Original Article

Institution of MECHANICAL ENGINEERS

Journal of SYSTEMS AND CONTROL ENGINEERING

Proc IMechE Part I:
J Systems and Control Engineering
2022, Vol. 236(3) 579–591

DOI: 10.1177/09596518211040285
journals.sagepub.com/home/pii
SAGE

Adaptive multi-dimensional Taylor network control for nonlinear stochastic systems with time-delay

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Abstract

In this article, the problem of adaptive multi-dimensional Taylor network control for strict-feedback nonlinear stochastic systems with time-delay is investigated. To overcome the control degradation resulting from the delay terms, the appropriate integral-type Lyapunov–Krasovskii functions are introduced. A novel adaptive multi-dimensional Taylor network control scheme is provided via backstepping technique. The proposed adaptive multi-dimensional Taylor network controller can ensure that all signals in the closed-loop system are bounded in probability and the tracking error eventually converges to a small neighborhood of the origin. Three simulation examples are given to demonstrate the effectiveness of the proposed control scheme.

Keywords

Adaptive control, Lyapunov–Krasovskii functions, nonlinear stochastic systems, multi-dimensional Taylor network

Date received: 31 December 2020; accepted: 29 July 2021

Introduction

It is well-known that stochastic disturbance often exists in many practical processes, such as economics, microelectronics, and biology.[1,2] Therefore, the study on nonlinear stochastic systems has attracted more and more attention.[3–6] Especially, the controller design of stochastic systems has become a research hotspot due to its important theoretical and practical values.[6] Through the efforts of researchers, many control methods of nonlinear systems have been naturally applied to nonlinear stochastic systems, such as fault tolerant control,[7] adaptive control,[8] backstepping technique,[9] and sliding mode control.[10] Especially, backstepping technique has become a common control method for nonlinear stochastic systems.[11,12] For example, this technique was successfully applied to the stochastic cases in Pan and Basar,[9] where a backstepping-based control scheme was proposed under a risk-sensitive cost criterion. Deng and Krstić[13] solve the stabilization problem of nonlinear stochastic systems with the quartic Lyapunov function via backstepping technique. Afterwards, many hybrid methods based on backstepping technique were extensively applied to the control problem of nonlinear stochastic systems.[14,15] Although the control of nonlinear stochastic systems has been obtained great progress, it is difficult to solve the problems such as unknown functions and uncertainties only by the aforementioned control methods.

In consideration of the influence of uncertainty and nonlinearity, the approximation-based control methods, such as fuzzy logic systems (FLSs) control methods and neural networks (NNs) control methods, have been widely used.[16–18] Because of the complexity of the control problems, many hybrid control methods have been proposed for stochastic systems, such as NN-based adaptive control[19–21] and FLS-based adaptive control.[22–24] Despite that many meaningful achievements have been obtained for nonlinear stochastic systems, these approximation-based control methods have some innate shortcomings.[25,26] For the NNs, the long training time leads to the poor real-time performance, and the selection of the NN structure is considered as a fuzzy and experiential process. For the FLSs, no

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applicable general techniques are available for stability analysis. In recent years, a novel multi-dimensional Taylor network (MTN)-based control method[27] has drawn wide attention.

The MTN is a special three-layer topological structure network and has excellent approximation ability to unknown functions.[28] Using the MTN-based control method, some excellent results have been obtained.[29–34] In Yan and Kang,[27] the asymptotic tracking and dynamic regulation were considered for single input single output (SISO) nonlinear systems with the help of the discrete MTN. In Yan et al.,[29] an optimal control problem was discussed for SISO nonlinear systems, and the proposed optimal MTN controller had excellent dynamic performance. After that, this control method had been naturally extended to nonlinear stochastic systems. For example, the MTN-based control approach was successfully applied to SISO nonlinear stochastic systems in Yan et al.,[31] where an optimal MTN control scheme was developed for the systems. An adaptive MTN controller was designed in Han et al.[30] for a class of nonlinear stochastic systems, and the real-time performance of the controller had been improved. Although many important MTN-based results have been obtained in the study of nonlinear stochastic systems, the research results of time-delay systems were still insufficient.

However, time-delay exist widely in practical systems, such as economical systems,[35] telerobotic systems,[36] and hydraulic systems.[37] And time-delay often reduces the control effect and even destroys the stability of the systems.[38] Therefore, more attentions have been paid to the control problem of nonlinear stochastic systems with time-delay, and some valuable results have been reported.[39–43] An adaptive NN-based controller was developed in Li et al.[40] for nonlinear stochastic systems with time-delay, and the exponential-type Lyapunov–Krasovskii functions were used to compensate the delay terms in the systems. However, the real-time performances of adaptive NN-based controllers were unsatisfactory due to the long parameter training time.[25] Si[41] proposed an adaptive observer-based control scheme for the uncertain interconnected nonlinear stochastic system with input time-delay. However, the above results neglected the time-delay of system state variables. Therefore, it is worth noting that the approximate-based MTN control method is rarely used in the control problem of nonlinear stochastic systems with time-delay, which motivates our research.

Based on the above observations, this article focuses on the adaptive control problem of nonlinear stochastic systems with time-delay. First, the time-delay terms are handled by the novel integral-type Lyapunov–Krasovskii functions. Second, the MTNs are used to estimate the input signal and the unknown functions. Then, based on the adaptive backstepping control method, a novel adaptive MTN control scheme is proposed. Finally, the simulation results show that the proposed control scheme can guarantee that all the signals in the closed-loop system remain bounded in probability and the tracking error eventually converges to a small neighborhood of the origin. Compared with the existing works, the main contributions of this article are as follows:

1. The MTN method is successfully used to deal with the control problem of nonlinear stochastic systems with time-delay. In the current MTN-based research results,[30,31,34,44] the control scheme based on the MTN for time-delay nonlinear systems[44] cannot be directly utilized to dispose of the problem of nonlinear stochastic systems with time-delay. In this article, we extend the MTN control method to nonlinear stochastic systems with time-delay.
2. The novel integral-type Lyapunov–Krasovskii functions are applied to handle the time-delay terms. Compared with the nonlinear stochastic system with time-delay,[41] time-delays that exist in the state variables are investigated in this article, which makes the developed results more applicable.
3. The unknown functions and the input signal are approximated by the MTNs. The real-time performance of the proposed control scheme has improved greatly owing to that the MTN has a pretty straightforward structure. Compared with the NN-based adaptive control schemes on nonlinear stochastic systems,[18,45,46] we may conclude that the proposed adaptive MTN control scheme has a good real-time performance.

The rest of this article is organized as follows. System descriptions and preliminaries are described in section "System descriptions and preliminaries." The MTN-based adaptive tracking control scheme and stability analysis are developed in section "Main results." Three simulation examples to validate the proposed theoretic results are provided in section "Simulation examples." This article is concluded in section "Conclusion."

System descriptions and preliminaries

Correlation theory

In order to introduce the definition and theorems of the stochastic system, consider the general nonlinear stochastic system as follows

$$d\boldsymbol{\chi}(t) = f(\boldsymbol{\chi}(t))dt + g(\boldsymbol{\chi}(t))d\boldsymbol{\omega} \quad (1)$$

where $\boldsymbol{\chi} \in R^n$ is the system state, $f : R^n \to R^n$ and $g : R^n \to R^{n\times r}$ denote the unknown continuous nonlinear functions and satisfy $f(\mathbf{0}) = \mathbf{0}$, $g(\mathbf{0}) = \mathbf{0}$, and $\boldsymbol{\omega}$ is an r-dimensional independent standard Wiener process.

Definition 1. Consider the system equation (1), for $\forall V(\boldsymbol{\chi}) \in C^2$, the differential operator $\mathcal{L}$ is defined by[47]

$$\mathcal{L}V(\boldsymbol{\chi}) = \frac{\partial V}{\partial \boldsymbol{\chi}} f + \frac{1}{2}\mathrm{Tr}\left\{g^{\mathrm{T}} \frac{\partial^2 V}{\partial \boldsymbol{\chi}^2} g\right\} \tag{2}$$

where C^2 denotes the set of all functions with continuous second partial derivatives.

Lemma 1. Associated with system equation (1), if there exists a positive definite, radially unbounded, twice continuously differentiable Lyapunov function $V(\boldsymbol{\chi}) : R^n \to R$, such that[48]

$$\mathcal{L}V(\boldsymbol{\chi}) \leqslant -aV(\boldsymbol{\chi}) + b$$

then the system is bounded in probability and has a unique solution almost surely, where $a > 0$, $b > 0$ are the design constants.

System descriptions

In this article, consider the nonlinear stochastic system with time-delay as follows

$$\begin{cases} d\chi_i(t) = (\chi_{i+1}(t) + f_i(\bar{\boldsymbol{\chi}}_i(t)) + h_i(\bar{\boldsymbol{\chi}}_i(t-\gamma_i)))dt + g_i(y)^{\mathrm{T}} d\omega \\ d\chi_n(t) = (u + f_n(\bar{\boldsymbol{\chi}}_n(t)) + h_n(\bar{\boldsymbol{\chi}}_n(t-\gamma_n)))dt + g_n(y)^{\mathrm{T}} d\omega \\ y(t) = \chi_1(t) \end{cases} \tag{3}$$

where $\bar{\boldsymbol{\chi}}_n = [\chi_1, \chi_2, \ldots, \chi_n]^{\mathrm{T}} \in \mathrm{R}^n$ denotes the state vectors, and $\bar{\boldsymbol{\chi}}_i = [\chi_1, \chi_2, \ldots, \chi_i]^{\mathrm{T}} \in \mathrm{R}^i$. $f_i(\cdot) : R^n \to R(1 \leqslant i \leqslant n)$ denote the unknown continuous functions and satisfy $f_i(\mathbf{0}) = 0$, $h_i(\cdot)(1 \leqslant i \leqslant n)$ denote the unknown continuous time-delay functions, $\gamma_i(1 \leqslant i \leqslant n)$ are the unknown constant delays, and $g_i(\cdot) : R \to R^r(1 \leqslant i \leqslant n)$ are the unknown continuous functions and satisfy $g_i(0) = 0$. $u \in R$ is the system input, and $y(t) \in R$ is the system output. ω is an r-dimensional independent standard Wiener process.

The purpose of this article is to realize the control of the system equation (3), such that the system output y tracks the given reference signal y_d, and all the signals in the closed-loop system remain bounded in probability.

Remark 1. The time-delay systems were discussed widely.[39–42] However, the problems such as complex structure and poor real-time performance still exist in the current control schemes. Therefore, developing an MTN control algorithm with simple structure for nonlinear stochastic systems with time-delay has great theoretical and practical significances.

Remark 2. $g_i(y)(1 \leqslant i \leqslant n)$ are the continuous functions and satisfy $g_i(0) = 0$, there exist unknown smooth functions $\bar{g}_i(y)$, such that

$$g_i(y) = y\bar{g}_i(y) \tag{4}$$

To introduce the design of the controller, define the coordinate transformation as follows

$$z_i = \chi_i - \alpha_{i-1} \quad (1 \leqslant i \leqslant n) \tag{5}$$

where $\alpha_0 = y_d(t)$, and $\alpha_{i-1}(2 \leqslant i \leqslant n)$ are the virtual control signals and will be designed later.

Assumption 1. The reference trajectory $y_d(t)$ and its up to nth order derivatives are bounded and continuous.[6]

Assumption 2. For $h_i(\bar{\boldsymbol{\chi}}_i)$, $1 \leqslant i \leqslant n$, there exist positive unknown continuous functions $\varpi_{ik}(\cdot)$, $k = 1, \ldots, i$, such that[49]

$$|h_i(\bar{\boldsymbol{\chi}}_i)| \leqslant \sum_{k=1}^{i} |z_i| \varpi_{ik}(\bar{z}_k + \bar{\alpha}_{k-1}) \tag{6}$$

Remark 3. Assumptions 1 and 2 are both normal assumptions for nonlinear stochastic systems which can be found in previous works.[6,21,26,49] These assumptions do not pose a strong restriction upon nonlinear stochastic systems.

MTN

The MTN is composed of a large number of polynomials, which includes the input, hidden, and output layers,[25] and the MTN approximation theorem has been proved in Yan and Kang.[27] In this article, the MTNs are used to estimate the unknown functions. The topology structure is shown in Figure 1.

Lemma 2. On a compact set Ω_Z, consider continuous nonlinear function $f(\mathbf{Z})$, for $\forall \varepsilon > 0$, there exists an MTN, such that[25]

$$f(\mathbf{Z}) = \boldsymbol{\varphi}^{*\mathrm{T}} S_{m_n}(\mathbf{Z}) + \delta(\mathbf{Z}) \tag{7}$$

where $\boldsymbol{Z} = [z_1, z_2, \ldots, z_n]$ and $\delta(\mathbf{Z})$ is the approximation error with $|\delta(\mathbf{Z})| \leqslant \varepsilon$. $\boldsymbol{\varphi}^* = [\varphi_1, \varphi_2, \ldots, \varphi_n]^{\mathrm{T}}$ is the optimal weight vector and defined by

$$\boldsymbol{\varphi}^* := \arg\min_{\boldsymbol{\varphi} \in R^N} \left\{ \sup_{Z \in \Omega_Z} \left| f(\mathbf{Z}) - \boldsymbol{\varphi}^{\mathrm{T}} S_{m_n}(\mathbf{Z}) \right| \right\} \tag{8}$$

Remark 4. As shown in Figure 1 and equation (7), the advantages of MTN can be summarized as follows: (1) the structure of MTN is simple, and the hidden layer is composed of addition and multiplication of polynomials, which is easy to compute and be applied in

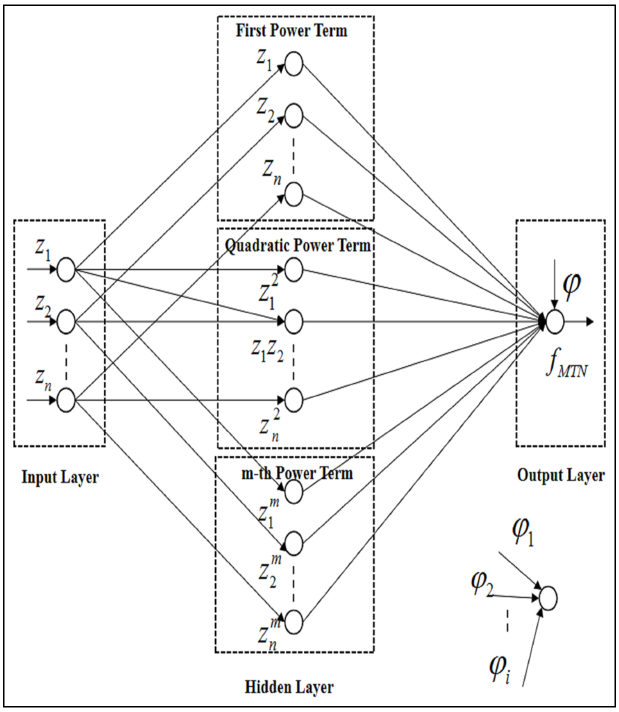

Figure 1. The structure of MTN.

engineering; (2) the structure of MTN can be confirmed by the number of inputs and the highest power of polynomials of the hidden layer; (3) the input–output characteristic of the MTN is complex and dynamic, which makes the MTN be suitable for actual dynastic systems.

Main results

This section contributes to develop an adaptive MTN control scheme.

Substituting equation (5) into equation (3), we obtain

$$\begin{cases} dz_i = (\chi_{i+1}(t) + f_i + h_i(\bar{\boldsymbol{\chi}}_i(t-\gamma_i)) - \Delta\alpha_{i-1})dt + \left(g_i(y) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\chi_j}g_j(y)\right)^{\mathrm{T}} d\omega \\ \qquad i = 1, \ldots, n-1 \\ dz_n = (u + f_n + h_n(\bar{\boldsymbol{\chi}}_n(t-\gamma_n)) - \Delta\alpha_{n-1})dt + \left(g_n(y) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\chi_j}g_j(y)\right)^{\mathrm{T}} d\omega \end{cases} \tag{9}$$

where

$$\Delta\alpha_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\chi_j}\left(\chi_{j+1}(t) + f_j + h_j(\bar{\boldsymbol{\chi}}_j(t-\gamma_j))\right) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\varphi}}_j}\dot{\hat{\boldsymbol{\varphi}}}_j + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial y_p\partial y_q}g_p^{\mathrm{T}}(y)g_q^{\mathrm{T}}(y)$$

Next, an adaptive MTN controller design procedure will be developed via backstepping technique.

Backstepping design based on MTN

Step 1: consider the stochastic Lyapunov function candidate as

$$V_1 = \frac{1}{2}\tilde{\boldsymbol{\varphi}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\tilde{\boldsymbol{\varphi}}_1 + \frac{1}{4}z_1^4 + \frac{n}{4}\int_{t-\gamma_1}^{t} z_1^4(\tau)\varpi_{11}^4(\bar{z}_1(\gamma) + \bar{\alpha}_0(\gamma))d\gamma \tag{10}$$

where $\tilde{\boldsymbol{\varphi}}_1 = \boldsymbol{\varphi}_1 - \hat{\boldsymbol{\varphi}}_1$ is the parameter error, $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_1^{\mathrm{T}} > 0$ is the symmetric positive definite matrix, and n is the order of the system.

According to equations (2), (4), (9), and (10), we have

$$\begin{aligned} \mathcal{L}V_1 = {} & z_1^3\left(\chi_2 + f_1 + h_1\left(\bar{\boldsymbol{\chi}}_{\gamma_1}\right) - \dot{y}_d\right) \\ & + \frac{3}{2}z_1^4\bar{g}_1(z_1)^{\mathrm{T}}\bar{g}_1(z_1) - \tilde{\boldsymbol{\varphi}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\varphi}}}_1 \\ & + \frac{n}{4}z_1^4(t)\varpi_{11}^4(\bar{z}_1(t) + \bar{\alpha}_0(t)) \\ & - \frac{n}{4}z_1^4(t-\gamma_1)\varpi_{11}^4(\bar{z}_1(t-\gamma_1) + \bar{\alpha}_0(t-\gamma_1)) \end{aligned} \tag{11}$$

By Young's inequality, we obtain

$$z_1^3h_1\left(\bar{\boldsymbol{\chi}}_{\gamma_1}\right) \leqslant \frac{1}{4}z_1^4(t-\gamma_1)\varpi_{11}^4(\bar{z}_1(t-\gamma_1) + \bar{\alpha}_0(t-\gamma_1)) + \frac{3}{4}z_1^4 \tag{12}$$

Substituting equation (12) into equation (11), the following inequality holds

$$\begin{aligned} \mathcal{L}V_1 \leqslant {} & z_1^3(\chi_2 + \bar{f}_1) \\ & - \frac{n-1}{4}z_1^4(t-\gamma_1)\varpi_{11}^4(\bar{z}_1(t-\gamma_1) + \bar{\alpha}_0(t-\gamma_1)) \\ & - \tilde{\boldsymbol{\varphi}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\varphi}}}_1 - \frac{3}{2}z_1^4 \end{aligned} \tag{13}$$

where $\bar{f}_1 = f_1 - \dot{y}_d + (n/4\ \varpi_{11}^4(z_1) + 3/2\ g_1(z_1)^{\mathrm{T}} g_1(z_1) + 9/4) z_1$.

$\bar{f}_1$ is obviously an unknown function, and we can use an MTN to estimate $\bar{f}_1$. By Lemma 2, for $\forall \varepsilon_1 > 0$, an MTN $\boldsymbol{\varphi}_1^{\mathrm{T}} S_{m_1}(\mathbf{z}_1)$ is applied to estimate $\bar{f}_1$, such that

$$\bar{f}_1 = \boldsymbol{\varphi}_1^{\mathrm{T}} S_{m_1}(\mathbf{z}_1) + \delta_1(\mathbf{z}_1), |\delta_1(\mathbf{z}_1)| \leqslant \varepsilon_1 \tag{14}$$

where $\mathbf{z}_1 = [z_1]^{\mathrm{T}}$ is the input of MTN, and $\delta_1(\mathbf{z}_1)$ is the approximation error.

Substituting equations (5) and (14) into equation (13), one has

$$\begin{aligned}\mathcal{L}V_1 \leqslant & z_1^3\left(z_2 + \alpha_1 + \boldsymbol{\varphi}_1^{\mathrm{T}} S_{m_1}(\mathbf{z}_1) + \delta_1\right) - \frac{3}{2} z_1^4 \\ & - \frac{n-1}{4} z_1^4(t-\gamma_1)\varpi_{11}^4(\bar{z}_1(t-\gamma_1) \\ & + \bar{\alpha}_0(t-\gamma_1)) - \tilde{\boldsymbol{\varphi}}_1^{\mathrm{T}} \boldsymbol{\Gamma}_1^{-1} \dot{\hat{\boldsymbol{\varphi}}}_1\end{aligned} \tag{15}$$

By Young's inequality, we obtain

$$z_1^3 z_2 \leqslant \frac{1}{4} z_2^4 + \frac{3}{4} z_1^4 \tag{16}$$

$$z_1^3 \delta_1 \leqslant \frac{3}{4} z_1^4 + \frac{1}{4} \varepsilon_1^4 \tag{17}$$

Then, substituting equations (16) and (17) into equation (15), we have

$$\begin{aligned}\mathcal{L}V_1 \leqslant & z_1^3\left(\alpha_1 + \boldsymbol{\varphi}_1^{\mathrm{T}} S_{m_1}(\mathbf{z}_1)\right) + \frac{1}{4} z_2^4 + \frac{3}{4} z_1^4 \\ & + \frac{3}{4} z_1^4 + \frac{1}{4} \varepsilon_1^4 - \frac{n-1}{4} z_1^4(t-\gamma_1)\varpi_{11}^4(\bar{z}_1(t-\gamma_1) \\ & + \bar{\alpha}_0(t-\gamma_1)) - \tilde{\boldsymbol{\varphi}}_1^{\mathrm{T}} \boldsymbol{\Gamma}_1^{-1} \dot{\hat{\boldsymbol{\varphi}}}_1 - \frac{3}{2} z_1^4\end{aligned} \tag{18}$$

Based on equation (18), design the intermediate control signal α_1 as

$$\alpha_1 = -k_1 z_1 - \hat{\boldsymbol{\varphi}}_1^{\mathrm{T}} S_{m_1}(\mathbf{z}_1) \tag{19}$$

where $k_1 > 0$ is a design constant.

Substituting equation (19) into equation (18), one has

$$\begin{aligned}\mathcal{L}V_1 \leqslant & -k_1 z_1^4 + \frac{1}{4} z_2^4 + \tilde{\boldsymbol{\varphi}}_1^{\mathrm{T}}\left(z_1^3 S_{m_1}(\mathbf{z}_1) - \boldsymbol{\Gamma}_1^{-1} \dot{\hat{\boldsymbol{\varphi}}}_1\right) \\ & + \frac{1}{4} \varepsilon_1^4 - \frac{n-1}{4} z_1^4(t-\gamma_1) \\ & \varpi_{11}^4(\bar{z}_1(t-\gamma_1) + \bar{\alpha}_0(t-\gamma_1))\end{aligned} \tag{20}$$

Step i $(2 \leqslant i \leqslant n-1)$: Consider the stochastic Lyapunov function candidate as

$$\begin{aligned}V_i = & V_{i-1} + \frac{1}{2} \tilde{\boldsymbol{\varphi}}_i^{\mathrm{T}} \boldsymbol{\Gamma}_i^{-1} \tilde{\boldsymbol{\varphi}}_i + \frac{1}{4} z_i^4 + \frac{n-i+1}{4} \\ & \int_{t-\gamma_i}^{t} \sum_{k=1}^{i} z_i^4(\gamma)\varpi_{ik}^4(\bar{z}_k(\gamma) + \bar{\alpha}_{k-1}(\gamma)) d\gamma\end{aligned} \tag{21}$$

where $\tilde{\boldsymbol{\varphi}}_i = \boldsymbol{\varphi}_i - \hat{\boldsymbol{\varphi}}_i$ is the parameter error, $\boldsymbol{\Gamma}_i = \boldsymbol{\Gamma}_i^{\mathrm{T}} > 0$ is the symmetric positive definite matrix.

Similar to Step 1, we obtain

$$\begin{aligned}\mathcal{L}V_i \leqslant & \mathcal{L}V_{i-1} + z_i^3\left(\chi_{i+1} + f_i + h_i\left(\bar{\boldsymbol{\chi}}_{\gamma_i}\right) - \Delta\alpha_{i-1}\right) \\ & + \frac{3}{2} z_i^4\left(\bar{g}_i(y) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \chi_j} \bar{g}_j(y)\right)^{\mathrm{T}} \\ & \left(\bar{g}_i(y) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \chi_j} \bar{g}_j(y)\right) - \tilde{\boldsymbol{\varphi}}_i^{\mathrm{T}} \boldsymbol{\Gamma}_i^{-1} \dot{\hat{\boldsymbol{\varphi}}}_i \\ & + \frac{n-i+1}{4} \sum_{k=1}^{i} z_i^4(t)\varpi_{ik}^4(\bar{z}_k(t) + \bar{\alpha}_{k-1}(t)) \\ & - \frac{n-i+1}{4} \sum_{k=1}^{i} z_i^4(t-\gamma_i)\varpi_{ik}^4(\bar{z}_k(t-\gamma_i) + \bar{\alpha}_{k-1}(t-\gamma_i))\end{aligned} \tag{22}$$

By Young's inequality, we obtain

$$\begin{aligned}z_i^3 h_i\left(\bar{\boldsymbol{\chi}}_{\gamma_i}\right) \leqslant & \frac{1}{4} \sum_{k=1}^{i} z_i^4(t-\gamma_i) \\ & \varpi_{ik}^4(\bar{z}_k(t-\gamma_i) + \bar{\alpha}_{k-1}(t-\gamma_i)) + \frac{3}{4} z_i^4\end{aligned} \tag{23}$$

$$\begin{aligned}- z_i^3 \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \chi_j} h_j\left(\bar{\boldsymbol{\chi}}_{\gamma_j}\right) \leqslant & \sum_{j=1}^{i-1} \frac{3}{4} z_i^4 \left(\frac{\partial \alpha_{i-1}}{\partial \chi_j}\right)^{\frac{4}{3}} + \frac{1}{4} \sum_{j=1}^{i-1} \sum_{k=1}^{j} \\ & \left(z_j^4(t-\gamma_j)\left(\varpi_{jk}^4\left(\bar{z}_k(t-\gamma_j) + \bar{\alpha}_{k-1}(t-\gamma_j)\right)\right)\right)\end{aligned} \tag{24}$$

Substituting equations (20), (23), and (24) into equation (22), the following inequality holds

$$\begin{aligned}\mathcal{L}V_i \leqslant & -k_1 z_1^4 - \sum_{j=2}^{i-1}\left(k_j - \frac{1}{4}\right) z_j^4 + \frac{1}{4} z_i^4 - \tilde{\boldsymbol{\varphi}}_i^{\mathrm{T}} \boldsymbol{\Gamma}_i^{-1} \dot{\hat{\boldsymbol{\varphi}}}_i - \frac{3}{2} z_i^4 \\ & + \sum_{j=1}^{i-1} \tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3 S_{m_j}(\mathbf{z}_j) - \boldsymbol{\Gamma}_j^{-1} \dot{\hat{\boldsymbol{\varphi}}}_j\right) + \frac{1}{4} \sum_{j=1}^{i-1} \varepsilon_j^4 + z_i^3(\chi_{i+1} + \bar{f}_i) \\ & - \frac{n-i}{4} \sum_{j=1}^{i-1} \sum_{k=1}^{j} z_j^4(t-\gamma_j)\varpi_{jk}^4(\bar{z}_k(t-\gamma_j) + \bar{\alpha}_{k-1}(t-\gamma_j)) \\ & - \frac{n-i}{4} \sum_{k=1}^{i} z_i^4(t-\gamma_i)\varpi_{ik}^4(\bar{z}_k(t-\gamma_i) + \bar{\alpha}_{k-1}(t-\gamma_i))\end{aligned} \tag{25}$$

where

$$\bar{f}_i = f_i - \left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\chi_j}\left(\chi_{j+1}+f_j\right) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\varphi}}_j}\dot{\hat{\boldsymbol{\varphi}}}_j + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial y_p\partial y_q}g_p^{\mathrm{T}}(y)g_q^{\mathrm{T}}(y)\right)$$
$$+\left(\frac{n-i+1}{4}\sum_{k=1}^{i}\varpi_{ik}^4(\bar{z}_k(\gamma)+\bar{\alpha}_{k-1}(\gamma)) + \frac{3}{2}\left(g_i(y)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\chi_j}g_j(y)\right)^{\mathrm{T}}\left(g_i(y)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\chi_j}g_j(y)\right)+\frac{9}{4}\right)z_1$$

$\bar{f}_i$ is also an unknown function, and we can use an MTN to estimate $\bar{f}_i$. In other words, for $\forall\varepsilon_i>0$, an MTN $\boldsymbol{\varphi}_i^{\mathrm{T}}S_{m_i}(\mathbf{z}_i)$ is applied to estimate $\bar{f}_i$, such that

$$\bar{f}_i = \boldsymbol{\varphi}_i^{\mathrm{T}}S_{m_i}(\mathbf{z}_i) + \delta_i(\mathbf{z}_i), |\delta_i(\mathbf{z}_i)| \le \varepsilon_i \tag{26}$$

where $\mathbf{z}_i = [z_1, z_2, \ldots, z_i]^{\mathrm{T}}$ is the input of MTN, and $\delta_i(\mathbf{z}_i)$ is the approximation error.

Substituting equations (5) and (26) into equation (25), one has

$$\begin{aligned}\mathcal{L}V_i \le& -k_1z_1^4 - \sum_{j=2}^{i-1}\left(k_j-\frac{1}{4}\right)z_j^4\\ &+\sum_{j=1}^{i-1}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3S_{m_j}(\mathbf{z}_j)-\boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right)+\frac{1}{4}z_i^4\\ &+z_i^3\left(z_{i+1}+\alpha_i+\boldsymbol{\varphi}_i^{\mathrm{T}}S_{m_i}(\mathbf{z}_i)+\delta_i\right)\\ &-\frac{n-i}{4}\cdot\sum_{j=1}^{i-1}\sum_{k=1}^{j}z_j^4(t-\gamma_j)\varpi_{jk}^4(\bar{z}_k(t-\gamma_j)+\bar{\alpha}_{k-1}(t-\gamma_j))\\ &-\frac{n-i}{4}\sum_{k=1}^{i}z_i^4(t-\gamma_i)\varpi_{ik}^4(\bar{z}_k(t-\gamma_i)+\bar{\alpha}_{k-1}(t-\gamma_i))\\ &-\tilde{\boldsymbol{\varphi}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\varphi}}}_i+\frac{1}{4}\sum_{j=1}^{i-1}\varepsilon_j^4-\frac{3}{2}z_i^4\end{aligned} \tag{27}$$

By Young's inequality, we obtain

$$z_i^3z_{i+1} \le \frac{1}{4}z_{i+1}^4 + \frac{3}{4}z_i^4 \tag{28}$$

$$z_i^3\delta_i \le \frac{3}{4}z_i^4 + \frac{1}{4}\varepsilon_i^4 \tag{29}$$

Then, substituting equations (28) and (29) into equation (27), we have

$$\begin{aligned}\mathcal{L}V_i \le& -k_1z_1^4 - \sum_{j=2}^{i-1}\left(k_j-\frac{1}{4}\right)z_j^4\\ &+\sum_{j=1}^{i-1}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3S_{m_j}(\mathbf{z}_j)-\boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right)+\frac{1}{4}z_i^4+\frac{1}{4}\sum_{j=1}^{i-1}\varepsilon_j^4\\ &+z_i^3\left(\alpha_i+\boldsymbol{\varphi}_i^{\mathrm{T}}S_{m_i}(\mathbf{z}_i)\right)-\tilde{\boldsymbol{\varphi}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\varphi}}}_i+\frac{1}{4}z_{i+1}^4\\ &-\frac{n-i}{4}\sum_{k=1}^{i}z_i^4(t-\gamma_i)\varpi_{ik}^4(\bar{z}_k(t-\gamma_i)+\bar{\alpha}_{k-1}(t-\gamma_i))\\ &-\frac{n-i}{4}\sum_{j=1}^{i-1}\sum_{k=1}^{j}z_j^4(t-\gamma_j)\varpi_{jk}^4(\bar{z}_k(t-\gamma_j)\\ &+\bar{\alpha}_{k-1}(t-\gamma_j))+\frac{3}{4}z_i^4+\frac{3}{4}z_i^4+\frac{1}{4}\varepsilon_i^4-\frac{3}{2}z_i^4\end{aligned} \tag{30}$$

Based on equation (30), design the intermediate control signal α_i as

$$\alpha_i = -k_iz_i - \hat{\boldsymbol{\varphi}}_i^{\mathrm{T}}S_{m_i}(\mathbf{z}_i) \tag{31}$$

where $k_i>0$ is a design constant.

Substituting equation (31) into equation (30), one has

$$\begin{aligned}\mathcal{L}V_i \le& -k_1z_1^4 - \sum_{j=2}^{i}\left(k_j-\frac{1}{4}\right)z_j^4+\frac{1}{4}z_{i+1}^4\\ &+\frac{1}{4}\sum_{j=1}^{i}\varepsilon_j^4+\sum_{j=1}^{i}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3S_{m_j}(\mathbf{z}_j)-\boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right)\\ &-\frac{n-i}{4}\sum_{j=1}^{i}\sum_{k=1}^{j}z_j^4(t-\gamma_j)\varpi_{jk}^4\\ &(\bar{z}_k(t-\gamma_j)+\bar{\alpha}_{k-1}(t-\gamma_j))\end{aligned} \tag{32}$$

Step n: consider the stochastic Lyapunov function candidate as

$$\begin{aligned}V_n =& V_{n-1}+\frac{1}{2}\tilde{\boldsymbol{\varphi}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\tilde{\boldsymbol{\varphi}}_n+\frac{1}{4}z_n^4+\frac{1}{4}\\ &\int_{t-\gamma_n}^{t}\sum_{k=1}^{n}z_n^4(\gamma)\varpi_{nk}^4(\bar{z}_k(\gamma)+\bar{\alpha}_{k-1})d\gamma\end{aligned} \tag{33}$$

where $\tilde{\boldsymbol{\varphi}}_n = \boldsymbol{\varphi}_n - \hat{\boldsymbol{\varphi}}_n$ is the parameter error, and $\boldsymbol{\Gamma}_n = \boldsymbol{\Gamma}_n^{\mathrm{T}} > 0$ is the symmetric positive definite matrix.

According to equations (2), (4), (9), and (33), we have

$$\begin{aligned}\mathcal{L}V_n \le& \mathcal{L}V_{n-1}+z_n^3\left(u+f_n+h_n\left(\bar{\boldsymbol{\chi}}_{\gamma_n}\right)-\Delta\alpha_{n-1}\right)\\ &+\frac{3}{2}z_n^4\cdot\left(\bar{g}_n(y)-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\chi_j}\bar{g}_j(y)\right)^{\mathrm{T}}\\ &\left(\bar{g}_i(y)-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\chi_j}\bar{g}_j(y)\right)\\ &-\tilde{\boldsymbol{\varphi}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\varphi}}}_n+\frac{1}{4}\sum_{k=1}^{n}z_n^4(t)\varpi_{nk}^4(\bar{z}_k(t)+\bar{\alpha}_{k-1}(t))\\ &-\frac{1}{4}\sum_{k=1}^{n}z_n^4(t-\tau_n)\varpi_{nk}^4(\bar{z}_k(t-\gamma_n)+\bar{\alpha}_{k-1}(t-\gamma_n))\end{aligned} \tag{34}$$

By Young's inequality, we obtain

$$z_n^3 h_n\left(\bar{\boldsymbol{\chi}}_{\gamma_n}\right) \leqslant \frac{1}{4}\sum_{k=1}^{n} z_n^4(t-\gamma_n) \varpi_{nk}^4(\bar{z}_k(t-\gamma_n)+\bar{\alpha}_{k-1}(t-\gamma_n)) + \frac{3}{4}z_n^4 \tag{35}$$

$$\begin{aligned} -z_n^3\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\chi_j}h_j\left(\bar{\boldsymbol{\chi}}_{\gamma_j}\right) \leqslant{}& \frac{3}{4}z_n^4\sum_{j=1}^{n-1}\left(\frac{\partial\alpha_{n-1}}{\partial\chi_j}\right)^{\frac{4}{3}} \\ &+ \frac{1}{4}\sum_{j=1}^{n-1}\sum_{k=1}^{j}\left(z_j^4(t-\tau_j)\cdot\varpi_{jk}^4\left(\bar{z}_k\left(t-\gamma_j\right)+\bar{\alpha}_{k-1}\left(t-\gamma_j\right)\right)\right) \end{aligned} \tag{36}$$

Substituting equations (32), (35), and (36) into equation (34), the following inequality holds

$$\begin{aligned} \mathcal{L}V_n \leqslant{}& -k_1z_1^4 - \sum_{j=2}^{n-1}\left(k_j-\frac{1}{4}\right)z_j^4 \\ &+ \sum_{j=1}^{n-1}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3S_{m_j}(\mathbf{z}_j)-\boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right) + \frac{1}{4}\sum_{j=1}^{n-1}\varepsilon_j^4 \\ &+ z_n^3(u+\bar{f}_n) - \tilde{\boldsymbol{\varphi}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\varphi}}}_n \\ &- \frac{n-n}{4}\sum_{j=1}^{n-1}\sum_{k=1}^{j}z_j^4\left(t-\gamma_j\right) \\ &\varpi_{jk}^4\left(\bar{z}_k\left(t-\gamma_j\right)+\bar{\alpha}_{k-1}\left(t-\gamma_j\right)\right) \\ &-\frac{3}{4}z_n^4 - \frac{n-n}{4}\sum_{k=1}^{n}z_n^4(t-\gamma_n) \\ &\varpi_{nk}^4(\bar{z}_k(t-\gamma_n)+\bar{\alpha}_{k-1}(t-\gamma_n)) \\ ={}& -k_1z_1^4 - \sum_{j=2}^{n-1}\left(k_j-\frac{1}{4}\right)z_j^4 \\ &+ \sum_{j=1}^{n-1}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3S_{m_j}(\mathbf{z}_j)-\boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right) \\ &+ \frac{1}{4}\sum_{j=1}^{n-1}\varepsilon_j^4 + z_n^3(u+\bar{f}_n) - \tilde{\boldsymbol{\varphi}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\varphi}}}_n - \frac{3}{4}z_n^4 \end{aligned} \tag{37}$$

where

Similar to functions $\bar{f}_1$ and $\bar{f}_i$, $\bar{f}_n$ is an unknown function, and we can use an MTN to estimate $\bar{f}_n$. Namely, for $\forall\varepsilon_n > 0$, an MTN $\boldsymbol{\varphi}_n^{\mathrm{T}}S_{m_n}(\mathbf{z}_n)$ is applied to estimate $\bar{f}_n$, such that

$$\bar{f}_n = \boldsymbol{\varphi}_n^{\mathrm{T}}S_{m_n}(\mathbf{z}_n) + \delta_n(\mathbf{z}_n), |\delta_n(\mathbf{z}_n)| \leqslant \varepsilon_n \tag{38}$$

where $\mathbf{z}_n = [z_1, z_2, \ldots, z_n]^{\mathrm{T}}$ is the input of MTN, and $\delta_n(\mathbf{z}_n)$ is the approximation error.

Substituting equation (38) into equation (37), one has

$$\begin{aligned} \mathcal{L}V_n \leqslant{}& -k_1z_1^4 - \sum_{j=2}^{n-1}\left(k_j-\frac{1}{4}\right)z_j^4 - \tilde{\boldsymbol{\varphi}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\varphi}}}_n \\ &+ \sum_{j=1}^{n-1}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3S_{m_j}(\mathbf{z}_j)-\boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right) \\ &+ \frac{1}{4}\sum_{j=1}^{n-1}\varepsilon_j^4 + z_n^3\left(u+\boldsymbol{\varphi}_n^{\mathrm{T}}S_{m_n}+\delta_n\right) - \frac{3}{4}z_n^4 \end{aligned} \tag{39}$$

By Young's inequality, we obtain

$$z_n^3\delta_n \leqslant \frac{3}{4}z_n^4 + \frac{1}{4}\varepsilon_n^4 \tag{40}$$

Then, substituting equation (40) into equation (39)

$$\begin{aligned} \mathcal{L}V_n \leqslant{}& -k_1z_1^4 - \sum_{j=2}^{n-1}\left(k_j-\frac{1}{4}\right)z_j^4 - \tilde{\boldsymbol{\varphi}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\varphi}}}_n \\ &+ \sum_{j=1}^{n-1}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3S_{m_j}(\mathbf{z}_j)-\boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right) \\ &+ \frac{1}{4}\sum_{j=1}^{n-1}\varepsilon_j^4 + z_n^3\left(u+\boldsymbol{\varphi}_n^{\mathrm{T}}S_{m_n}\right) + \frac{1}{4}\varepsilon_n^4 \end{aligned} \tag{41}$$

The control law u is defined as

$$u = -k_nz_n - \hat{\boldsymbol{\varphi}}_n^{\mathrm{T}}S_{m_n}(\mathbf{z}_n) \tag{42}$$

where $k_n > 0$ is a design constant.

Substituting equation (42) into equation (41), the following inequality holds

$$\begin{aligned} \bar{f}_n = f_n - \Bigg(&\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\chi_j}\left(\chi_{j+1}+f_j\right) + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol{\varphi}}_j}\dot{\hat{\boldsymbol{\varphi}}}_j + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \frac{1}{2}\sum_{p,q=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial y_p\partial y_q}g_p^{\mathrm{T}}(y)g_q^{\mathrm{T}}(y)\Bigg) \\ &+ \left(\frac{7}{4} + \frac{1}{4}\sum_{k=1}^{n}\varpi_{nk}^4(\bar{z}_k(\gamma)+\bar{\alpha}_{k-1}(\gamma)) + \frac{3}{2}\left(g_n(y)-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}g_j(y)\right)^{\mathrm{T}}\left(g_i(y)-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}g_j(y)\right)\right)z_n \end{aligned}$$

$$\mathcal{L}V_n \leqslant -k_1 z_1^4 - \sum_{j=2}^{n}\left(k_j - \frac{1}{4}\right)z_j^4 + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 + \sum_{j=1}^{n}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3 S_{m_j}(\mathbf{z}_j) - \boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right) \tag{43}$$

Stability analysis

The main results of this article can be summarized by the following theorem.

Theorem 1. Consider the nonlinear stochastic system equation (3), the MTN-based control algorithm is constructed as follows

$$u = -k_n z_n - \hat{\boldsymbol{\varphi}}_n^{\mathrm{T}} S_{m_n}(\mathbf{z}_n) \tag{44}$$

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\varphi}}_i^{\mathrm{T}} S_{m_i}(\mathbf{z}_i), 1 \leqslant i \leqslant n-1 \tag{45}$$

$$\dot{\hat{\boldsymbol{\varphi}}}_i = \boldsymbol{\Gamma}_i S_{m_i}(\mathbf{z}_i) z_i^3 - \boldsymbol{\Gamma}_i \eta_i \hat{\boldsymbol{\varphi}}_i,\ 1 \leqslant i \leqslant n \tag{46}$$

where $k_i >$ (1/4), and $\eta_i > 0$ are the design parameters. Then, for any bounded initial condition, all the signals in the closed-loop system are bounded in probability, and the tracking error converges to a small neighborhood around the origin.

Proof. Consider the following stochastic Lyapunov function for the nonlinear stochastic system with time-delay

$$V = V_n = \frac{1}{4}\sum_{i=1}^{n} z_i^4 + \frac{1}{2}\sum_{i=1}^{n}\tilde{\boldsymbol{\varphi}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\varphi}}_i + \sum_{i=1}^{n}\left(\frac{n-i+1}{4}\cdot\int_{t-\gamma_i}^{t}\sum_{k=1}^{i} z_i^4(\gamma)\varpi_{ik}^4(\bar{z}_k(\gamma) + \bar{\alpha}_{k-1}(\gamma))d\gamma\right) \tag{47}$$

According to equations (33) and (43), we can obtain

$$\mathcal{L}V \leqslant -k_1 z_1^4 - \sum_{j=2}^{n}\left(k_j - \frac{1}{4}\right)z_j^4 + \sum_{j=1}^{n}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\left(z_j^3 S_{m_j}(\mathbf{z}_j) - \boldsymbol{\Gamma}_j^{-1}\dot{\hat{\boldsymbol{\varphi}}}_j\right) + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 \tag{48}$$

Substituting equation (46) into equation (48), the following inequality holds

$$\mathcal{L}V \leqslant -k_1 z_1^4 - \sum_{j=2}^{n}\left(k_j - \frac{1}{4}\right)z_j^4 + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 + \sum_{j=1}^{n}\eta_i\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\hat{\boldsymbol{\varphi}}_i \tag{49}$$

According to the definition of $\hat{\boldsymbol{\varphi}}_i$, we have

$$\eta_j\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\hat{\boldsymbol{\varphi}}_i = \eta_j\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}(\boldsymbol{\varphi}_j - \tilde{\boldsymbol{\varphi}}_j) \leqslant -\frac{\eta_j}{2}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\tilde{\boldsymbol{\varphi}}_j + \frac{\eta_j}{2}\|\boldsymbol{\varphi}_j\|^2 \tag{50}$$

with

$$-\frac{\eta_i}{2}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\tilde{\boldsymbol{\varphi}}_j \leqslant -\frac{\eta_i}{2\lambda_{\max}\left(\boldsymbol{\Gamma}_j^{-1}\right)}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\boldsymbol{\Gamma}_j^{-1}\tilde{\boldsymbol{\varphi}}_j \tag{51}$$

Substituting equations (50) and (51) into equation (49), the following inequality holds

$$\mathcal{L}V \leqslant -k_1 z_1^4 - \sum_{j=2}^{n}\left(k_j - \frac{1}{4}\right)z_j^4 + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 - \frac{{}^{-}\eta_j}{2}\sum_{j=1}^{n}\tilde{\boldsymbol{\varphi}}_j^{\mathrm{T}}\boldsymbol{\Gamma}_j^{-1}\tilde{\boldsymbol{\varphi}}_j + \frac{1}{2}\sum_{j=1}^{n}\eta_j\|\boldsymbol{\varphi}_j\|^2 \tag{52}$$

where ${}^{-}\eta_j = \min\{(\eta_j/(\lambda_{\max}(\boldsymbol{\Gamma}_j^{-1})))|j = 1, 2, \ldots, n\}$.

Let

$$a = \min\left\{4k_1, 4\left(k_i - \frac{1}{4}\right)|2 \leqslant i \leqslant n, {}^{-}\eta_j|1 \leqslant j \leqslant n\right\},$$
$$b = \frac{1}{2}\sum_{j=1}^{n}\eta_j\|\boldsymbol{\varphi}_j\|^2 + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4$$

We can have

$$\mathcal{L}V \leqslant -aV + b$$

Thus, according to Lemma 1, we can obtain that all signals in the close-loop system are bounded in probability, and the tracking error converges to a small neighborhood around the origin.

In addition, the design procedure of the MTN-based controller is shown in Figure 2.

Simulation examples

In this section, three simulation examples are given to demonstrate the effectiveness of the proposed scheme in this article.

Example 1. Numerical example. Consider the second-order nonlinear stochastic system with time-delay

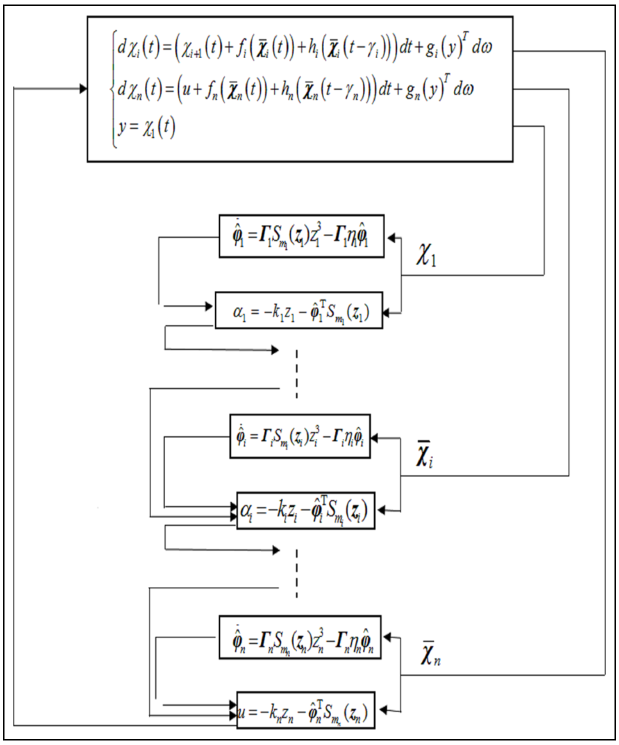

Figure 2. Block diagram of control system.

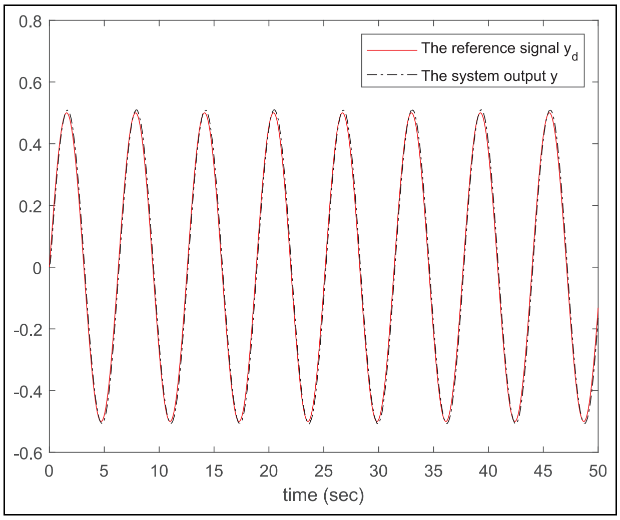

Figure 3. The trajectories of system output y and the reference signal y_d.

Figure 6 depicts that the tracking error $y - y_d$ converges to a small neighborhood around the origin.

$$\begin{cases} d\chi_1 = \left(\chi_2 + 0.1\chi_1 e^{-0.5\chi_1^2}\sin(\chi_1) + \dfrac{\chi_1^3(t-\gamma_1)}{1+\chi_1^2(t-\gamma_1)}\right)dt + 0.2\chi_1 d\omega \\ d\chi_2 = \left(u + \chi_2 \sin\left(\dfrac{0.2}{1+\chi_1^2}\right) + \dfrac{\chi_1(t-\gamma_2)\sin(\chi_2)}{1+\chi_1^2(t-\gamma_2)}\right)dt + 0.3\chi_1 d\omega \\ y = \chi_1 \end{cases} \tag{53}$$

with the initial state $[\chi_1(0), \chi_2(0)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, and $y_d = 0.5\sin t$ is the given reference signal.

By Theorem 1, the actual control law, the intermediate control signal, and the adaptive laws are chosen, respectively, as

$$\begin{aligned} u &= -k_2 z_2 - \hat{\boldsymbol{\varphi}}_2^{\mathrm{T}} S_{m_2}(\mathbf{z}_2) \\ \alpha_1 &= -k_1 z_1 - \hat{\boldsymbol{\varphi}}_1^{\mathrm{T}} S_{m_1}(\mathbf{z}_2) \\ \dot{\hat{\boldsymbol{\varphi}}}_i &= \boldsymbol{\Gamma}_i S_{m_i}(\mathbf{z}_i) z_i^3 - \boldsymbol{\Gamma}_i \eta_i \hat{\boldsymbol{\varphi}}_i, \quad i = 1, 2 \end{aligned}$$

where $\mathbf{z}_1 = [z_1]^{\mathrm{T}}$, $\mathbf{z}_2 = [z_1, z_2]^{\mathrm{T}}$, $z_1 = \chi_1 - y_d$, and $z_2 = \chi_2 - \alpha_1$.

Through the continuous optimization of parameters, a set of ideal parameters are empirically determined, which ensure that a good tracking performance can be achieved. The design parameters are taken as follows: $\eta_1 = 10$, $\eta_2 = 10$, $k_1 = 20$, $k_2 = 60$, $\boldsymbol{\Gamma}_1 = 5\mathbf{I}_5$, $\boldsymbol{\Gamma}_2 = 10\mathbf{I}_9$, and the time-delays are chosen as $\gamma_1 = \gamma_2 = 0.05$ s. The simulation results of Example 1 are shown in Figures 3–6, respectively. Figure 3 illustrates the trajectories of reference signal y_d and output y. Figure 4 displays the response of control signal u. Figure 5 shows that the state variable χ_2 is bounded.

Example 2. Practical example. Considering a one-link manipulator system with the influence of stochastic disturbance and time-delay. According to Wang et al.,[50] the system can be described as

$$\begin{cases} d\chi_1 = (\chi_2 + f_1(\chi_1))dt + 0.1\chi_1 d\omega \\ d\chi_2 = (\chi_3 + f_2(\bar{\boldsymbol{\chi}}_2) + x_1(t-\gamma_2))dt + 0.1\chi_1 d\omega \\ d\chi_3 = (u + f_3(\bar{\boldsymbol{\chi}}_3) + 0.1\chi_3(t-\gamma_3))dt + 0.1\chi_1 d\omega \\ y = \chi_1 \end{cases} \tag{54}$$

where χ_1 denotes the link angular position, χ_2 denotes the velocity, and χ_3 denotes the acceleration, and the initial state $[\chi_1(0), \chi_2(0), \chi_3(0)]^{\mathrm{T}} = [0, 0, 0]^{\mathrm{T}}$. $f_1(\chi_1) = 0$, $f_2(\bar{\boldsymbol{\chi}}_2) = -N\chi_2 \sin\chi_1$, and $f_3(\bar{\boldsymbol{\chi}}_3) = -(K_m/\mathrm{MD})\chi_2$. $N = 0.1$ is a positive constant related to the mass of the load and the coefficient of gravity, K_m is the back electromotive force coefficient, M is the armature inductance, and D is the mechanical inertia, and their values are selected as $K_m = 10$, $M = 10$, and $D = 10$. $y_d = 0.5\sin t$ is the given reference signal.

By Theorem 1, the actual control law, the intermediate control signals, and the adaptive laws are chosen, respectively, as

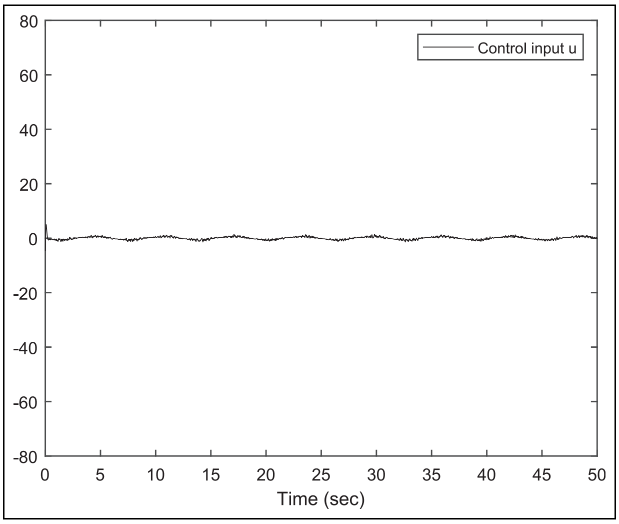

Figure 4. The trajectory of the input u.

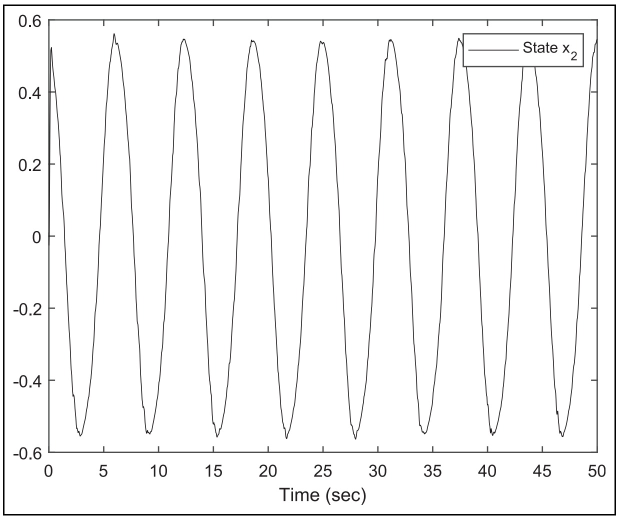

Figure 5. The trajectory of state variable χ_2.

$$u = -k_3 z_3 - \hat{\boldsymbol{\varphi}}_3^{\mathrm{T}} S_{m_3}(\mathbf{z}_3)$$
$$\alpha_j = -k_j z_j - \hat{\boldsymbol{\varphi}}_j^{\mathrm{T}} S_{m_j}(\mathbf{z}_j), \quad j = 1, 2$$
$$\dot{\hat{\boldsymbol{\varphi}}}_i = \boldsymbol{\Gamma}_i S_{m_i}(\mathbf{z}_i) z_i^3 - \boldsymbol{\Gamma}_i \eta_i \hat{\boldsymbol{\varphi}}_i, \quad 1 \leqslant i \leqslant 3$$

where $\mathbf{z}_1 = [z_1]^{\mathrm{T}}$, $\mathbf{z}_2 = [z_1, z_2]^{\mathrm{T}}$, $\mathbf{z}_3 = [z_1, z_2, z_3]^{\mathrm{T}}$, $z_1 = \chi_1 - y_d$, $z_2 = \chi_2 - \alpha_1$, and $z_3 = \chi_3 - \alpha_2$.

The design parameters are taken as follows: $\eta_1 = 8$, $\eta_2 = 10$, $\eta_3 = 100$, $k_1 = 6$, $k_2 = 10$, $k_3 = 120$, $\boldsymbol{\Gamma}_1 = 0.5\boldsymbol{I}_5$, $\boldsymbol{\Gamma}_2 = 10\boldsymbol{I}_9$, $\boldsymbol{\Gamma}_3 = 5\boldsymbol{I}_9$, and the time-delays are chosen as $\gamma_1 = \gamma_2 = \gamma_3 = 0.025\,\mathrm{s}$. The simulation results are shown in Figures 7–10, respectively. Figure 7 illustrates the trajectories of reference signal y_d and output y. Figure 8 displays the response of control signal u. Figure 9 shows that the state variables χ_2 and χ_3 are bounded. Figure 10 illustrates the trajectory of the tracking error $y - y_d$. According to the simulation results of the practical example, we conclude that the

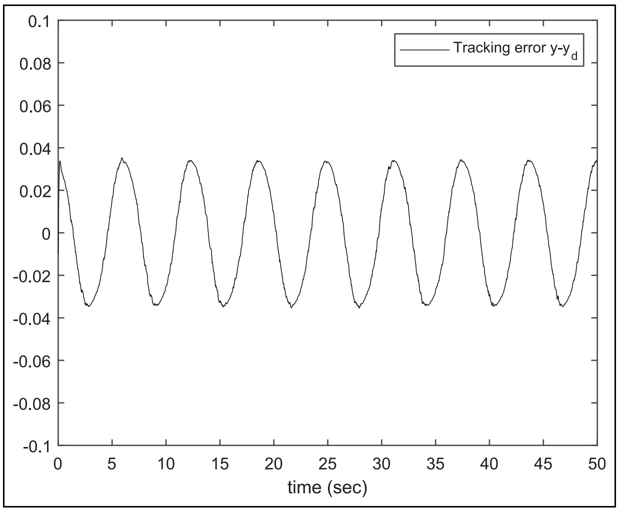

Figure 6. The trajectory of tracking error $y - y_d$.

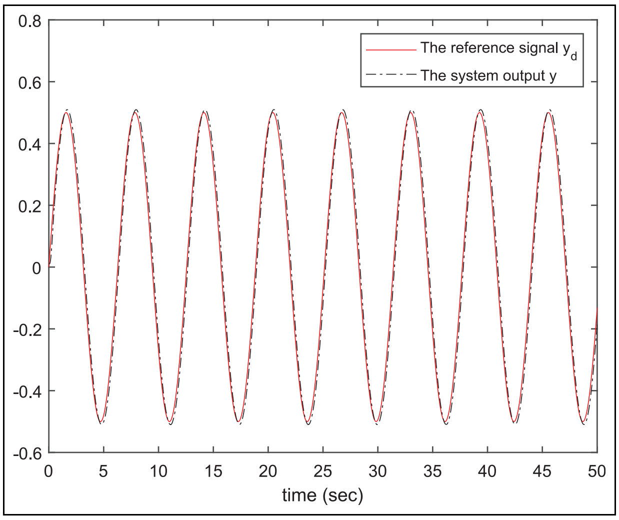

Figure 7. The trajectories of system output y and the reference signal y_d.

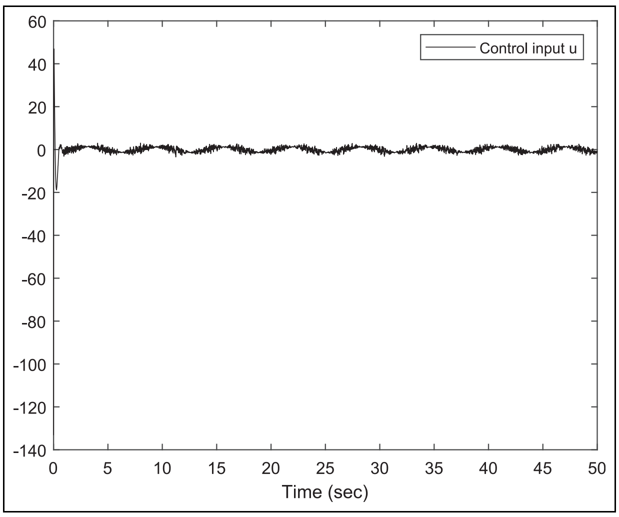

Figure 8. The trajectory of the input u.

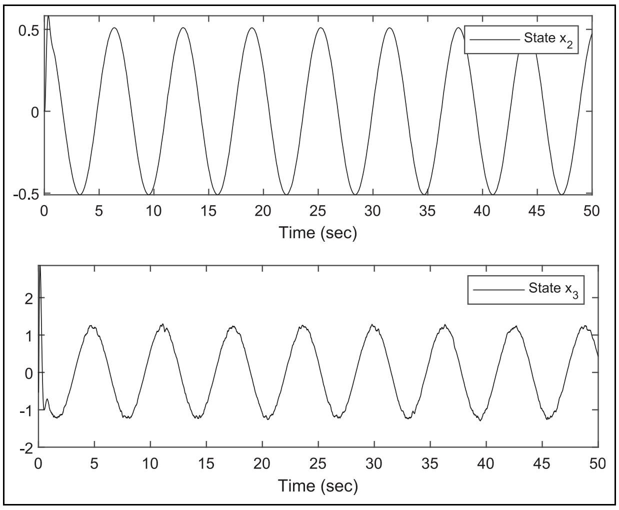

Figure 9. The trajectories of state variables χ_2 and χ_3.

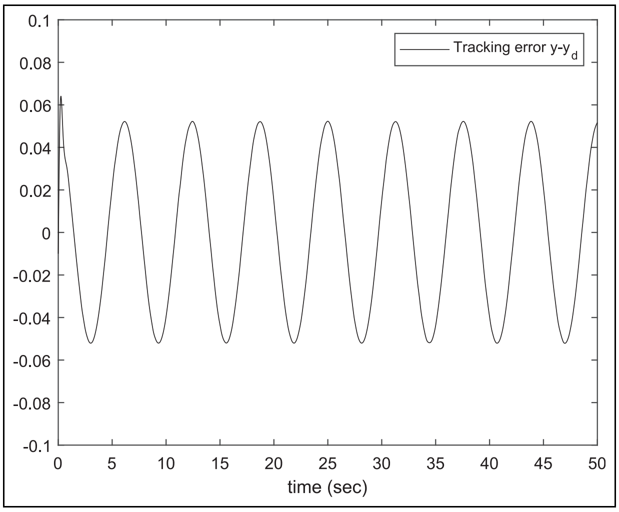

Figure 10. The trajectory of tracking error $y - y_d$.

proposed approach can realize the control objective of the one-link manipulator stochastic system with time-delay.

Example 3. Comparative example. To further demonstrate the superiority of the proposed control scheme, a comparative experiment between the adaptive MTN and radial basis function neural network (RBFNN) control methods was carried out on the basis of Example 1. The simulation comparison results are illustrated in Figure 11.

As we can see in Figure 11, although both MTN-based controller and RBFNN-based controller can realize the tracking control, the former possesses lower computational complexity and more satisfactory performance than the latter, as shown in the local enlarged diagram of Figure 11. The comparative experiment results further verify the effectiveness of the proposed control scheme.

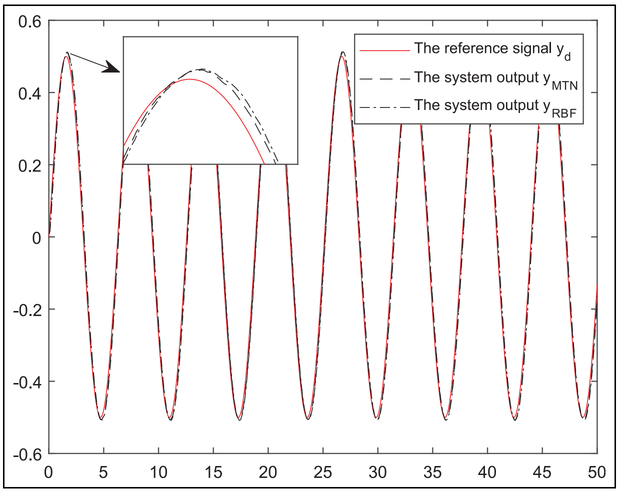

Figure 11. The tracking performances of the MTN and RBFNN.

Conclusion

In this article, a new adaptive MTN backstepping control scheme is developed to deal with the problem of tracking control for nonlinear stochastic systems with time-delay. The MTNs are applied to handle the unknown functions, which greatly improve the real-time performance of the controller because the MTN has a pretty simple structure. The novel integral-type Lyapunov–Krasovskii functions are successfully applied to overcome the unknown time-delay functions. It is shown that the proposed controller can ensure that all signals of the closed-loop system are bounded in probability and the tracking error converges to a small neighborhood of the origin. Finally, the simulation results of three examples demonstrate the effectiveness of the proposed control approach in this article.

In addition, in view of many signal processing and control applications of simulation process are realized by digital computers, sampled-data filtering problem has attracted a lot of attention.[1,2] Therefore, in our future research, we will focus on generalizing the results of this article to nonlinear systems with stochastic disturbances under the sampling-data filtering framework.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the Shandong Provincial Natural Science Foundation, China (no. ZR2020QF055) and the Open Fund of the Key Laboratory of Ocean Circulation and Waves, Chinese Academy of Sciences (no. KLOCW2003).

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