

Multi-dimensional Taylor network-based control for a class of nonlinear stochastic systems with full state time-varying constraints and the finite-time output constraint

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Abstract

In this paper, the adaptive multi-dimensional Taylor network (MTN) control problem is investigated for nonlinear stochastic systems with full state time-varying constraints and the finite-time output constraint. By combining the MTN-based approximation method and the adaptive backstepping control method, a novel adaptive MTN control scheme is provided by constructing the time-varying barrier Lyapunov function (TVBLF). To implement the finite-time output constraint, the finite-time performance function (FTPF) is introduced in the control scheme. The proposed scheme can ensure that the tracking error finally converges to a small neighborhood of the origin in the finite-time and all signals in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB) in probability. Finally, two simulation examples are presented to show the effectiveness of the provided control scheme.

KEYWORDS

adaptive control, finite-time performance function, multi-dimensional Taylor network, nonlinear stochastic systems, time-varying barrier Lyapunov function

1 | INTRODUCTION

Stochastic disturbances exist widely in many practical systems, and they may reduce system performance or cause instability [1,2]. Driven by these problems and the practical engineering demand, the control problem for nonlinear stochastic systems has been paid an increasingly attention, and many significant methods have been proposed [3–12]. On the whole, the existing control methods for nonlinear stochastic systems can be mainly divided into two categories according to different stochastic Lyapunov functions. By using the quadratic Lyapunov function, Pan and Basar [3] proposed a backstepping-based control scheme for nonlinear stochastic systems via the risk-sensitive cost criterion. Afterwards, a lot of control problems of nonlinear stochastic systems have been

further investigated based on the above functions [5–7]. As another alternative, the quartic Lyapunov function was introduced to handle some special terms in the Lyapunov analysis which arise due to the differentiation rule in the process of designing controllers for nonlinear stochastic systems [8]. At present, this method is widely applied to several different problems, such as tracking control [4], decentralized control [10,11], and control of high-order systems [9,12]. However, it is difficult to solve the problems such as unknown functions and uncertainties only by the aforementioned control methods.

In consideration of the above problems, many approximation methods, such as fuzzy logic systems (FLSs) methods [13,14], neural network (NN) methods [15–19], and multi-dimensional Taylor network (MTN) methods [20,21], have been widely used in the control problems of

nonlinear stochastic systems. Compared with NN or FLSs, the MTN has the advantages of the simple structure, the good real-time performance, and the low computational cost. Therefore, the MTN control method has received increasing attention, and many achievements have been reported [22–30]. For example, by using the MTN-based control methods, the asymptotically tracking problem and the optimal control problem were considered for nonlinear systems [22,23], respectively. At the moment, this control method has been naturally extended to nonlinear stochastic systems, such as the stochastic system with input dead-zone [24], the uncertain nonlinear stochastic system [25,26], and the large-scale nonlinear stochastic system [27]. Although many important MTN-based results have been obtained in the study of nonlinear stochastic systems, the research results on full state constraints systems are still insufficient.

As we know, full state constraints problems are ubiquitous in many practical control areas, and violating constraints may cause poor performance of the control systems [31,32]. Therefore, the full state constraints problem cannot be ignored in the controller design process. To resolve this problem, the barrier Lyapunov function (BLF) has been proposed in [33]. At the moment, several different styles of BLFs have been widely used to solve control problems of full state constraints systems, such as log-type [34], integral-type [35], and tan-type [36]. Especially, Zhu et al. [37] recently proposed a tracking control scheme based on the time-varying barrier Lyapunov function (TVBLF) for state constraints systems. However, it is worth noting that the stochastic disturbances have not been taken into account in [32]. On the other hand, practical systems often require that the tracking error converges to a small neighborhood of the origin in the finite-time [38–40]. Therefore, many finite-time control schemes have been obtained for several different systems [41–48], such as nonlinear stochastic systems [41], and full state constraints stochastic systems [44], non-triangular stochastic systems [47], and high-order stochastic systems [48]. However, there are few results on finite-time output constraint of nonlinear stochastic systems with full state constraints, which motivate our research.

Based on the above observations, this paper investigates the adaptive control problem for nonlinear stochastic systems with full state time-varying constraints and the finite-time output constraint. An adaptive MTN control scheme is developed by constructing the proper TVBLF. In this scheme, the MTNs are employed to approximate the unknown functions in the process of the controller design, and a finite-time performance function (FTPF) is introduced to implement the finite-time output constraint. The proposed control scheme can guarantee

that the tracking error finally converges to a small neighborhood of the origin in the finite-time and all the signals in the closed-loop system are bounded in probability. Finally, the simulation results illustrate the effectiveness of the provided control scheme. Compared with the existing works, the main contributions of this paper are as follows:

- i. A novel adaptive MTN scheme is first presented for nonlinear stochastic systems with full state time-varying constraints and the finite-time output constraint. In the current MTN-based research results [20–30], the existing MTN control schemes cannot be directly utilized to dispose of the control problem of the systems considered in this paper. The MTN method is successfully extended to the stochastic system in this paper.
- ii. In this paper, the stochastic disturbances, the full state constraints and the finite-time output constraint of stochastic systems are considered at the same time. Compared with the research results of state constraints systems [32–36] and the existing finite-time control results [42–45], the proposed scheme can guarantee that the state variables of stochastic systems satisfy time-varying constraints and the tracking error converges to a small neighborhood of the origin at a settling time.
- iii. The online computation burden of the proposed control scheme has greatly alleviated thanks to the simple structure of the MTN. Therefore, we can draw a conclusion that the proposed MTN-based control scheme has excellent practical value.

2 | SYSTEM DESCRIPTION AND PRELIMINARIES

2.1 | Problem description

In this paper, consider the nonlinear stochastic system as follows:

$$\begin{cases} dx_i(t) = [f_i(\bar{\mathbf{x}}_i(t)) + h_i(\bar{\mathbf{x}}_i(t))x_{i+1}(t)]dt + g_i^T(\bar{\mathbf{x}}_i(t))d\omega \\ \quad 1 \leq i \leq n-1 \\ dx_n(t) = [f_n(\bar{\mathbf{x}}_n(t)) + h_n(\bar{\mathbf{x}}_n(t))u]dt + g_n^T(\bar{\mathbf{x}}_n(t))d\omega \\ \quad y = x_1(t) \end{cases} \quad (1)$$

where $\bar{\mathbf{x}}_n(t) = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ denotes state vectors of the system and $\bar{\mathbf{x}}_i(t) = [x_1, x_2, \dots, x_i]^T \in \mathbb{R}^i$. $f_i(\cdot) : \mathbb{R}^i \rightarrow \mathbb{R}$ and $h_i(\cdot) : \mathbb{R}^i \rightarrow \mathbb{R}$ are the unknown nonlinear

functions with $f_i(\mathbf{0}) = \mathbf{0}$. $u \in R$ and $y \in R$ are the input and output of the system, respectively. $g_i(\cdot) : R^i \rightarrow R^r$ are the unknown functions and satisfy $g_i(\mathbf{0}) = \mathbf{0}$. ω is an independent r -dimensional standard Wiener process. For the system (1), all states x_i are restricted in the sets $\Omega_{x_i} = \{x_i \in R \mid |x_i| < k_{c_i}(t)\}$, where $k_{c_i}(t)$ are time-varying continuous functions.

The control objective of this paper is to design a novel adaptive MTN control scheme for the stochastic system (1), which shall meet the following requirements: (i) the system output y can track the given reference signal y_d and the tracking error finally converges to a small neighborhood of the origin in the finite-time; (ii) all the signals in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB) in probability and the system states satisfy the given limited conditions.

2.2 | Preliminaries

In order to introduce the definitions and theorems of the nonlinear stochastic system, consider the following system

$$d\mathbf{x}(t) = f(\mathbf{x}(t))dt + g(\mathbf{x}(t))d\omega \quad (2)$$

where $\mathbf{x} \in R^n$ is the system state, ω is an independent r -dimensional standard Wiener process, and $f : R^n \rightarrow R^n$, $g : R^n \rightarrow R^{n \times r}$ stand for unknown smooth nonlinear functions and satisfy $f(\mathbf{0}) = \mathbf{0}$, $g(\mathbf{0}) = \mathbf{0}$.

Definition 1. [37]: Consider the nonlinear stochastic system (2), for any function $V(\mathbf{x}, t) \in C^2$, the differential operator \mathcal{L} is defined as follows:

$$\mathcal{L}V(\mathbf{x}, t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \mathbf{x}}f + \frac{1}{2}\text{Tr}\left\{g^T \frac{\partial^2 V}{\partial \mathbf{x}^2} g\right\} \quad (3)$$

where C^2 denotes the set of all functions with twice continuous partial derivative.

Lemma 1. [33]: For the nonlinear stochastic system (2), there exists a Lyapunov function $V(\mathbf{x})$, for constants $a > 0$ and $b > 0$, such that

$$\begin{cases} \phi_1(|\mathbf{x}|) \leq V(\mathbf{x}) \leq \phi_2(|\mathbf{x}|) \\ \mathcal{L}V(\mathbf{x}) \leq -aV(\mathbf{x}) + b \end{cases}$$

where ϕ_1 and ϕ_2 are K_∞ -functions, then, the system (2) has a unique strong solution and the following inequality holds

$$E(V(t)) \leq V(0)e^{-at} + \frac{b}{a}$$

2.3 | Finite-time output constraint

To achieve the control objective (i), the following definition is introduced.

Definition 2. [49]: A continuous function $v(t)$ is named as the finite-time performance function (FTPF), if it satisfies the following properties:

1. $v(0) > 0$;
2. $\dot{v}(t) \leq 0$;
3. $\lim_{t \rightarrow T_f} v(t) = v_{T_f} > 0$ and $v(t) = v_{T_f}$ for any $t \geq T_f$ where v_{T_f} and T_f are the arbitrarily small constant and settling time, respectively.

With the consideration of the properties in Definition 2, a FTPF is defined as

$$v(t) = \begin{cases} \left(v_0 - \frac{t}{T_f}\right)e^{1 - \frac{t}{T_f}} + v_{T_f}, & t \in [0, T_f] \\ v_{T_f}, & t \in [T_f, +\infty) \end{cases} \quad (4)$$

where $v_0 > 1$ and $v_{T_f} > 0$ are parameters to be designed.

Remark 1. The function $v(t)$ satisfies all the properties in Definition 2, and the initial condition of $v(t)$ is $v(0) = v_0 + v_{T_f}$. Apparently, $v(t)$ is a continuous function.

2.4 | Relative lemmas and assumptions

For the controller design, the following lemmas and assumptions is presented.

Lemma 2. [37]: For any $|z| < k_b(t)$, the following inequality holds

$$\log \frac{k_b^4(t)}{k_b^4(t) - z^4} \leq \frac{z^4}{k_b^4(t) - z^4}$$

where $z \in R$, $k_b(t) > 0$ is a time-varying function.

$$\frac{z_1^2(3v^4(t) + z_1^4)}{2(v^4(t) - z_1^4)^2} g_1^T g_1 \leq \frac{z_1^4(3v^4(t) + z_1^4)^2}{8(v^4(t) - z_1^4)^4} \|g_1\|^4 + \frac{1}{2} \quad (11)$$

Substituting (11) into (10), we have

$$\begin{aligned} \mathcal{L}V_1 \leq & -\frac{z_1^4}{(v^4(t) - z_1^4)} \frac{\dot{v}(t)}{v(t)} + \frac{z_1^3}{v^4(t) - z_1^4} [\bar{f}_1 + h_1 x_2] + \frac{1}{2} \\ & - h_{10} \tilde{\varphi}_1^T \Gamma_1^{-1} \dot{\hat{\varphi}}_1 - \frac{h_{10}}{2} \frac{z_1^6}{(v^4(t) - z_1^4)^2} - \frac{3z_1^4 h_1}{4(v^4(t) - z_1^4)^{4/3}} \end{aligned} \quad (12)$$

where $\bar{f}_1 = f_1 - \dot{y}_d + \frac{z_1(3v^4(t) + z_1^4)^2}{8(v^4(t) - z_1^4)^3} \|g_1\|^4 + \frac{z_1^3}{2(v^4(t) - z_1^4)} + \frac{3z_1}{4(v^4(t) - z_1^4)^{1/3}}$.

As we all know, \bar{f}_1 is a unknown function. By Lemma 3, we can use a MTN $\varphi_1^T S_{m_1}(\mathbf{z}_1)$ to estimate \bar{f}_1 . In other words, for $\forall \bar{\delta}_1 > 0$, a MTN $\varphi_1^T S_{m_1}(\mathbf{z}_1)$ is applied to approach \bar{f}_1 , such that

$$\bar{f}_1 = \varphi_1^T S_{m_1}(\mathbf{z}_1) + \delta_1(\mathbf{z}_1), |\delta_1(\mathbf{z}_1)| \leq \bar{\delta}_1 \quad (13)$$

where $\mathbf{z}_1 = [z_1]^T$ is the input of the MTN and $\delta_1(\mathbf{z}_1)$ is the approximation error.

By Young's Inequality, we obtain

$$\frac{z_1^3}{v^4(t) - z_1^4} \delta_1(\mathbf{z}_1) \leq \frac{h_{10}}{2} \frac{z_1^6}{(v^4(t) - z_1^4)^2} + \frac{1}{2h_{10}} \bar{\delta}_1^2 \quad (14)$$

$$\frac{z_1^3 h_1 z_2}{v^4(t) - z_1^4} \leq \frac{3z_1^4 h_1}{4(v^4(t) - z_1^4)^{4/3}} + \frac{1}{4} z_2^4 h_1 \quad (15)$$

Substituting (7), (13), (14) and (15) into (12), we have

$$\begin{aligned} \mathcal{L}V_1 \leq & -\frac{z_1^4}{(v^4(t) - z_1^4)} \frac{\dot{v}(t)}{v(t)} + \frac{z_1^3}{v^4(t) - z_1^4} [\varphi_1^T S_{m_1}(\mathbf{z}_1) + h_1 \alpha_1] + \frac{1}{2} \\ & - h_{10} \tilde{\varphi}_1^T \Gamma_1^{-1} \dot{\hat{\varphi}}_1 + \frac{1}{2h_{10}} \bar{\delta}_1^2 + \frac{1}{4} z_2^4 h_1 \end{aligned} \quad (16)$$

Based on (16), taking the intermediate control signal α_1 as

$$\alpha_1 = -k_1 z_1 - \frac{1}{h_{10}} \bar{k}_1 z_1 - \hat{\varphi}_1^T S_{m_1}(\mathbf{z}_1) \quad (17)$$

where $\bar{k}_1 = \sqrt{(\dot{v}(t)/v(t))^2 + \sigma_1}$, $k_1 > 0$ and $\sigma_1 > 0$ are design parameters.

By the definition of \bar{k}_1 , we have

$$\bar{k}_1 + \frac{\dot{v}(t)}{v(t)} > 0 \quad (18)$$

According to Assumption 1 and (17), we obtain

$$\begin{aligned} \frac{z_1^3 h_1 \alpha_1}{v^4(t) - z_1^4} \leq & -k_1 h_{10} \frac{z_1^4}{v^4(t) - z_1^4} - \bar{k}_1 \frac{z_1^4}{v^4(t) - z_1^4} \\ & - \frac{z_1^3 h_{10} \hat{\varphi}_1^T S_{m_1}(\mathbf{z}_1)}{v^4(t) - z_1^4} \end{aligned} \quad (19)$$

Substituting (17), (18) and (19) into (16), we have

$$\begin{aligned} \mathcal{L}V_1 \leq & -k_1 h_{10} \frac{z_1^4}{v^4(t) - z_1^4} + \frac{1}{4} z_2^4 h_1 + \frac{\bar{\delta}_1^2}{2h_{10}} \\ & + \frac{1}{2} + \tilde{\varphi}_1^T h_{10} \left[\frac{z_1^3 S_{m_1}(\mathbf{z}_1)}{(v^4(t) - z_1^4)^2} - \Gamma_1^{-1} \dot{\hat{\varphi}}_1 \right] \end{aligned} \quad (20)$$

Step i : Consider the stochastic Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{4} \log \frac{k_{b_i}^4(t)}{k_{b_i}^4(t) - z_i^4} + \frac{1}{2} h_{i0} \tilde{\varphi}_i^T \Gamma_i^{-1} \tilde{\varphi}_i \quad (21)$$

where $\tilde{\varphi}_i = \varphi_i - \hat{\varphi}_i$ is the parameter error vector, $\Gamma_i = \mathbf{I}_i^T > 0$ is a symmetric positive definite matrix.

According to Definition 1 and (21), we have

$$\begin{aligned} \mathcal{L}V_i = \mathcal{L}V_{i-1} & - \frac{z_i^4}{(k_{b_i}^4(t) - z_i^4)} \frac{\dot{k}_{b_i}}{k_{b_i}} \\ & + \frac{z_i^3}{k_{b_i}^4(t) - z_i^4} [f_i + h_i x_{i+1} - \Delta \alpha_{i-1}] \\ & + \frac{z_i^2 (3k_{b_i}^4(t) + z_i^4)}{2(k_{b_i}^4(t) - z_i^4)^2} \tilde{g}_i^T \tilde{g}_i + h_{i0} \tilde{\varphi}_i^T \Gamma_i^{-1} \dot{\hat{\varphi}}_i \end{aligned} \quad (22)$$

where $\tilde{g}_i = g_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} g_j$.

By Young's Inequality, we obtain

$$\frac{z_i^2 (3k_{b_i}^4(t) + z_i^4)}{2(k_{b_i}^4(t) - z_i^4)^2} \tilde{g}_i^T \tilde{g}_i \leq \frac{z_i^4 (3k_{b_i}^4(t) + z_i^4)^2}{8(k_{b_i}^4(t) - z_i^4)^4} \|\tilde{g}_i\|^4 + \frac{1}{2} \quad (23)$$

Substituting (23) into (22), we have

$$\begin{aligned} \mathcal{L}V_i \leq & \mathcal{L}V_{i-1} - \frac{z_i^4 \dot{k}_{b_i}}{(k_{b_i}^4(t) - z_i^4) k_{b_i}} \\ & + \frac{z_i^3}{k_{b_i}^4(t) - z_i^4} [\bar{f}_i + h_i x_{i+1}] - \frac{h_{i0} z_i^6}{2(k_{b_i}^4(t) - z_i^4)^2} \\ & - \frac{3z_i^4 h_i}{4(k_{b_i}^4(t) - z_i^4)^{4/3}} - h_{i0} \tilde{\varphi}_i^T \Gamma_i^{-1} \dot{\hat{\varphi}}_i - \frac{z_i^4 h_{i-1}}{4} + \frac{1}{2} \end{aligned} \quad (24)$$

where $\bar{f}_i = f_i - \Delta\alpha_{i-1} + \frac{(k_{b_i}^4(t) - z_i^4)}{4} z_i h_{i-1} + \frac{z_i (3k_{b_i}^4(t) + z_i^4)^2}{8(k_{b_i}^4(t) - z_i^4)^3} \|\tilde{g}_i\|^4$

$$+ \frac{z_i^3}{2(k_{b_i}^4(t) - z_i^4)} + \frac{3z_i}{4(k_{b_i}^4(t) - z_i^4)^{1/3}}.$$

\bar{f}_i is also a unknown function, and we can use a MTN $\varphi_i^T S_{m_i}(\mathbf{z}_i)$ to estimate \bar{f}_i . In other words, for $\forall \bar{\delta}_i > 0$, a MTN $\varphi_i^T S_{m_i}(\mathbf{z}_i)$ is applied to approach \bar{f}_i , such that

$$\bar{f}_i = \varphi_i^T S_{m_i}(\mathbf{z}_i) + \delta_i(\mathbf{z}_i), |\delta_i(\mathbf{z}_i)| \leq \bar{\delta}_i \quad (25)$$

where $\mathbf{z}_i = [z_1, \dots, z_i]^T$ is the input of the MTN and $\delta_i(\mathbf{z}_i)$ is the approximation error.

According to Young's Inequality, we obtain

$$\frac{z_i^3}{k_{b_i}^4(t) - z_i^4} \delta_i(\mathbf{z}_i) \leq \frac{h_{i0}}{2} \frac{z_i^6}{(k_{b_i}^4(t) - z_i^4)^2} + \frac{1}{2h_{i0}} \bar{\delta}_i^2 \quad (26)$$

$$\frac{z_i^3 h_i z_{i+1}}{k_{b_i}^4(t) - z_i^4} \leq \frac{3z_i^4 h_i}{4(k_{b_i}^4(t) - z_i^4)^{4/3}} + \frac{1}{4} z_{i+1}^4 h_i \quad (27)$$

Substituting (7), (25), (26) and (27) into (24), we have

$$\begin{aligned} \mathcal{L}V_i \leq & \mathcal{L}V_{i-1} - \frac{z_i^4}{(k_{b_i}^4(t) - z_i^4)} \frac{\dot{k}_{b_i}}{k_{b_i}} \\ & + \frac{z_i^3}{k_{b_i}^4(t) - z_i^4} [h_i \alpha_i + \varphi_i^T S_{m_i}(\mathbf{z}_i)] \\ & + \frac{1}{2} - h_{i0} \tilde{\varphi}_i^T \Gamma_i^{-1} \dot{\hat{\varphi}}_i + \frac{1}{2h_{i0}} \bar{\delta}_i^2 - \frac{z_i^4 h_{i-1}}{4} + \frac{1}{4} z_{i+1}^4 h_i \end{aligned} \quad (28)$$

Based on (28), taking the intermediate control signal α_i as

$$\alpha_i = -k_i z_i - \frac{1}{h_{i0}} \bar{k}_i z_i - \hat{\varphi}_i^T S_{m_i}(\mathbf{z}_i) \quad (29)$$

where $\bar{k}_i = \sqrt{(\dot{k}_{b_i}/k_{b_i})^2 + \sigma_i}$, $k_i > 0$ and $\sigma_i > 0$ are design parameters.

By the definition of \bar{k}_i , we have

$$\bar{k}_i + \frac{\dot{k}_{b_i}}{k_{b_i}} > 0 \quad (30)$$

According to Assumption 1 and (29), we obtain

$$\begin{aligned} \frac{z_i^3 h_i \alpha_i}{k_{b_i}^4(t) - z_i^4} \leq & -k_i h_{i0} \frac{z_i^4}{k_{b_i}^4(t) - z_i^4} - \bar{k}_i \frac{z_i^4}{k_{b_i}^4(t) - z_i^4} \\ & - \frac{z_i^3 h_i \hat{\theta}_i^T S_{m_i}(\mathbf{z}_i)}{k_{b_i}^4(t) - z_i^4} \end{aligned} \quad (31)$$

Substituting (20), (29), (30) and (31) into (28), we have

$$\begin{aligned} \mathcal{L}V_i \leq & - \sum_{j=1}^i k_j h_{j0} \frac{z_j^4}{k_{b_j}^4(t) - z_j^4} + \frac{1}{4} z_{i+1}^4 h_i + \frac{i}{2} + \sum_{j=1}^i \frac{\bar{\delta}_j^2}{2h_{j0}} \\ & - \sum_{j=1}^i \tilde{\varphi}_j^T h_{j0} \left[\frac{z_j^3 S_{m_j}(\mathbf{z}_j)}{(k_{b_j}^4(t) - z_j^4)^2} - \Gamma_j^{-1} \dot{\hat{\varphi}}_j \right] \end{aligned} \quad (32)$$

Step n : Consider the stochastic Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{4} \log \frac{k_{b_n}^4(t)}{k_{b_n}^4(t) - z_n^4} + \frac{1}{2} h_{n0} \tilde{\varphi}_n^T \Gamma_n^{-1} \tilde{\varphi}_n \quad (33)$$

where $\tilde{\varphi}_n = \varphi_n - \hat{\varphi}_n$ is the parameter error vector, $\Gamma_n = \Gamma_n^T > 0$ is a symmetric positive definite matrix.

According to Definition 1 and (33), we have

$$\begin{aligned} \mathcal{L}V_n = & \mathcal{L}V_{n-1} - \frac{z_n^4}{(k_{b_n}^4(t) - z_n^4)} \frac{\dot{k}_{b_n}}{k_{b_n}} \\ & + \frac{z_n^3}{k_{b_n}^4(t) - z_n^4} [f_n + h_n u - \Delta\alpha_{n-1}] \\ & + \frac{z_n^2 (3k_{b_n}^4(t) + z_n^4)}{2(k_{b_n}^4(t) - z_n^4)^2} \tilde{g}_n^T \tilde{g}_n - h_{n0} \tilde{\varphi}_n^T \Gamma_n^{-1} \dot{\hat{\varphi}}_n \end{aligned} \quad (34)$$

where $\tilde{g}_n = g_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial x_j} g_j$.

By Young's Inequality, we obtain

$$\frac{z_n^2(3k_{b_n}^4(t) + z_n^4)}{2(k_{b_n}^4(t) - z_n^4)^2} \tilde{g}_n \leq \frac{z_n^4(3k_{b_n}^4(t) + z_n^4)^2}{8(k_{b_n}^4(t) - z_n^4)^4} \|\tilde{g}_n\|^4 + \frac{1}{2} \quad (35)$$

Substituting (35) into (34), we have

$$\begin{aligned} \mathcal{L}V_n \leq & \mathcal{L}V_{n-1} - \frac{z_n^4}{(k_{b_n}^4(t) - z_n^4)} \frac{\dot{k}_{b_n}}{k_{b_n}} \\ & + \frac{z_n^3}{k_{b_n}^4(t) - z_n^4} [\bar{f}_n + h_n u] + \frac{1}{2} - h_{n0} \tilde{\varphi}_n^T \Gamma_n^{-1} \dot{\hat{\varphi}}_n \\ & - \frac{h_{n0}}{2} \frac{z_n^6}{(k_{b_n}^4(t) - z_n^4)^2} - \frac{z_n^4 h_{n-1}}{4} \end{aligned} \quad (36)$$

where $\bar{f}_n = f_n - \Delta\alpha_{n-1} + \frac{(k_{b_n}^4(t) - z_n^4)}{4} z_n h_{n-1} + \frac{z_n^3}{2(k_{b_n}^4(t) - z_n^4)} + \frac{z_n(3k_{b_n}^4(t) + z_n^4)^2}{8(k_{b_n}^4(t) - z_n^4)^3} \|\tilde{g}_n\|^4$.

In view of \bar{f}_n is a unknown function. By Lemma 3, for $\forall \bar{\delta}_n > 0$, a MTN $\varphi_n^T S_{m_n}(\mathbf{z}_n)$ can be applied to estimate \bar{f}_n , such that

$$\bar{f}_n = \varphi_n^T S_{m_n}(\mathbf{z}_n) + \delta_n(\mathbf{z}_n), |\delta_n(\mathbf{z}_n)| \leq \bar{\delta}_n \quad (37)$$

where $\mathbf{z}_n = [z_1, \dots, z_n]^T$ is the input of the MTN and $\delta_n(\mathbf{z}_n)$ is the approximation error.

According to Young's Inequality and (35), we obtain

$$\frac{z_n^3}{k_{b_n}^4(t) - z_n^4} \delta_n(\mathbf{z}_n) \leq \frac{h_{n0}}{2} \frac{z_n^6}{(k_{b_n}^4(t) - z_n^4)^2} + \frac{1}{2h_{n0}} \bar{\delta}_n^2 \quad (38)$$

Substituting (7), (37) and (38) into (36), we have

$$\begin{aligned} \mathcal{L}V_n \leq & \mathcal{L}V_{n-1} - \frac{z_n^4}{(k_{b_n}^4(t) - z_n^4)} \frac{\dot{k}_{b_n}}{k_{b_n}} + \frac{z_n^3}{k_{b_n}^4(t) - z_n^4} [h_n u + \varphi_n^T S_{m_n}(\mathbf{z}_n)] \\ & + \frac{1}{2} - h_{n0} \tilde{\varphi}_n^T \Gamma_n^{-1} \dot{\hat{\varphi}}_n + \frac{1}{2h_{n0}} \bar{\delta}_n^2 - \frac{z_n^4 h_{n-1}}{4} \end{aligned} \quad (39)$$

Taking the control input u as

$$u = -k_n z_n - \frac{1}{h_{n0}} \bar{k}_n z_n - \hat{\varphi}_n^T S_{m_n}(\mathbf{z}_n) \quad (40)$$

where $\bar{k}_n = \sqrt{(\dot{k}_{b_n}/k_{b_n})^2 + \sigma_n}$, $k_n > 0$ and $\sigma_n > 0$ are design parameters.

By definition of \bar{k}_n , we have

$$\bar{k}_n + \frac{\dot{k}_{b_n}}{k_{b_n}} > 0 \quad (41)$$

According to Assumption 1 and (40), we have

$$\begin{aligned} \frac{z_n^3 h_n u}{k_{b_n}^4(t) - z_n^4} \leq & -k_n h_{n0} \frac{z_n^4}{k_{b_n}^4(t) - z_n^4} - \bar{k}_n \frac{z_n^4}{k_{b_n}^4(t) - z_n^4} \\ & - \frac{z_n^3 h_n \tilde{\varphi}_n^T S_{m_n}(\mathbf{z}_n)}{k_{b_n}^4(t) - z_n^4} \end{aligned} \quad (42)$$

Substituting (32), (40), (41) and (42) into (39), we have

$$\begin{aligned} \mathcal{L}V_n \leq & - \sum_{j=1}^n k_j h_{j0} \frac{z_j^4}{k_{b_j}^4(t) - z_j^4} + \frac{n}{2} + \sum_{j=1}^n \frac{\bar{\delta}_j^2}{2h_{j0}} \\ & - \sum_{j=1}^n \tilde{\varphi}_j^T h_{j0} \left[\frac{z_j^3 S_{m_j}(\mathbf{z}_j)}{(k_{b_j}^4(t) - z_j^4)^2} - \Gamma_j^{-1} \dot{\hat{\varphi}}_j \right] \end{aligned} \quad (43)$$

Select the adaptation laws $\dot{\hat{\varphi}}_j$ as

$$\dot{\hat{\varphi}}_j = \Gamma_j \frac{z_j^3 S_{m_j}(\mathbf{z}_j)}{(k_{b_j}^4(t) - z_j^4)^2} - \Gamma_j \eta_j \hat{\varphi}_j, \quad j = 1, 2, \dots, n \quad (44)$$

where $\eta_j > 0$ are designed parameters.

Substituting (44) into (43), we have

$$\mathcal{L}V_n \leq - \sum_{j=1}^n k_j h_{j0} \frac{z_j^4}{k_{b_j}^4(t) - z_j^4} - \sum_{j=1}^n h_{j0} \eta_j \tilde{\varphi}_j^T \hat{\varphi}_j + \frac{n}{2} + \sum_{j=1}^n \frac{\bar{\delta}_j^2}{2h_{j0}} \quad (45)$$

By Young's Inequality, we obtain

$$h_{j0} \eta_j \tilde{\varphi}_j^T \hat{\varphi}_j \leq - \frac{h_{j0} \eta_j}{2} \tilde{\varphi}_j^T \tilde{\varphi}_j + \frac{h_{j0} \eta_j}{2} \|\varphi_j\|^2 \quad (46)$$

with

$$- \frac{h_{j0} \eta_j}{2} \tilde{\varphi}_j^T \tilde{\varphi}_j \leq - \frac{h_{j0} \eta_j}{2\lambda_{\max}(\Gamma_j^{-1})} \tilde{\varphi}_j^T \Gamma_j^{-1} \tilde{\varphi}_j \quad (47)$$

Substituting (46) and (47) into (45), we have

$$\begin{aligned} \mathcal{L}V_n \leq & - \sum_{j=1}^n k_j h_{j0} \frac{z_j^4}{k_{b_j}^4(t) - z_j^4} - \frac{1}{2} \bar{\eta}_j \sum_{j=1}^n \tilde{\boldsymbol{\varphi}}_j^T \boldsymbol{\Gamma}_j^{-1} \tilde{\boldsymbol{\varphi}}_j + \frac{n}{2} \\ & + \sum_{j=1}^n \frac{\bar{\delta}_j^2}{2h_{j0}} + \sum_{j=1}^n \frac{h_{j0}\eta_j}{2} \|\boldsymbol{\varphi}_j\|^2 \end{aligned} \quad (48)$$

where $\bar{\eta}_j = \min \left\{ \left(\left(h_{j0}\eta_j \right) / \left(\lambda_{\max} \left(\boldsymbol{\Gamma}_j^{-1} \right) \right) \right) \mid j = 1, 2, \dots, n \right\}$.

Let $a = \min \left\{ k_j h_{j0}, \bar{\eta}_j, j = 1, 2, \dots, n \right\}$, $b = \frac{n}{2} + \sum_{j=1}^n \frac{\bar{\delta}_j^2}{2h_{j0}} + \sum_{j=1}^n \frac{h_{j0}\eta_j}{2} \|\boldsymbol{\varphi}_j\|^2$, yields

$$\mathcal{L}V_n \leq -aV + b \quad (49)$$

4 | STABILITY ANALYSIS

Based on the above control scheme and design process, we can generalize the following theorem:

Theorem 1. *Under Assumptions 1–2, consider the closed-loop control system, which comprises the system (1), the control input (40), the intermediate control signals (17) and (29), and the adaptive law (44). For any initial condition, the following requirements are achieved:*

- (i) The system output y can track the given reference signal y_d and the tracking error finally converges to a small neighborhood of the origin in the finite-time;
- (ii) All the signals in the closed-loop system are SGUUB in probability and the system states satisfy the given limited conditions.

Proof. From (9), (21) and (33), we have

$$V_n = \frac{1}{4} \sum_{i=1}^n \log \frac{k_{b_i}^4(t)}{k_{b_i}^4(t) - z_i^4} + \frac{1}{2} \sum_{i=1}^n h_{i0} \tilde{\boldsymbol{\varphi}}_i^T \boldsymbol{\Gamma}_i^{-1} \tilde{\boldsymbol{\varphi}}_i \quad (50)$$

Combining (49) with Lemma 1, we have

$$E(V_n(t)) \leq V_n(0)e^{-at} + \frac{b}{a} \quad (51)$$

The above inequality means that $E(V_n(t))$ is eventually bounded. Recalling (49), we obtain that all the signals in the closed-loop system are SGUUB in probability.

From (50) and (51), we obtain

$$\frac{1}{4} \log \frac{k_{b_i}^4(t)}{k_{b_i}^4(t) - z_i^4} \leq V_n(0)e^{-at} + \frac{b}{a} \quad (52)$$

Taking exponentials on both sides of the (52), we have

$$|z_i| \leq k_{b_i} \sqrt[4]{1 - e^{-4[V_n(0)e^{-at} + b/a]}} \quad (53)$$

From (53), we obtain that $|z_1| < k_{b_1} = v(t)$. According to Definition 2, we can conclude that the tracking error finally converges to a small neighborhood of the origin in the finite-time.

According to (7), we know that $x_1 = z_1 + y_d(t)$, since $|z_1| < k_{b_1}$ and $y_d(t) \leq A_1$, then, $|x_1| \leq |z_1| + |y_d(t)| < k_{b_1} + A_1 < k_{c_1}$. Similarly, we obtain that $x_i = z_i + \alpha_{i-1}$, $i = 2, \dots, n$, since $|z_i| < k_{b_i}$ and $\alpha_{i-1} \leq A_i$, then $|x_i| \leq |z_i| + |\alpha_{i-1}| < k_{b_i} + A_i < k_{c_i}$. Thus, the system constraints are not violated.

The proof of Theorem 1 is completed.

5 | SIMULATION RESULTS

In this section, two simulation examples are given to show the effectiveness of the proposed scheme in this paper.

Example 1. (Numerical example): Consider the second-order nonlinear stochastic system with time-varying state and finite-time output constraint

$$\begin{cases} dx_1 = (x_1^2 \sin(x_1) + (1 + x_1^2)x_2)dt + 0.5x_1 d\boldsymbol{\omega} \\ dx_2 = (x_1x_2^2 + 2u)dt + 0.5x_2 d\boldsymbol{\omega} \\ y = x_1 \end{cases} \quad (54)$$

with the initial state $[x_1(0), x_2(0)]^T = [0, 0]^T$, and $y_d = \sin t$ is the given reference signal. The time-varying state constraint is $|x_2| \leq 1.35 + 0.3\sin(1.3t)$.

By the controller design procedure, the actual control law, the intermediate control signal and the adaptive laws are chosen respectively as

$$u = -k_2 z_2 - \frac{1}{h_{20}} \bar{k}_2 z_2 - \hat{\boldsymbol{\varphi}}_2^T S_{m_2}(\mathbf{z}_2)$$

$$\alpha_1 = -k_1 z_1 - \frac{1}{h_{10}} \bar{k}_1 z_1 - \hat{\boldsymbol{\varphi}}_1^T S_{m_1}(\mathbf{z}_1)$$

$$\dot{\hat{\boldsymbol{\varphi}}}_j = \boldsymbol{\Gamma}_j \frac{z_j^3 S_{m_j}(\mathbf{z}_j)}{\left(k_{b_j}^4(t) - z_j^4\right)^2} - \boldsymbol{\Gamma}_j \eta_j \hat{\boldsymbol{\varphi}}_j$$

where $j = 1, 2$, $\mathbf{z}_1 = [z_1]^T$, $\mathbf{z}_2 = [z_1, z_2]^T$, $z_1 = x_1 - y_d$, and $z_2 = x_2 - \alpha_1$. The FTPF is chosen in the following form

$$v(t) = \begin{cases} \left(v_0 - \frac{t}{T_f}\right) e^{1 - \frac{T_f}{T_f - t}} + v_{T_f}, & t \in [0, T_f) \\ v_{T_f}, & t \in [T_f, +\infty) \end{cases}$$

where $v_0 = 2$, $v_{T_f} = 0.1$, $T_f = 3$, and $v(0) = v_0 + T_f = 2.1$.

The design parameters are taken as follows: $k_1 = 10$, $k_2 = 8.5$, $\Gamma_1 = 9\mathbf{I}_5$, $\Gamma_2 = 5.5\mathbf{I}_9$. The simulation results of Example 1 are shown in Figures 1–4, respectively. Figure 1 illustrates trajectories of the reference signal y_d and the system output y . Figure 2 shows the response of control signal u . Figure 3 displays that the state variable x_2 satisfies the given constraints. Figure 4 depicts that the tracking error $y - y_d$ converges to a small neighborhood around the origin in the finite-time. The presented

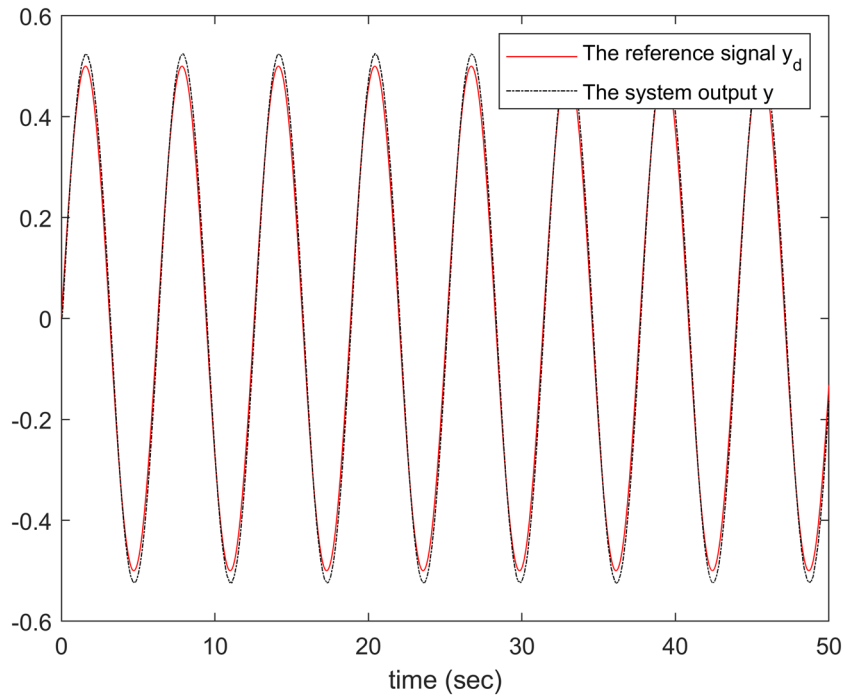


FIGURE 1 The trajectories of system output y and the reference signal y_d [Color figure can be viewed at wileyonlinelibrary.com]

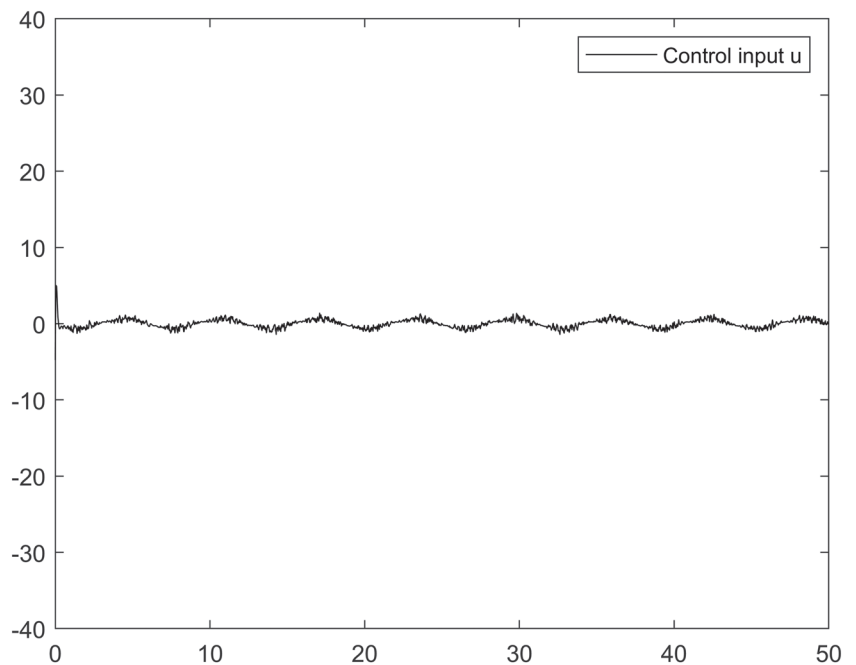


FIGURE 2 The trajectory of system control input u

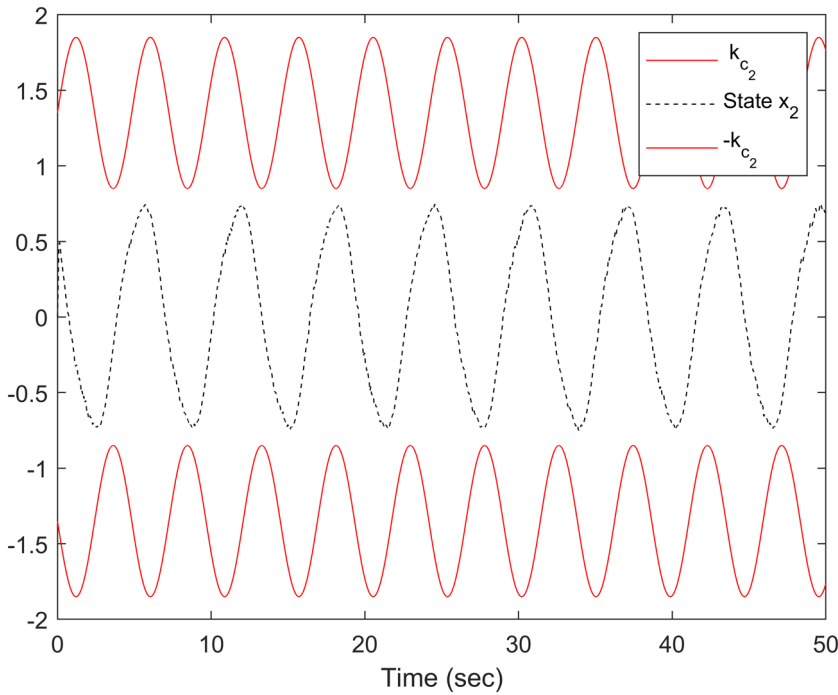


FIGURE 3 The trajectories of system state variable x_2 and constraint conditions [Color figure can be viewed at wileyonlinelibrary.com]

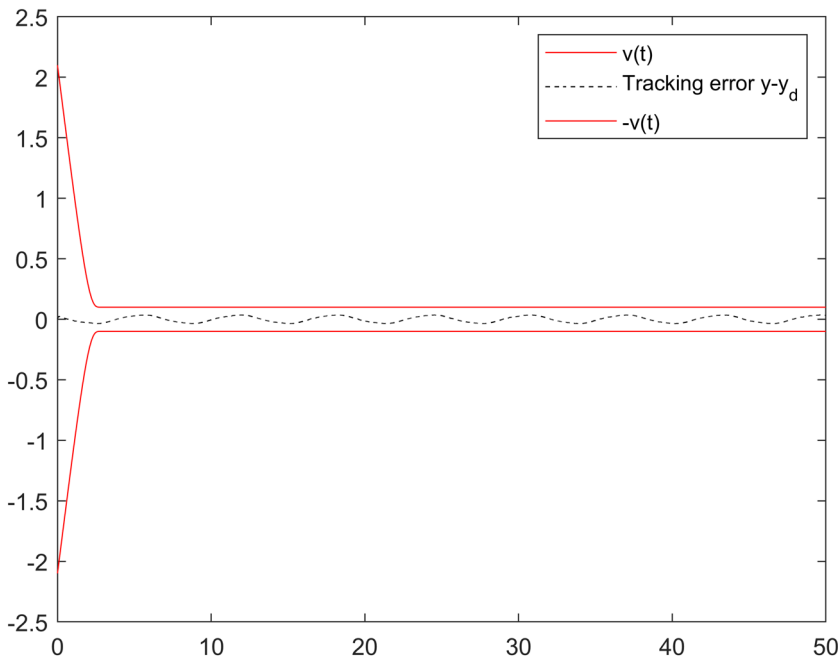


FIGURE 4 The trajectories of $v(t)$ and tracking error $y - y_d$ [Color figure can be viewed at wileyonlinelibrary.com]

simulation results illustrate the effectiveness of the proposed approach in this paper.

Example 2. (Practical example): Considering a one-link manipulator system with time-varying state constraints and finite-time output constraint. According to [50], the system can be described by

$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = (x_3 - 2\sin(x_1) - x_2) dt + \sin(x_1^2) d\omega \\ dx_3 = (u - 2x_2 - x_3) dt \\ y = x_1 \end{cases} \quad (55)$$

with the initial state $[x_1(0), x_2(0), x_3(0)]^T = [0, 0, 0]^T$. x_1 denotes the link angular position, x_2 denotes

velocity, and x_3 denotes acceleration. $y_d = 0.5\sin t$ is the given reference signal. The time-varying state constraints are $|x_2| \leq 1 + 0.1\sin(1.5t)$ and $|x_3| \leq 4 + 0.3\sin(1.1t)$.

By the controller design procedure, the actual control law, the intermediate control signals and the adaptive laws are chosen, respectively, as

$$u = -k_3 z_3 - \frac{1}{h_{30}} \bar{k}_3 z_3 - \hat{\varphi}_3^T S_{m_3}(\mathbf{z}_3)$$

$$\alpha_j = -k_j z_j - \frac{1}{h_{j0}} \bar{k}_j z_j - \hat{\varphi}_j^T S_{m_j}(\mathbf{z}_j)$$

$$\dot{\hat{\varphi}}_i = \Gamma_i \frac{z_i^3 S_{m_i}(\mathbf{z}_i)}{(k_{b_j}^4(t) - z_j^4)^2} - \Gamma_i \eta_i \hat{\varphi}_i$$

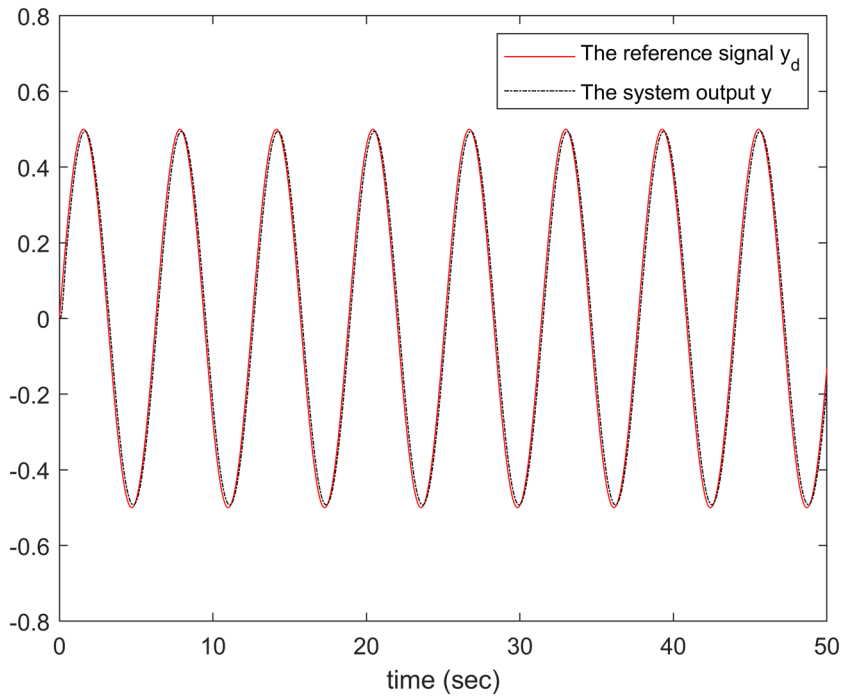


FIGURE 5 The trajectories of system output y and the reference signal y_d [Color figure can be viewed at wileyonlinelibrary.com]

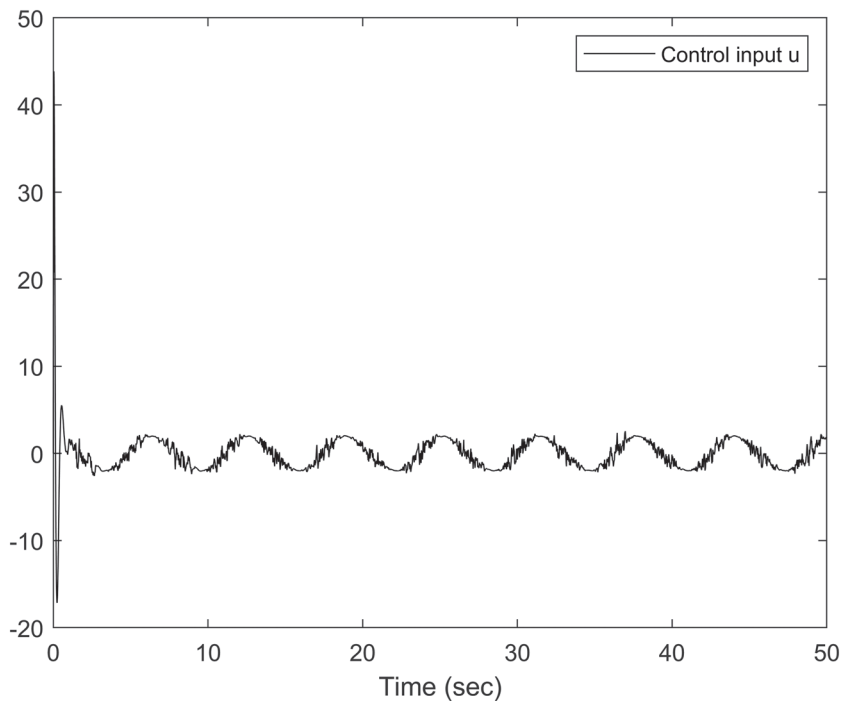


FIGURE 6 The trajectory of system control input u

where $j = 1, 2, 1 \leq i \leq 3, \mathbf{z}_1 = [z_1]^T, \mathbf{z}_2 = [z_1, z_2]^T, \mathbf{z}_3 = [z_1, z_2, z_3]^T, z_1 = x_1 - y_d, z_2 = x_2 - \alpha_1,$ and $z_3 = x_3 - \alpha_2.$ The FTPF is chosen in the following form

$$v(t) = \begin{cases} \left(v_0 - \frac{t}{T_f}\right) e^{1 - \frac{T_f}{T_f - t}} + v_{T_f}, & t \in [0, T_f) \\ v_{T_f}, & t \in [T_f, +\infty) \end{cases}$$

where $v_0 = 2, v_{T_f} = 0.1, T_f = 3,$ and $v(0) = v_0 + T_f = 2.1.$

The design parameters are taken as follows: $k_1 = 80, k_2 = 100, k_3 = 100, \Gamma_1 = 5\mathbf{I}_5, \Gamma_2 = 10\mathbf{I}_9, \Gamma_3 = 10\mathbf{I}_9.$ The simulation results of Example 2 are shown in Figures 5–9, 9, respectively. The results show that the tracking control performance is still fairly satisfactory, which further verifies the effectiveness of the proposed control method in this paper.

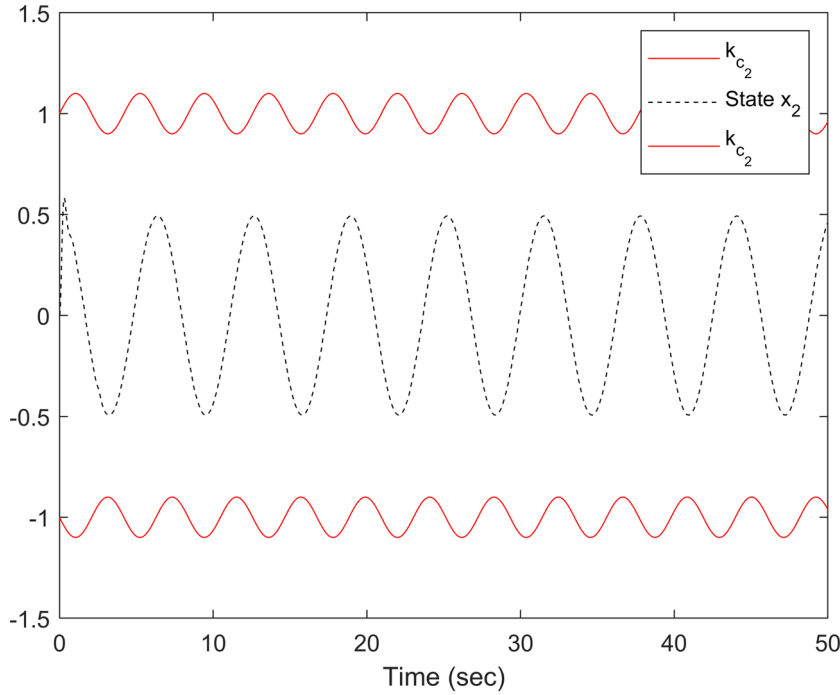


FIGURE 7 The trajectories of system state variables x_2 and constraint conditions [Color figure can be viewed at wileyonlinelibrary.com]

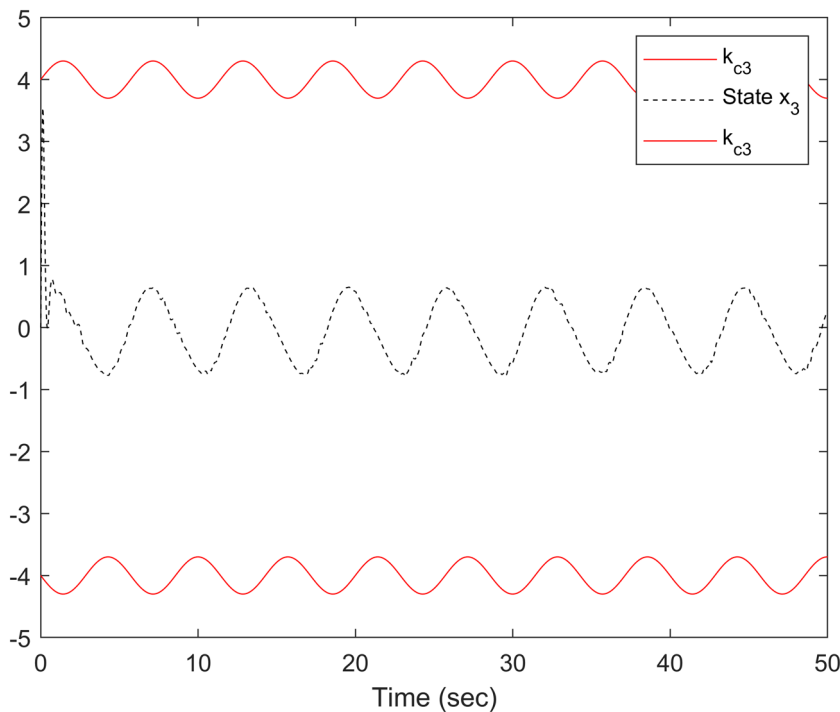
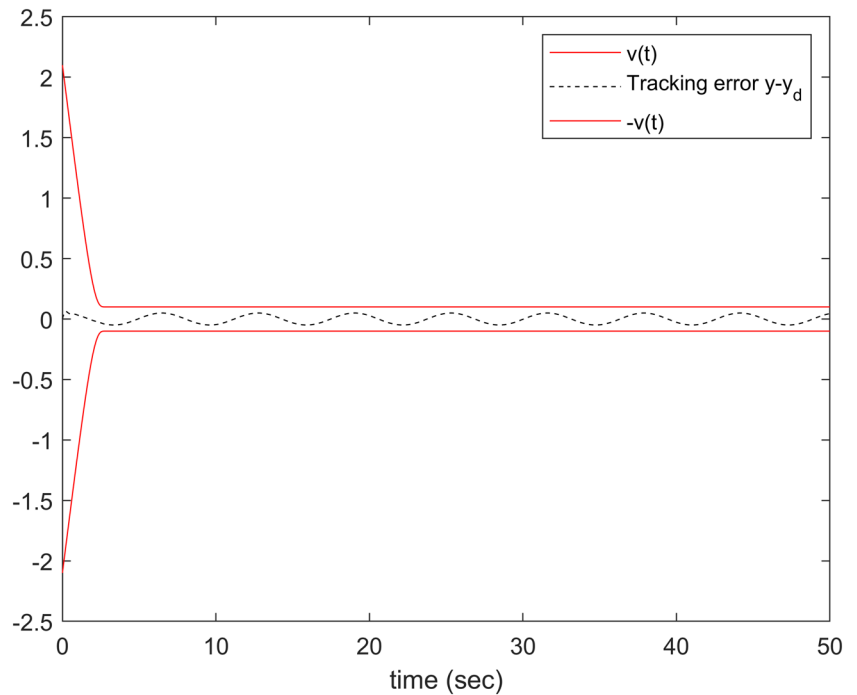


FIGURE 8 The trajectories of system state variables x_3 and constraint conditions [Color figure can be viewed at wileyonlinelibrary.com]

FIGURE 9 The trajectories of $v(t)$ and tracking error $y - y_d$ [Color figure can be viewed at wileyonlinelibrary.com]



Remark 3. In order to obtain good tracking performance, design parameters should be selected appropriately. In general, the tracking error converges to a small residual set near the origin by properly adjusting the parameters k_i and matrix Γ_i .

6 | CONCLUSION

In this paper, a new adaptive MTN backstepping control scheme is developed to solve the problem of tracking control for nonlinear stochastic systems with full state time-varying constraints and the finite-time output constraint. The MTNs are applied to handle the unknown functions, which greatly reduces the computation complexity of the proposed control scheme. The FTPF and the TVBLFs are introduced to achieve the finite-time output constraint and full state time-varying constraints, respectively. Different from the existing finite-time results, the proposed scheme can guarantee that the tracking error converges to an arbitrarily small region at a settling time. Finally, the simulation results of two examples demonstrate the effectiveness of the proposed control scheme in this paper.

In our future research, we could further focus on the problem of globally uniformly ultimately bounded

(GUUB) in probability of the nonlinear stochastic system of this paper and the adaptive finite-time MTN control problems for the other nonlinear stochastic systems such as switch systems and input dead-zone systems.



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AUTHOR CONTRIBUTIONS

Ming-Xin Wang: Conceptualization, formal analysis, visualization. **Shan-Liang Zhu:** Methodology, project administration, supervision. **Yu-Qun Han:** Conceptualization, formal analysis, funding acquisition, supervision.

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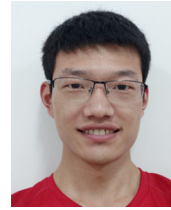
REFERENCES

1. H. Mukaidani, *The guaranteed cost control for uncertain nonlinear large-scale stochastic systems via state and static output feedback*, J. Math. Anal. Appl. **359** (2009), no. 2, 527–535.
2. S. Li and Z. R. Xiang, *Sampled-data decentralized output feedback control for a class of switched large-scale stochastic nonlinear systems*, IEEE Syst. J. **14** (2020), no. 2, 1602–1610.

3. Z. G. Pan and T. Basar, *Backstepping controller design for nonlinear stochastic systems under a risk-sensitive cost criterion*, SIAM J. Control Optim. **37** (1999), no. 3, 957–995.
4. H. B. Ji and H. S. Xi, *Adaptive output-feedback tracking of stochastic nonlinear systems*, IEEE Trans. Autom. Control **51** (2006), no. 2, 355–360.
5. Z. Pan, K. Ezal, A. J. Krener, and P. V. Kokotovic, *Backstepping design with local optimality matching*, IEEE Trans. Autom. Control **46** (2001), no. 7, 1014–1027.
6. Y. G. Liu, Z. G. Pan, and S. J. Shi, *Output feedback control design for strict-feedback stochastic nonlinear systems under a risk-sensitive cost*, IEEE Trans. Autom. Control **48** (2003), no. 3, 509–513.
7. Y. G. Liu and J. F. Zhang, *Practical output-feedback risk-sensitive control for stochastic nonlinear systems with stable zero-dynamics*, SIAM J. Control Optim. **45** (2006), no. 3, 885–926.
8. H. Deng and M. Krstic, *Stochastic nonlinear stabilization-I: A backstepping design*, Syst. Control Lett. **32** (1997), no. 3, 143–150.
9. X. J. Xie and J. Tian, *Adaptive state-feedback stabilization of high-order stochastic systems with nonlinear parameterization*, Automatica **45** (2009), no. 1, 126–133.
10. S. J. Liu, J. F. Zhang, and Z. P. Jiang, *Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems*, Automatica **43** (2007), no. 2, 238–251.
11. S. L. Xie and L. H. Xie, *Decentralized stabilization of a class of interconnected stochastic nonlinear systems*, IEEE Trans. Autom. Control **45** (2000), no. 1, 132–137.
12. X. J. Xie and J. Tian, *State-feedback stabilization for high-order stochastic nonlinear systems with stochastic inverse dynamics*, Int. J. Robust Nonlinear Control **17** (2007), no. 14, 1343–1362.
13. Y. M. Li, S. Sui, and S. C. Tong, *Adaptive fuzzy control design for stochastic nonlinear switched systems with arbitrary switchings and unmodeled dynamics*, IEEE Trans. Cybern. **47** (2017), no. 2, 403–414.
14. J. M. Li and H. Y. Yue, *Adaptive fuzzy tracking control for stochastic nonlinear systems with unknown time-varying delays*, Appl. Math Comput. **256** (2015), 514–528.
15. H. Q. Wang, B. Chen, X. P. Liu, K. F. Liu, and C. Lin, *Adaptive neural tracking control for stochastic nonlinear strict-feedback systems with unknown input saturation*, Inform. Sci. **269** (2014), 300–315.
16. C. C. Hua, L. L. Zhang, and X. P. Guan, *Decentralized output feedback adaptive NN tracking control for time-delay stochastic nonlinear systems with prescribed performance*, IEEE Trans. Neural Netw. Learn. Syst. **26** (2015), no. 11, 2749–2759.
17. Z. Namadchian and M. Rouhani, *Observer-based adaptive neural control for switched stochastic pure-feedback systems with input saturation*, Neurocomputing **375** (2020), 80–90.
18. J. Wu, W. S. Chen, D. Zhao, and J. Li, *Globally stable direct adaptive backstepping NN control for uncertain nonlinear strict-feedback systems*, Neurocomputing **122** (2013), 134–147.
19. W. S. Chen, S. Z. S. Ge, J. Wu, and M. G. Gong, *Globally stable adaptive backstepping neural network control for uncertain strict-feedback systems with tracking accuracy known a priori*, IEEE Trans. Neural Netw. Learn. Syst. **26** (2015), no. 9, 1842–1854.
20. H. S. Yan, Y. Q. Han, and Q. M. Sun, *Optimal output-feedback tracking of SISO stochastic nonlinear systems using multi-dimensional Taylor network*, Trans. Inst. Measure. Control **40** (2018), no. 10, 3049–3058.
21. Y. Q. Han and H. S. Yan, *Observer-based multi-dimensional Taylor network decentralised adaptive tracking control of large-scale stochastic nonlinear systems*, Int. J. Control **93** (2020), no. 7, 1605–1618.
22. H. S. Yan, Q. M. Sun, and B. Zhou, *Multidimensional Taylor network optimal control of SISO nonlinear systems for tracking by output feedback*, Optimal Control Appl. Methods **39** (2018), no. 2, 919–932.
23. H. S. Yan and A. M. Kang, *Asymptotic tracking and dynamic regulation of SISO non-linear system based on discrete multi-dimensional Taylor network*, IET Control Theory Appl. **11** (2017), no. 10, 1619–1626.
24. Y. Q. Han, S. L. Zhu, and S. G. Yang, *Adaptive multi-dimensional Taylor network tracking control for a class of stochastic nonlinear systems with unknown input dead-zone*, IEEE Access **6** (2018), 34543–34554.
25. Y. Q. Han and H. S. Yan, *Adaptive multi-dimensional Taylor network tracking control for SISO uncertain stochastic nonlinear systems*, IET Control Theory Appl. **12** (2018), no. 8, 1107–1115.
26. Y. Q. Han, *Output-feedback adaptive tracking control of stochastic nonlinear systems using multi-dimensional Taylor network*, Int. J. Adapt. Control Signal Process. **32** (2018), no. 3, 494–510.
27. H. S. Yan and Y. Q. Han, *Decentralized adaptive multi-dimensional Taylor network tracking control for a class of large-scale stochastic nonlinear systems*, Int. J. Adapt. Control Signal Process. **33** (2019), no. 4, 664–683.
28. Y. Q. Han, *Design of decentralized adaptive control approach for large-scale nonlinear systems subjected to input delays under prescribed performance*, Nonlinear Dyn. **106** (2021), no. 1, 565–582.
29. L. Chu, T. Gao, M. X. Wang, Y. Q. Han, and S. L. Zhu, *Adaptive decentralized control for large-scale nonlinear systems with finite-time output constraints by multi-dimensional Taylor network*, Asian J. Control (2021). <https://doi.org/10.1002/asjc.2571>
30. Y. Q. Han, *Adaptive control of a class of stochastic nonlinear systems with full state constraints and input saturation using multi-dimensional Taylor network*, Asian J. Control (2021). <https://doi.org/10.1002/asjc.2551>
31. Z. T. Chen, Z. J. Li, and C. L. P. Chen, *Adaptive neural control of uncertain MIMO nonlinear systems with state and input constraints*, IEEE Trans. Neural Netw. Learn. Syst. **28** (2017), no. 6, 1318–1330.
32. H. Ma, H. J. Liang, and G. W. Dong, *Adaptive fuzzy event-triggered control for stochastic nonlinear systems with full state constraints and actuator faults*, IEEE Trans. Fuzzy Syst. **27** (2019), no. 11, 2242–2254.
33. Y. J. Liu, S. M. Lu, S. C. Tong, X. K. Chen, C. L. P. Chen, and D. J. Li, *Adaptive control-based barrier Lyapunov functions for a class of stochastic nonlinear systems with full state constraints*, Automatica **87** (2018), 83–93.
34. K. P. Tee, S. S. Ge, and E. H. Tay, *Barrier Lyapunov functions for the control of output-constrained nonlinear systems*, Automatica **45** (2009), no. 4, 918–927.

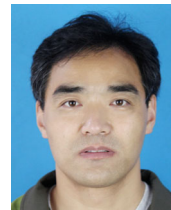
35. Z. L. Tang, S. Z. Ge, K. P. Tee, and W. He, *Adaptive neural control for an uncertain robotic manipulator with joint space constraints*, *Int. J. Control* **89** (2016), no. 7, 1428–1446.
36. X. Jin, *Adaptive fault tolerant control for a class of input and state constrained MIMO nonlinear systems*, *Int. J. Robust Nonlinear Control* **26** (2016), no. 2, 286–302.
37. Q. D. Zhu, Y. C. Liu, and G. X. Wen, *Adaptive neural network control for time-varying state constrained nonlinear stochastic systems with input saturation*, *Inform. Sci.* **527** (2020), 191–209.
38. H. Y. Li, S. Y. Zhao, W. He, and R. Q. Lu, *Adaptive finite-time tracking control of full state constrained nonlinear systems with dead-zone*, *Automatica* **100** (2019), 99–107.
39. Y. Liu, X. P. Liu, and Y. W. Jing, *Adaptive neural networks finite-time tracking control for non-strict feedback systems via prescribed performance*, *Inform. Sci.* **468** (2018), 29–46.
40. W. C. Zou, P. Shi, Z. R. Xiang, and Y. Shi, *Finite-time consensus of second-order switched nonlinear multi-agent systems*, *IEEE Trans. Neural Netw. Learn. Syst.* **31** (2020), no. 5, 1757–1762.
41. F. Wang, B. Chen, Y. M. Sun, and C. Lin, *Finite time control of switched stochastic nonlinear systems*, *Fuzzy Set. Syst.* **365** (2019), 140–152.
42. Z. B. Song and J. Y. Zhai, *Finite-time adaptive control for a class of switched stochastic uncertain nonlinear systems*, *J. Franklin Inst.* **354** (2017), no. 12, 4637–4655.
43. G. H. Zhao, J. C. Li, and S. J. Liu, *Finite-time stabilization of weak solutions for a class of non-local Lipschitzian stochastic nonlinear systems with inverse dynamics*, *Automatica* **98** (2018), 285–295.
44. J. Zhang, J. W. Xia, W. Sun, G. M. Zhuang, and Z. Wang, *Finite-time tracking control for stochastic nonlinear systems with full state constraints*, *Appl. Math Comput.* **338** (2018), 207–220.
45. S. Li, C. K. Ahn, and Z. R. Xiang, *Command-filter-based adaptive fuzzy finite-time control for switched nonlinear systems using state-dependent switching method*, *IEEE Trans. Fuzzy Syst.* **29** (2021), no. 4, 833–845.
46. J. L. Yin, S. Y. Khoo, Z. H. Man, and X. H. Yu, *Finite-time stability and instability of stochastic nonlinear systems*, *Automatica* **47** (2011), no. 12, 2671–2677.
47. S. Sui, C. L. P. Chen, and S. C. Tong, *Fuzzy adaptive finite-time control design for nontriangular stochastic nonlinear systems*, *IEEE Trans. Fuzzy Syst.* **27** (2019), no. 1, 172–184.
48. H. Wang and Q. X. Zhu, *Finite-time stabilization of high-order stochastic nonlinear systems in strict-feedback form*, *Automatica* **54** (2015), 284–291.
49. H. Q. Wang, S. W. Liu, and W. Bai, *Adaptive neural tracking control for non-affine nonlinear systems with finite-time output constraint*, *Neurocomputing* **397** (2020), 60–69.
50. F. Wang, Z. Liu, and G. Y. Lai, *Fuzzy adaptive control of nonlinear uncertain plants with unknown dead zone output*, *Fuzzy Set. Syst.* **263** (2015), 27–48.

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