. Besides, $f_i\left(\bar{\boldsymbol{x}}_i\right) : \mathbb{R}^i \to \mathbb{R}$ and $g_i\left(\bar{\boldsymbol{x}}_i\right) : \mathbb{R}^i \to \mathbb{R}^r$ are unknown smooth nonlinear functions with $f_i\left(\mathbf{0}\right) = 0$ and $g_i\left(\mathbf{0}\right) = 0$.

This paper aims to design a control strategy to achieve the following three objectives

1) The system output y tracks the given reference signal y_d.
2) All signals of the closed-loop system are SGUUB in terms of probability.
3) All system states cannot violate the specified constraints, that is, for any $i = 1, ..., n$, there are $|x_i| < k_{i,c}$, where $k_{i,c}$ are positive constants.

Assumption 1: The reference signal y_d and its i-th derivative with respect to time are continuous and bounded, where $i = 1, 2, ..., n$.

2.2. Stability theory

In order to introduce the necessary concepts and lemmas, consider the following stochastic nonlinear system

$$d\boldsymbol{x} = f\left(\boldsymbol{x}\right)dt + g\left(\boldsymbol{x}\right)d\omega, \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the system state vector, $f\left(\boldsymbol{x}\right) : \mathbb{R}^n \to \mathbb{R}^n$ and $g\left(\boldsymbol{x}\right) : \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are local Lipschitz functions. In addition, $f\left(\boldsymbol{x}\right)$ and $g\left(\boldsymbol{x}\right)$ are meet the conditions $f\left(\mathbf{0}\right) = 0$ and $g\left(\mathbf{0}\right) = 0$.

Definition 1 [26,27]**:** Considering the system (2), for any two continuously differentiable functions $V\left(\boldsymbol{x}\right)$, define a differential operator L as follows:

$$LV\left(\boldsymbol{x}\right) = \frac{\partial V\left(\boldsymbol{x}\right)}{\partial \boldsymbol{x}} f\left(\boldsymbol{x}\right) + \frac{1}{2} Tr\left\{g^{\mathrm{T}} \frac{\partial^2 V\left(\boldsymbol{x}\right)}{\partial \boldsymbol{x}^2} g\right\}, \tag{3}$$

where $Tr\{\bullet\}$ is the trace of $\bullet$.

Definition 2 [26,27]**:** For any initial condition $\boldsymbol{x}_0 = \boldsymbol{x}(0)$ and a compact set defined in $\mathbb{R}^n$, the solution $\{\boldsymbol{x}(t), t \geqslant 0\}$ of the stochastic nonlinear system (2) is said to be SGUUB in p-th moment, if there exists a time constant $T = T\left(\varepsilon, \boldsymbol{x}_0\right)$, with $\varepsilon > 0$, satisfies $E\left[|\boldsymbol{x}(t)|^p\right] < \varepsilon$, for all $t > t_0 + T$. In particular, when $p = 2$, it is usually calld SGUUB in mean square, where $E[\bullet]$ stands for the expectation of stochastic variable $\bullet$.

Lemma 1 [26,27]**:** For system (2), if there exists a Lyapunov function $V(\boldsymbol{x})$: $\mathbb{R}^n \to \mathbb{R}$ satisfies the following properties: $V(\boldsymbol{x})$ is second order continuous differentiable, positive definite and radically unbounded, such that the following inequality holds

$$LV(\boldsymbol{x}) \leqslant -a_0 V(\boldsymbol{x}) + b_0, \tag{4}$$

where a_0 and b_0 are positive constants. Then the system (2) is bounded in probability, and has a unique solution.

Assumption 2: The reference signal y_d satisfy $|y_d| \leqslant \rho_0 \leqslant k_{i,c}$ and $|y_d| \leqslant \varpi_i$, where $\rho_0 > 0$ and $\varpi_i > 0$ are constants.

2.3. Transformation of input delay

As mentioned in [38], the problem of input delay can be solved by combining the Laplace transform technique with the Padé approximation method, also described as below:

Firstly, define a Laplace variable μ, and by the Laplace transform, one has

$$l\{u(t-\tau)\} = e^{-\tau\mu} l\{u(t)\} = \frac{e^{-\tau\mu/2}}{e^{\tau\mu/2}} l\{u(t)\}, \tag{5}$$

where $l\{\bullet\}$ denotes the Laplace transform of $\bullet$.

Secondly, the term $\left(e^{-\tau\mu/2}/e^{\tau\mu/2}\right)$ is transformed by the Padé approximation method into the form as follows

$$\frac{e^{-\tau\mu/2}}{e^{\tau\mu/2}} \approx \frac{1-\tau\mu/2}{1+\tau\mu/2}. \tag{6}$$

Thirdly, based on (5) and (6), define a new variable x_{n+1} that satisfies

$$\frac{1-\tau\mu/2}{1+\tau\mu/2} l\{u(t)\} = l\{x_{n+1}(t)\} - l\{u(t)\}. \tag{7}$$

Then, let $\bar{\tau} = \frac{2}{\tau}$, there has

$$u(t-\tau) = x_{n+1}(t) - u(t), \tag{8}$$

and the derivative of x_{n+1} can be expressed as

$$\dot{x}_{n+1} = -\bar{\tau} x_{n+1} + 2\bar{\tau} u. \tag{9}$$

Finally, substituting (8) and (9) into system (1), one has

$$\begin{cases} dx_1 = (x_2 + f_1(\bar{\boldsymbol{x}}_1))\,dt + g_1^{\mathrm{T}}(\bar{\boldsymbol{x}}_1)\,d\omega, \\ dx_2 = (x_3 + f_2(\bar{\boldsymbol{x}}_2))\,dt + g_2^{\mathrm{T}}(\bar{\boldsymbol{x}}_2)\,d\omega, \\ \qquad \vdots \\ dx_{n-1} = (x_n + f_{n-1}(\bar{\boldsymbol{x}}_{n-1}))\,dt + g_{n-1}^{\mathrm{T}}(\bar{\boldsymbol{x}}_{n-1})\,d\omega, \\ dx_n = (x_{n+1} - u + f_n(\bar{\boldsymbol{x}}_n))\,dt + g_n^{\mathrm{T}}(\bar{\boldsymbol{x}}_n)\,d\omega, \\ dx_{n+1} = (-\bar{\tau} x_{n+1} + 2\bar{\tau} u)\,dt, \\ y = x_1. \end{cases} \tag{10}$$

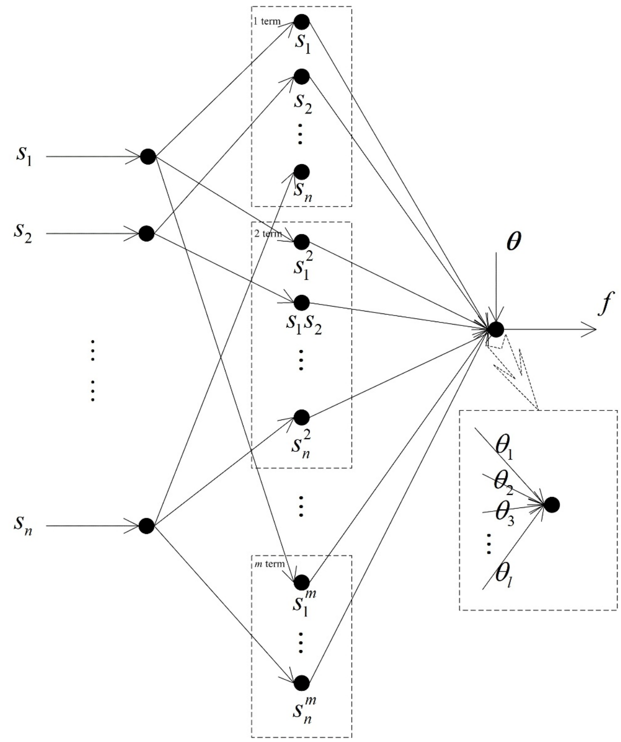

Fig. 1. The structure block diagram of MTN.

Remark 1 [38]**:** Especially note that the Padé approximation method is only suitable for the case of small delay.

2.4. Multi-dimensional Taylor network

The structure of MTN is given in Fig. 1, where $[s_1, \ldots, s_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the input vector of the system, $[\theta_1, \ldots, \theta_l]^{\mathrm{T}} \in \mathbb{R}^l$ is the weight vector and $[s_1, \ldots, s_n, s_1^2, s_1 s_2, \ldots, s_1 s_n, s_2 s_3, \ldots, s_n^2, s_1^m, \ldots, s_n^m]^{\mathrm{T}} \in \mathbb{R}^l$ is the middle layer.

The following Lemmas are very important for the design of control scheme of this paper.

Lemma 2 [31]**:** In the compact set Ω, if $f(\boldsymbol{S}) : \mathbb{R}^n \to \mathbb{R}$ is a continuous function, then for any $\varepsilon > 0$, there must exists a MTN as follows:

$$f(\boldsymbol{S}) = \boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(\boldsymbol{S}) + \delta(\boldsymbol{S}), \tag{11}$$

where $s_1, \cdots, s_n$ are the inputs of MTN with $\boldsymbol{S} = [s_1, \ldots, s_n]^{\mathrm{T}} \in \mathbb{R}^n$, and $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_l]^{\mathrm{T}} \in \mathbb{R}^l$ denotes the weight vector of MTN. $P_{m_n}(\boldsymbol{S}) = [s_1, \ldots, s_n, s_1^2, \ldots, s_1 s_n, s_2 s_3, \ldots, s_n^2, s_1^m, \ldots, s_n^m]^{\mathrm{T}}$ denotes the polynomial combination of the middle input layer, where n is input number and m is the highest power. $\delta(\boldsymbol{S})$ is the error between $f(\boldsymbol{S})$ and $\boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(\boldsymbol{S})$, and it satisfies $|\delta(\boldsymbol{S})| \leqslant \varepsilon$.

Lemma 3 [43]**:** For any positive constant k_b and any $z \in R$, if $|z| < k_b$, the following inequality holds

$$\log \frac{k_b^{2p}}{k_b^{2p} - z^{2p}} < \frac{z^{2p}}{k_b^{2p} - z^{2p}}, \tag{12}$$

where $\log(\bullet)$ is a logarithm of $(\bullet)$, p is a positive constant.

3. MAIN RESULTS

3.1. Controller design

First, define the following coordinate transformation:

$$\begin{cases} z_1 = x_1 - y_d, \\ z_i = x_i - \alpha_{i-1},\ i = 1,\ ...,\ n-1, \\ z_n = x_n - \alpha_{n-1} + (1/\bar{\tau})x_{n+1}, \end{cases} \tag{13}$$

where α_{i-1} is intermediate virtual control signal, and it will be designed later via backstepping.

Step 1: The time derivative with respect to z_1 from $z_1 = x_1 - y_d$ is as follows:

$$dz_1 = (x_2 + f_1 - \dot{y}_d)\,dt + g_1^{\mathrm{T}} d\omega. \tag{14}$$

Then, define the first BLF as follows:

$$V_1 = \frac{1}{4}\log\frac{k_1^4}{k_1^4 - z_1^4} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_1, \tag{15}$$

where $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$ is parameter error, $\hat{\boldsymbol{\theta}}_1$ is the estimated value of $\boldsymbol{\theta}_1$, $k_1 = k_{1,c} - \rho_1$, and ρ_1 is a positive constant.

In the compact set Ω, according to formula (3) in Definition 1, one has

$$\begin{aligned} LV_1 =& \frac{z_1^2}{2(k_1^4 - z_1^4)^2}\left(3k_1^4 + z_1^4\right)\|g_1\|^2 \\ &+ \frac{z_1^3}{k_1^4 - z_1^4}(x_2 + f_1 - \dot{y}_d) - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_1. \end{aligned} \tag{16}$$

By using Young's inequality, one has

$$\begin{aligned} &\frac{z_1^2}{2(k_1^4 - z_1^4)^2}\left(3k_1^4 + z_1^4\right)\|g_1\|^2 \\ &\leqslant \frac{a_1^2}{4} + \frac{1}{4a_1^2}\frac{z_1^4}{(k_1^4 - z_1^4)^4}\left(3k_1^4 + z_1^4\right)^2\|g_1\|^4, \end{aligned} \tag{17}$$

where a_1 is a positive constant.

Then, denote $\tilde{f}_1 = f_1 - \dot{y}_d + \frac{1}{4a_1^2}\frac{z_1}{(k_1^4 - z_1^4)^3}\left(3k_1^4 + z_1^4\right)^2\|g_1\|^4 + \frac{3}{4}z_1\left(\xi_1^{\frac{4}{3}} + \zeta_1^{\frac{4}{3}}\right)\left(k_1^4 - z_1^4\right)^{-\frac{1}{3}}$,where $\xi_1 > 0$ and $\zeta_1 > 0$ are constants.

The following inequality is obtained by substituting (17) into (16)

$$\begin{aligned} LV_1 =& \frac{z_1^3}{k_1^4 - z_1^4}\left(x_2 + \tilde{f}_1\right) + \frac{a_1^2}{4} - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_1 \\ &- \frac{3}{4}z_1^4\zeta_1^{\frac{4}{3}}\left(k_1^4 - z_1^4\right)^{-\frac{4}{3}} - \frac{3}{4}z_1^4\xi_1^{\frac{4}{3}}\left(k_1^4 - z_1^4\right)^{-\frac{4}{3}}. \end{aligned} \tag{18}$$

Since the structure of $\tilde{f}_1$ is not sufficient to directly apply it to the design of intermediate virtual control signals, according to Lemma 2, MTN in the form of $\boldsymbol{\theta}^{\mathrm{T}}P_{m_1}(z)$ can be used to approximate $\tilde{f}_1$. In particular, for any $\varepsilon_1 > 0$, there is

$$\tilde{f}_1 = \boldsymbol{\theta}_1^{\mathrm{T}}P_{m_1}(z_1) + \delta_1(z_1),\ \ |\delta_1(z_1)| \leqslant \varepsilon_1, \tag{19}$$

where $z_1 = [z_1]^{\mathrm{T}}$ is the input vector of MTN.

On the basis of (13), one has $z_2 = x_2 - \alpha_1$. Then, the following inequality can be obtained by substituting (19) into (18)

$$\begin{aligned} LV_1 \leqslant& \frac{z_1^3}{k_1^4 - z_1^4}\left(z_2 + \alpha_1 + \boldsymbol{\theta}_1^{\mathrm{T}}P_{m_1}(z_1) + \delta_1(z_1)\right) \\ &- \frac{3}{4}z_1^4\xi_1^{\frac{4}{3}}\left(k_1^4 - z_1^4\right)^{-\frac{4}{3}} - \frac{3}{4}z_1^4\zeta_1^{\frac{4}{3}}\left(k_1^4 - z_1^4\right)^{-\frac{4}{3}} \\ &+ \frac{a_1^2}{4} - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_1. \end{aligned} \tag{20}$$

According to (20), the first intermediate virtual control signal α_1 can be designed as follows:

$$\alpha_1 = -r_1 z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}}P_{m_1}, \tag{21}$$

where r_1 is a positive constant.

According to Young's inequality, the following two inequalities are established:

$$\frac{z_1^3}{k_1^4 - z_1^4}\delta_1 \leqslant \frac{3}{4}z_1^4\zeta_1^{\frac{4}{3}}\left(k_1^4 - z_1^4\right)^{-\frac{4}{3}} + \frac{1}{4\zeta_1^4}\varepsilon_1^4, \tag{22}$$

$$\frac{z_1^3}{k_1^4 - z_1^4}z_2 \leqslant \frac{3}{4}z_1^4\xi_1^{\frac{4}{3}}\left(k_1^4 - z_1^4\right)^{-\frac{4}{3}} + \frac{1}{4\xi_1^4}z_2^4. \tag{23}$$

Substituting (21), (22) and (23) into (20), one has

$$\begin{aligned} LV_1 \leqslant& -\frac{r_1 z_1^4}{k_1^4 - z_1^4} + \frac{1}{4\zeta_1^4}\varepsilon_1^4 + \frac{1}{4\xi_1^4}z_2^4 + \frac{a_1^2}{4} \\ &+ \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\left(P_{m_1}\frac{z_1^3}{k_1^4 - z_1^4} - \dot{\hat{\boldsymbol{\theta}}}_1\right). \end{aligned} \tag{24}$$

Constructing the adaptive law from (24) as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_1 = -\eta_1\hat{\boldsymbol{\theta}}_1 + \frac{z_1^3}{k_1^4 - z_1^4}P_{m_1}, \tag{25}$$

where $\eta_i > 0$, $i = 1, 2, ..., n$ are constants.

According to Lemma 3, when $|z_1| < k_1$, the following inequality holds

$$\log\frac{k_1^4}{k_1^4 - z_1^4} < \frac{z_1^4}{k_1^4 - z_1^4}. \tag{26}$$

Then plugging (25) and (26) into (24), the derivative of V_1 can be obtained as follows:

$$\begin{aligned} LV_1 \leqslant& -r_1\log\frac{k_1^4}{k_1^4 - z_1^4} + \frac{1}{4\zeta_1^4}\varepsilon_1^4 \\ &+ \frac{1}{4\xi_1^4}z_2^4 + \frac{a_1^2}{4} + \eta_1\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\hat{\boldsymbol{\theta}}_1. \end{aligned} \tag{27}$$

Step 2: According to $z_2 = x_2 - \alpha_1$, the derivative of z_2 can be obtained as follows:

$$dz_2 = (x_3 + f_2 - \nabla\alpha_1)\,dt + \tilde{g}_2^{\mathrm{T}} d\omega, \tag{28}$$

where $\nabla\alpha_1 = \frac{\partial\alpha_1}{\partial x_1}(x_2+f_1) + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(i)}} y_d^{(i+1)} + \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}_1}\dot{\hat{\boldsymbol{\theta}}}_1 + \frac{1}{2}\frac{\partial^2\alpha_1}{\partial x_1^2} g_1^{\mathrm{T}} g_1$, and $\tilde{g}_2 = g_2 - \frac{\partial\alpha_1}{\partial x_1} g_1$.

Then, define the second BLF as follows:

$$V_2 = \frac{1}{4}\log\frac{k_2^4}{k_2^4 - z_2^4} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_2 + V_1, \tag{29}$$

where $\tilde{\boldsymbol{\theta}}_2 = \boldsymbol{\theta}_2 - \hat{\boldsymbol{\theta}}_2$ represents the parameter error, $k_2 = k_{2,c} - \rho_2$, $|\alpha_1| \leqslant \rho_2$, and ρ_2 is a positive constant.

In compact set Ω, according to Definition 1, there is

$$\begin{aligned} LV_2 =& LV_1 + \frac{z_2^3}{k_2^4 - z_2^4}(x_3 + f_2 - \nabla\alpha_1) \\ &+ \frac{z_2^2}{2(k_2^4 - z_2^4)^2}\left(3k_2^4 + z_2^4\right)\|\tilde{g}_2\|^2 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_2. \end{aligned} \tag{30}$$

By using Young's inequality, the following inequality is easily obtained

$$\begin{aligned} &\frac{z_2^2}{2(k_2^4 - z_2^4)^2}\left(3k_2^4 + z_2^4\right)\|\tilde{g}_2\|^2 \\ &\leqslant \frac{1}{4a_2^2}\frac{z_2^4}{(k_2^4 - z_2^4)^4}\left(3k_2^4 + z_2^4\right)^2\|\tilde{g}_2\|^4 + \frac{a_2^2}{4}, \end{aligned} \tag{31}$$

where $a_2 > 0$ is a constant.

Substituting (31) into (30) and let $\tilde{f}_2 = f_2 - \nabla\alpha_1 + \frac{1}{4a_2^2}\frac{z_2}{(k_2^4 - z_2^4)^3}\left(3k_2^4 + z_2^4\right)^2\|\tilde{g}_2\|^4 + \frac{3}{4}z_2\left(\xi_2^{\frac{4}{3}} + \zeta_2^{\frac{4}{3}}\right)\left(k_2^4 - z_2^4\right)^{-\frac{1}{3}} + \frac{1}{4\xi_1^4}z_2\left(k_2^4 - z_2^4\right)$, where $\xi_2 > 0$ and $\zeta_2 > 0$ are constants.

Then one has

$$\begin{aligned} LV_2 =& \frac{z_2^3}{k_2^4 - z_2^4}\left(x_3 + \tilde{f}_2\right) - \frac{3}{4}z_2^4\xi_2^{\frac{4}{3}}\left(k_2^4 - z_2^4\right)^{-\frac{4}{3}} \\ &- \frac{3}{4}z_2^4\zeta_2^{\frac{4}{3}}\left(k_2^4 - z_2^4\right)^{-\frac{4}{3}} - \frac{1}{4\xi_1^4}z_2^4 + \frac{a_2^2}{4} \\ &- \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_2 + LV_1. \end{aligned} \tag{32}$$

Because the structure of $\tilde{f}_2$ is not enough to be used to design intermediate virtual control signals, according to Lemma 2, MTN in the form of $\boldsymbol{\theta}^{\mathrm{T}} P_{m_2}(z)$ can be used to approximate $\tilde{f}_2$. Especially, for arbitrary $\varepsilon_2 > 0$, there is

$$\tilde{f}_2 = \boldsymbol{\theta}_2^{\mathrm{T}} P_{m_2}(\mathbf{z}_2) + \delta_2(\mathbf{z}_2),\ |\delta_2(\mathbf{z}_2)| \leqslant \varepsilon_2, \tag{33}$$

where $\mathbf{z}_2 = [z_1, z_2]^{\mathrm{T}}$ is the input vector of MTN.

Obviously, it can be seen from the front that $z_3 = x_3 - \alpha_2$, the following inequality can be obtained by substituting (33) into (32)

$$\begin{aligned} LV_2 \leqslant& \frac{z_2^3}{k_2^4 - z_2^4}\left(z_3 + \alpha_2 + \boldsymbol{\theta}_2^{\mathrm{T}} P_{m_2}(\mathbf{z}_2) + \delta_2(\mathbf{z}_2)\right) \\ &- \frac{1}{4\xi_1^4}z_2^4 + \frac{a_2^2}{4} - \frac{3}{4}z_2^4\xi_2^{\frac{4}{3}}\left(k_2^4 - z_2^4\right)^{-\frac{4}{3}} \\ &- \frac{3}{4}z_2^4\zeta_2^{\frac{4}{3}}\left(k_2^4 - z_2^4\right)^{-\frac{4}{3}} - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_2 + LV_1. \end{aligned} \tag{34}$$

According to (34), the intermediate virtual control signal α_2 is designed as follows:

$$\alpha_2 = -r_2 z_2 - \hat{\boldsymbol{\theta}}_2^{\mathrm{T}} P_{m_2},\ r_2 > 0. \tag{35}$$

By using Young's inequality, the following two inequalities are correct

$$\frac{z_2^3}{k_2^4 - z_2^4}\delta_2 \leqslant \frac{3}{4}z_2^4\zeta_2^{\frac{4}{3}}\left(k_2^4 - z_2^4\right)^{-\frac{4}{3}} + \frac{1}{4\zeta_2^4}\varepsilon_2^4, \tag{36}$$

$$\frac{z_2^3}{k_2^4 - z_2^4}z_3 \leqslant \frac{3}{4}z_2^4\xi_2^{\frac{4}{3}}\left(k_2^4 - z_2^4\right)^{-\frac{4}{3}} + \frac{1}{4\xi_2^4}z_3^4. \tag{37}$$

Substituting (35), (36) and (37) into (34), one has

$$\begin{aligned} LV_2 \leqslant& -r_1\log\frac{k_1^4}{k_1^4 - z_1^4} + \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\left(P_{m_2}\frac{z_2^3}{k_2^4 - z_2^4} - \dot{\hat{\boldsymbol{\theta}}}_2\right) \\ &+ \sum_{j=1}^{2}\frac{1}{4\zeta_j^4}\varepsilon_j^4 - \frac{r_2 z_2^4}{k_2^4 - z_2^4} + \sum_{j=1}^{2}\frac{a_j^2}{4} \\ &+ \eta_1\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\hat{\boldsymbol{\theta}}_1 + \frac{1}{4\xi_2^4}z_3^4. \end{aligned} \tag{38}$$

According to (38), the adaptive law is constructed as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_2 = -\eta_2\hat{\boldsymbol{\theta}}_2 + \frac{z_2^3}{k_2^4 - z_2^4}P_{m_2}. \tag{39}$$

According to Lemma 3, when $|z_2| < k_2$, one has

$$\log\frac{k_2^4}{k_2^4 - z_2^4} < \frac{z_2^4}{k_2^4 - z_2^4}. \tag{40}$$

Substituting (39) and (40) into (38), one has

$$\begin{aligned} LV_2 \leqslant& -\sum_{j=1}^{2} r_j\log\frac{k_j^4}{k_j^4 - z_j^4} + \sum_{j=1}^{2}\frac{1}{4\zeta_j^4}\varepsilon_j^4 \\ &+ \frac{1}{4\xi_2^4}z_3^4 + \sum_{j=1}^{2}\frac{a_j^2}{4} + \sum_{j=1}^{2}\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j. \end{aligned} \tag{41}$$

Step $3 \leqslant i \leqslant n-1$: According to $z_i = x_i - \alpha_{i-1}$, the derivative of z_i can be obtained as follows:

$$dz_i = (x_{i+1} + f_i - \nabla\alpha_{i-1})\,dt + \tilde{g}_i^{\mathrm{T}} d\omega, \tag{42}$$

where $\nabla\alpha_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1} + f_j) + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}g_p^{\mathrm{T}}g_q$, and $\tilde{g}_i = g_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_j$.

Define the i-th BLF as follows:

$$V_i = \frac{1}{4}\log\frac{k_i^4}{k_i^4 - z_i^4} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i + V_{i-1}, \tag{43}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ is the parameter error, $k_i = k_{i,c} - \rho_i$, $|\alpha_{i-1}| \leqslant \rho_i$ and ρ_i is a positive constant.

In the compact set Ω, repeating the similar steps in Step 2, one has

$$LV_i \leqslant -\sum_{j=1}^{i} r_j \log \frac{k_j^4}{k_j^4 - z_j^4} + \sum_{j=1}^{i} \frac{1}{4\zeta_j^4}\varepsilon_j^4 + \frac{1}{4\xi_i^4} z_{i+1}^4 + \sum_{j=1}^{i} \frac{a_j^2}{4} + \sum_{j=1}^{i} \eta_j \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} \hat{\boldsymbol{\theta}}_j, \tag{44}$$

where $\zeta_i > 0$, $\xi_i > 0$ and $a_i > 0$ are constants.

The intermediate virtual control signal α_i and the adaptive law $\dot{\hat{\theta}}_i$ are designed as follows:

$$\alpha_i = -r_i z_i - \hat{\theta}_i^{\mathrm{T}} P_{m_i},\ r_i > 0, \tag{45}$$

$$\dot{\hat{\theta}}_i = -\eta_i \hat{\theta}_i + \frac{z_i^3}{k_i^4 - z_i^4} P_{m_i}, \tag{46}$$

where $r_i > 0$ and $\eta_i > 0$ are design parameters.

Step *n*: Let $z_n = x_n - \alpha_{n-1} + (1/\bar{\tau}) x_{n+1}$ to get the derivative of z_n as follows:

$$dz_n = (u + f_n - \nabla\alpha_{n-1})\, dt + \tilde{g}_n^{\mathrm{T}} d\omega, \tag{47}$$

where $\nabla\alpha_{n-1} = \sum_{i=1}^{n-1} \frac{\partial\alpha_{n-1}}{\partial x_i}(x_{i-1} + f_i) + \sum_{i=0}^{n-1} \frac{\partial\alpha_{n-1}}{\partial y_d^{(i)}} y_d^{(i+1)} + \sum_{i=1}^{n-1} \frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol{\theta}}_i}\dot{\hat{\boldsymbol{\theta}}}_i + \frac{1}{2}\sum_{p,q=1}^{n-1} \frac{\partial^2\alpha_{n-1}}{\partial x_p \partial x_q} g_p^{\mathrm{T}} g_q$, and $\tilde{g}_n = g_n - \sum_{i=1}^{n-1} \frac{\partial\alpha_{n-1}}{\partial x_i} g_i$.

Then we define the n-th BLF as follows:

$$V_n = \frac{1}{4}\log\frac{k_n^4}{k_n^4 - z_n^4} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_n + V_{n-1}, \tag{48}$$

where $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n - \hat{\boldsymbol{\theta}}_n$ is the parameter error, $k_n = k_{n,c} - \rho_n$, $|\alpha_{n-1}| \leqslant \rho_n$ and ρ_n is a positive constant.

In the compact set Ω, according to Definition 1, there is

$$LV_n = LV_{n-1} + \frac{z_n^3}{k_n^4 - z_n^4}(u + f_n - \nabla\alpha_{n-1}) + \frac{z_n^2}{2(k_n^4 - z_n^4)^2}(3k_n^4 + z_n^4)\|\tilde{g}_n\|^2 - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_n. \tag{49}$$

Using Young's inequality, the following inequality is easily obtained

$$\frac{z_n^2}{2(k_n^4 - z_n^4)^2}(3k_n^4 + z_n^4)\|\tilde{g}_n\|^2 \leqslant \frac{a_n^2}{4} + \frac{1}{4a_n^2}\frac{z_n^4}{(k_n^4 - z_n^4)^4}(3k_n^4 + z_n^4)^2\|\tilde{g}_n\|^4, \tag{50}$$

where $a_n > 0$ is a constant.

Let

$$\tilde{f}_n = f_n - \nabla\alpha_{n-1} + \frac{1}{4a_n^2}\frac{z_n}{(k_n^4 - z_n^4)^3}(3k_n^4 + z_n^4)^2\|\tilde{g}_n\|^4 + \frac{3}{4} z_n \zeta_n^{\frac{4}{3}} (k_n^4 - z_n^4)^{-\frac{1}{3}} + \frac{1}{4\xi_{n-1}^4} z_n (k_n^4 - z_n^4),$$

where $\xi_n > 0$ and $\zeta_n > 0$ are constant, and then we have

$$LV_n \leqslant LV_{n-1} + \frac{z_n^3}{k_n^4 - z_n^4}(u + \tilde{f}_n) - \frac{1}{4\xi_{n-1}^4} z_n^4 - \frac{3}{4} z_n^4 \zeta_n^{\frac{4}{3}} (k_n^4 - z_n^4)^{-\frac{4}{3}} + \frac{a_n^2}{4} - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_n. \tag{51}$$

According to Lemma 2, MTN of form $\boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(z)$ is used to approximate $\tilde{f}_n$, and in particular, for any $\varepsilon_n > 0$, there is

$$\tilde{f}_n = \boldsymbol{\theta}_n^{\mathrm{T}} P_{m_n}(z_n) + \delta_n(z_n),\ |\delta_n(z_n)| \leqslant \varepsilon_n, \tag{52}$$

where $z_n = [z_1, z_2, ..., z_n]^{\mathrm{T}}$ is represents the input vector of MTN.

From this, the following inequation is obtained

$$LV_n \leqslant LV_{n-1} + \frac{z_n^3}{k_n^4 - z_n^4}(u + \boldsymbol{\theta}_n^{\mathrm{T}} P_{m_n} + \delta_n) - \frac{1}{4\xi_{n-1}^4} z_n^4 - \frac{3}{4} z_n^4 \zeta_n^{\frac{4}{3}} (k_n^4 - z_n^4)^{-\frac{4}{3}} + \frac{a_n^2}{4} - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_n. \tag{53}$$

Meanwhile, the following inequality is true according to Young's inequality

$$\frac{z_n^3}{k_n^4 - z_n^4}\delta_n \leqslant \frac{3}{4} z_n^4 \zeta_n^{\frac{4}{3}} (k_n^4 - z_n^4)^{-\frac{4}{3}} + \frac{1}{4\zeta_n^4}\varepsilon_n^4. \tag{54}$$

Then, substituting (54) into (53), the following inequality holds

$$LV_n \leqslant \frac{z_n^3}{k_n^4 - z_n^4}(u + \boldsymbol{\theta}_n^{\mathrm{T}} P_{m_n}) + \frac{a_n^2}{4} - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_n + \frac{1}{4\zeta_n^4}\varepsilon_n^4 + LV_{n-1} - \frac{1}{4\xi_{n-1}^4} z_n^4. \tag{55}$$

Selecting the actual control input as follows:

$$u = -r_n z_n - \hat{\boldsymbol{\theta}}_n^{\mathrm{T}} P_{m_n},\ r_n > 0. \tag{56}$$

So we have

$$LV_n \leqslant LV_{n-1} - \frac{r_n z_n^4}{k_n^4 - z_n^4} + \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\left(\frac{z_n^3}{k_n^4 - z_n^4} P_{m_n} - \dot{\hat{\boldsymbol{\theta}}}_n\right) + \frac{a_n^2}{4} + \frac{1}{4\zeta_n^4}\varepsilon_n^4 - \frac{1}{4\xi_{n-1}^4} z_n^4. \tag{57}$$

According to (57), the adaptive law is constructed as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_n = -\eta_n \hat{\boldsymbol{\theta}}_n + \frac{z_n^3}{k_n^4 - z_n^4} P_{m_n}. \tag{58}$$

From Lemma 3, when $|z_n| < k_n$, one has

$$\log\frac{k_n^4}{k_n^4 - z_n^4} < \frac{z_n^4}{k_n^4 - z_n^4}. \tag{59}$$

Finally, substituting (44), (58), and (59) into (57), one has

$$LV_n \leqslant -\sum_{i=1}^{n} r_i \log \frac{k_i^4}{k_i^4 - z_i^4} + \sum_{i=1}^{n} \eta_i \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \hat{\boldsymbol{\theta}}_i + \sum_{i=1}^{n} \frac{1}{4\zeta_i^4}\varepsilon_i^4 + \sum_{i=1}^{n} \frac{a_i^2}{4}. \tag{60}$$

3.2. Stability analysis

Theorem 1: Under Assumptions 1-2, consider that the closed-loop system consists of system (1), intermediate virtual control signals (21), (35), (45), actual control input (56) and adaptive law (25), (39), (46) and (58). For any initial conditions, the following three properties are true

1) All signals of closed-loop system are SGUUB in probability.
2) All system states $x_1, x_2, ..., x_n$ meet the constraints.
3) The system output y tracking of the continuous reference signal y_d.

Proof: For the closed-loop system, the Lyapunov function is defined as follows:

$$V = \frac{1}{4}\sum_{i=1}^{n} \log \frac{k_i^4}{k_i^4 - z_i^4} + \frac{1}{2}\sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_i. \tag{61}$$

According to (60), we have the differential of V as follows:

$$LV \leqslant -\sum_{i=1}^{n} r_i \log \frac{k_i^4}{k_i^4 - z_i^4} + \sum_{i=1}^{n} \eta_i \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \hat{\boldsymbol{\theta}}_i + \sum_{i=1}^{n} \frac{a_i^2}{4} + \sum_{i=1}^{n} \frac{1}{4\zeta_i^4}\varepsilon_i^4. \tag{62}$$

In addition, for any $i = 1, ..., n$, there is $\eta_i \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \hat{\boldsymbol{\theta}}_i \leqslant -\frac{1}{2}\eta_i \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_i + \frac{1}{2}\eta_i \boldsymbol{\theta}_i^{\mathrm{T}} \boldsymbol{\theta}_i$, then one has

$$LV \leqslant -\sum_{i=1}^{n} r_i \log \frac{k_i^4}{k_i^4 - z_i^4} - \frac{1}{2}\sum_{i=1}^{n} \eta_i \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_i + \frac{1}{2}\sum_{i=1}^{n} \eta_i \boldsymbol{\theta}_i^{\mathrm{T}} \boldsymbol{\theta}_i + \sum_{i=1}^{n} \frac{1}{4\zeta_i^4}\varepsilon_i^4 + \sum_{i=1}^{n} \frac{a_i^2}{4}. \tag{63}$$

Moreover, let $l = \frac{1}{2}\sum_{i=1}^{n} \eta_i \boldsymbol{\theta}_i^{\mathrm{T}} \boldsymbol{\theta}_i + \sum_{i=1}^{n} \frac{1}{4\zeta_i^4}\varepsilon_i^4 + \sum_{i=1}^{n} \frac{a_i^2}{4}$ and $\lambda = \min\{4r_i,\ \eta_i : i = 1, ..., n\}$, then (63) can be expressed as the following form

$$LV \leqslant -\lambda V + l. \tag{64}$$

Then, on the basis of Definition 1, for any $t \geqslant 0$, the following inequality is correct

$$0 \leqslant E[V(t)] \leqslant V(0)e^{-\lambda t} + \frac{l}{\lambda}. \tag{65}$$

On the one hand, using the similarly discuss in [23], it concluded that all system states $x_1, x_2, ..., x_n$ meet the constraints.
On the other hand, we can see from the above equation that l/λ is a bound on $E[V(t)]$. Therefore, according to (61), it can be concluded that the signals of all closed-loop systems are ultimately uniformly bounded in probability, and the output of the system tracks the given reference signal. □

4. SIMULATION EXAMPLE

In this section, two examples are given to verify the effectiveness of the proposed control method.

Example 1: Consider third-order stochastic nonlinear system with full-state constraints and input delay as follows:

$$\begin{cases} dx_1 = x_2 dt + 0.5 x_1 d\omega, \\ dx_2 = (x_3 - 2\sin x_1)\,dt + 0.1\sin x_3 d\omega, \\ dx_3 = (u(t-\tau) + x_1 x_2 x_3)\,dt + 0.1\sin x_3 d\omega, \\ y_1 = x_1. \end{cases} \tag{66}$$

According to Theorem 1, the control strategies of the system (66) are as follows:

$$\alpha_i = -r_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} P_{m_i},\ r_i > 0,\ i = 1,\ 2,$$
$$u = -r_3 z_3 - \hat{\boldsymbol{\theta}}_3^{\mathrm{T}} P_{m_3},\ r_3 > 0,$$
$$\dot{\hat{\boldsymbol{\theta}}}_i = -\eta_i \hat{\boldsymbol{\theta}}_i + \frac{z_i^3}{k_i^4 - z_i^4} P_{m_i},\ i = 1,\ 2,\ 3.$$

The initial condition of the system is $x_1(0) = x_2(0) = x_3(0) = 0$, the reference signal is selected as $y_d = \sin t$, the input delay is selected as $\tau = 0.01$, the design parameters of the system controller are $r_1 = 5$, $r_2 = 8$, $r_3 = 9$, $\eta_1 = 0.05$, $\eta_2 = \eta_3 = 0.01$.

Fig. 2 shows the trajectories of the system output y and the specified reference signal y_d, it is easy to see that the tracking effect is satisfactory. Fig. 3 shows the tracking error $y - y_d$ of the system. It can be seen from the Fig. 3 that the tracking error converges at the small neighborhood of origin. The responses of the control output u, the state x_2, x_3 are described in Figs. 4 and 5, respectively. It is clear that x_2, x_3 are limited within the specified range. From the above simulation results, we can see that all the states of the system are controlled in the given limited range.

Example 2: In order to further verify the effectiveness of the proposed method, we consider a class of Duffing-Holmes stochastic system [43], and the specific form is given by the following formula

$$\begin{cases} dx_1 = x_1 dt, \\ dx_2 = (u(t-\tau) + f(\bar{\boldsymbol{x}}_2))\,dt + g(\bar{\boldsymbol{x}}_2)\,d\omega, \\ y = x_1, \end{cases} \tag{67}$$

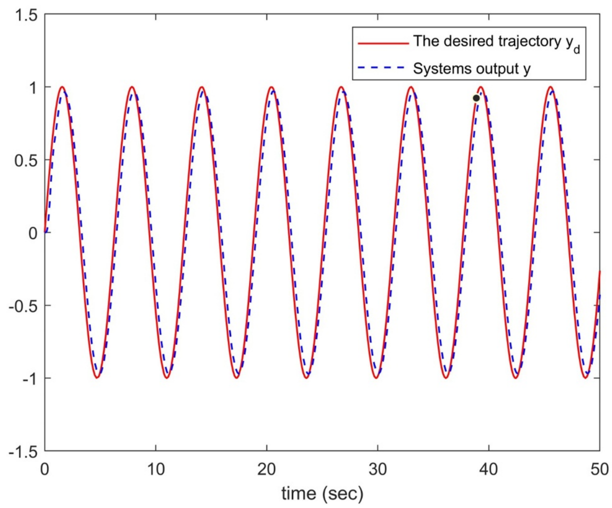

Fig. 2. Reference signal y_d and system output y.

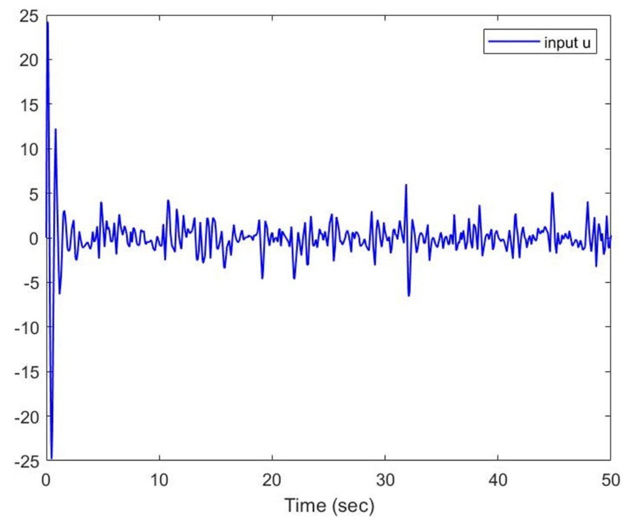

Fig. 4. The system input u.

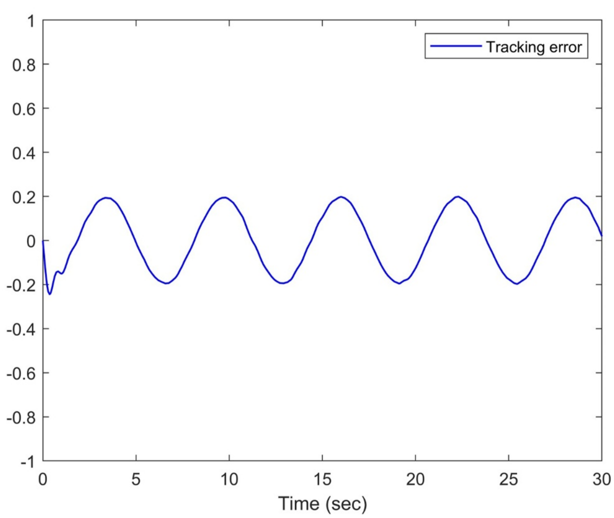

Fig. 3. The tracking error of system.

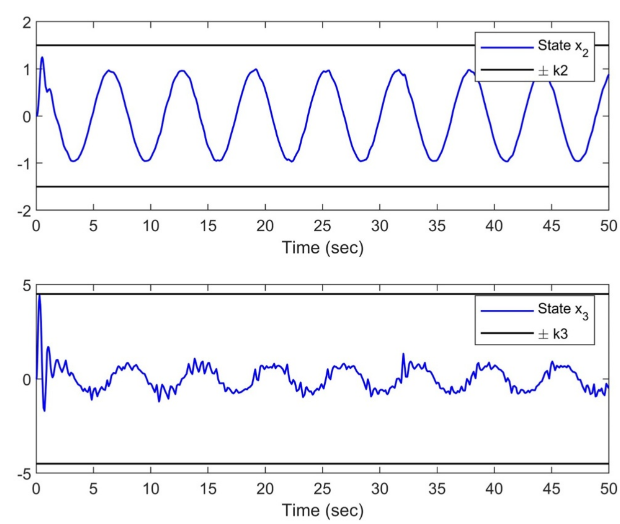

Fig. 5. The trajectories of x_2 and x_3.

where $f(\bar{\boldsymbol{x}}_2) = x_1 - x_2 - x_1^3 + 0.5\text{cost}$, $g(\bar{\boldsymbol{x}}_2) = 0.2x_2^2 \cos x_1$. The states are constrained as $|x_1| < 1$ and $|x_2| < 10$.

According to Theorem 1, the control strategies of the system (67) are as follows:

$$\alpha_1 = -r_1 z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}} P_{m_1},\ r_1 > 0,$$
$$u = -r_2 z_2 - \hat{\boldsymbol{\theta}}_2^{\mathrm{T}} P_{m_2},\ r_2 > 0,$$
$$\dot{\hat{\boldsymbol{\theta}}}_i = -\eta_i \hat{\boldsymbol{\theta}}_i + \frac{z_i^3}{k_i^4 - z_i^4} P_{m_i},\ i = 1,\ 2.$$

The initial condition of the system is $x_1(0) = x_2(0) = 0$, the reference signal is selected as $y_d = 0.5(\sin t + \sin(0.5t))$, the input delay is selected as $\tau = 0.01$, the selection of system controller design parameters are $r_1 = r_2 = 10$, $\eta_1 = 0.1$, $\eta_2 = 1$, $k_1 = k_2 = 1$.

The simulation results are represented in Figs. 6-9. It can be clearly seen from Figs. 6-9 that a satisfactory tracking results have been obtained.

Example 3 (Comparative example)**:** In order to further demonstrate the effectiveness of the proposed method, based on Example 1, we compare the adaptive control method based on MTN with the NN method. The specific way is to replace the MTNs with NNs in the control strategy proposed in Theorem 1, respectively. The comparison results are shown in Fig. 10.

It is obvious from the figure that although both the MTN based controller and the NN controller can achieve tracking effect, it is obvious that the former has better approximation ability than the latter. This comparative experiment further verifies the effectiveness of the proposed scheme.

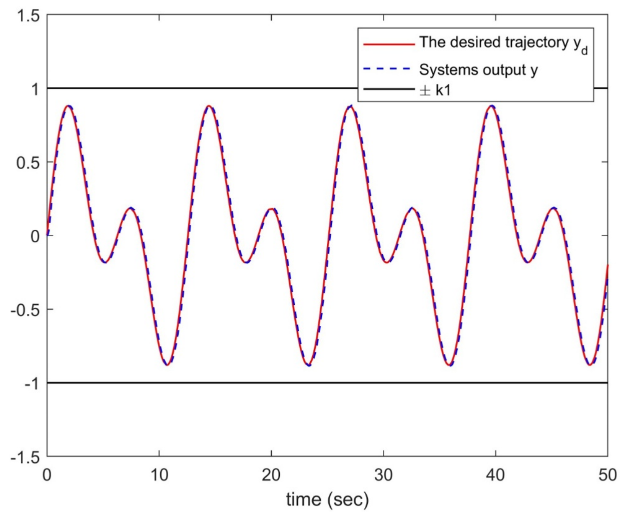

Fig. 6. Reference signal y_d and system output y.

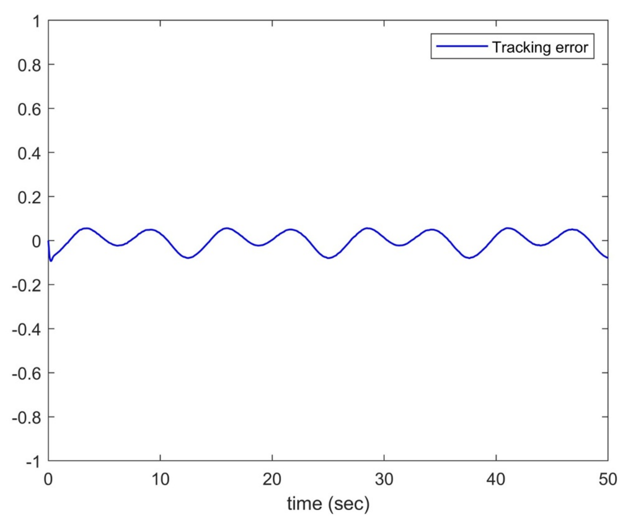

Fig. 7. The tracking error of system.

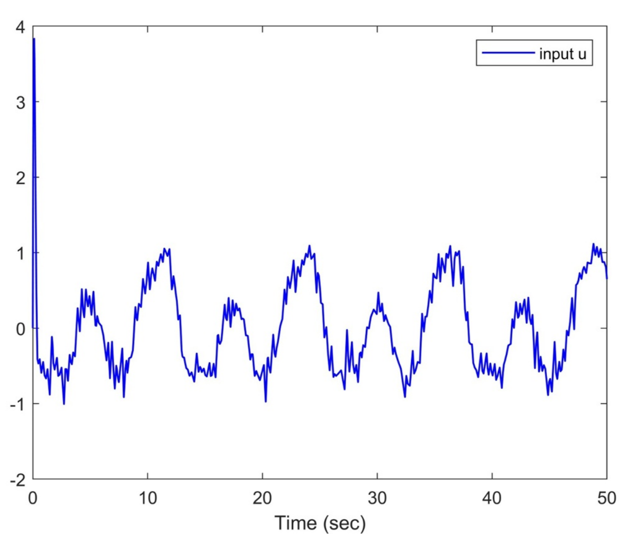

Fig. 8. The system input u.

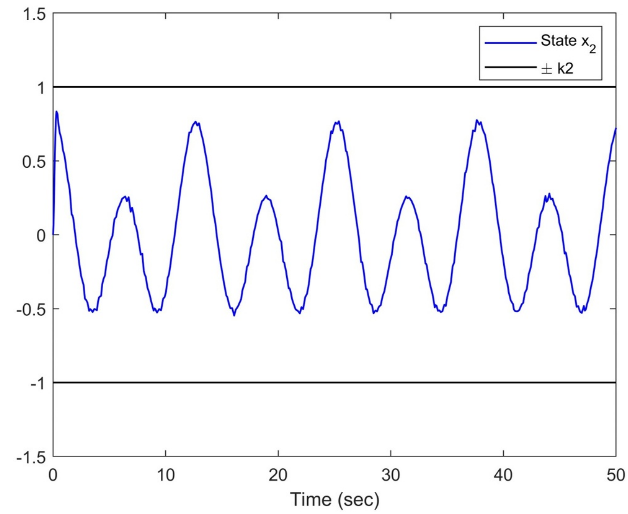

Fig. 9. The trajectories of x_2 and k_2.

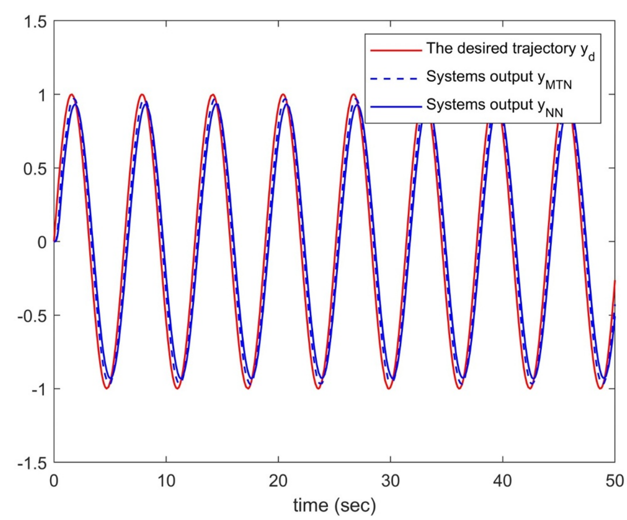

Fig. 10. The tracking trajectories of system (66) under two methods.

5. CONCLUSION

For the first time, the input delay and full state constraints of stochastic nonlinear systems are studied in a unified framework using MTN approach, and a new adaptive controller based on the MTN is proposed, which has the advantages of good approximation performance, simple system structure and low computational complexity. Two simulation examples verify the effectiveness of the designed controller. The proposed control scheme guarantees the boundedness of all signals in the closed-loop system, and all states meet the specified constraints and achieve satisfactory tracking performance.

Since the method developed in this paper is not suitable for the case of big input delay, our future work will be transferred to extend the proposed methodology to a broader case of input delay, including input big delay and input time-varying delay.

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