Received: 10 December 2022 | Revised: 9 April 2023 | Accepted: 17 May 2023
DOI: 10.1002/rnc.6803

RESEARCH ARTICLE

WILEY

Switching threshold-based event-triggered adaptive asymptotic tracking control for stochastic nonlinear systems with full-state constraints

Yang Du | Shan-Liang Zhu | Lian-Lian Zhai | Yu-Qun Han

School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao, China

Correspondence
Yu-Qun Han, School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao, 266061, China.
Email: yuqunhan@163.com

Funding information
Shandong Provincial Natural Science Foundation, China, Grant/Award Number: ZR2020QF055

Abstract
In this article, the problem of event-triggered adaptive asymptotic tracking control (ATC) for stochastic nonlinear systems with unknown control directions (UCDs) and full state constraints is concerned. It must be said that the controller design and system analysis is more complex and difficult since the existence of stochastic disturbances, UCDs and full state constraints simultaneously. By introducing the lower bound of the UCDs into the barrier Lyapunov functions, an event-triggered adaptive MTN ATC scheme is proposed based on the boundary estimation method and a new event-triggered control (ETC) strategy, which can achieve satisfactory asymptotic tracking performance and control performance of the system, while reduce the occupation of network resources. The simulation results not only verify the effectiveness of the proposed control scheme, but also present different tracking performances between three ETC strategies for comparison, further confirming the superiority of the proposed ETC strategy in achieving asymptotic tracking performance.

KEYWORDS
asymptotic tracking control, boundary estimation method, event-triggered control, multi-dimensional Taylor network, stochastic nonlinear systems

1 | INTRODUCTION

It is widely known that stochastic disturbances are important factor affecting system instability. Therefore, the research of stochastic nonlinear systems has become a hot topic in recent years, and many effective methods have been reported, such as adaptive backstepping control,[1,2] robust control,[3] fault-tolerant control,[4] and sliding mode control.[5] However, it is very difficult to achieve the tracking control purpose only by relying on the above methods when the nonlinear structure in the systems has strong nonlinearity and uncertainty. As some alternative methods, the approximation-based adaptive control methods have been reported, such as neural networks (NNs) control,[6-9] fuzzy logic systems (FLSs) control[10,11] and multi-dimensional Taylor network (MTN) control.[12-14] Among them, the MTN is taken as a new type of NN, which has received a growing number of attention, and many MTN-based adaptive tracking control schemes have been proposed.[15,16] However, the above research results only ensure that the tracking error bounded near the origin rather than asymptotic tracking. In addition, the resources limitation is another difficulty to be overcome when the network as most control systems transmission medium.

The traditional time-triggered control methods may lead to unnecessary occupation of network resources including communication channel bandwidth and computing abilities due to frequent transmission and execution of control signals. In order to avoid the above situation, the event-triggered control (ETC) was proposed.[17-21] The underlying idea of ETC is to optimize the control input of the systems. Specifically, the control signals will be transmitted and acted on the system only when the pre-given trigger condition is reached, thus achieving the balance between saving network resources and maintaining the system performance. By using the approach of ETC, many interesting research has been conducted for different systems, such as multi-agent nonlinear systems,[22,23] Markov jump systems,[24,25] and multiple-input multiple-output nonlinear systems.[26,27] More recently, many asymptotic tracking control (ATC) approaches based on ETC for nonlinear systems with unknown control directions (UCDs) have been proposed.[28,29] On this basis, the ATC has become a major research topic due to its unique advantages in the field of tracking control. For example, by introducing an ETC strategy based on relative threshold strategy (RTS), authors in Reference 30 investigated an adaptive fuzzy ATC method for pure-feedback nonlinear systems with uncertain external disturbances. Based on FLSs, authors in References 31 and 32 proposed some novel event-triggered adaptive ATC strategies for nonlinear systems with UCDs and uncertain external disturbances.

Despite that many ETC strategies have been obtained for stochastic nonlinear systems, the mentioned results ignore the issue of state constraints. Recently, an increasing attention has been focused on state constraints due to the fact that state constraints exist in most actual systems. The existence of those constraints may reduce the system performance, and even lead to the system instability. In fact, the problem of state constraints has been solved well by constructing barrier Lyapunov functions (BLFs), and different types of BLFs have been proposed, including tangent-BLFs,[33] integral-BLFs[34] and logarithmic-BLFs.[35,36] Furthermore, by combining BLFs and FLSs or NNs, many adaptive tracking control methods have been researched for different nonlinear systems with full-state constraints, such as nonlinear systems,[37-39] switched nonlinear systems,[40] nonlinear time-delay systems,[41] high-order nonlinear time-delay systems[42] and stochastic nonlinear systems.[43] It is worth pointing out that authors in Reference 44 and 45 investigated the tracking control issues of stochastic nonlinear systems with full-state constraints, and proposed a novel MTN-based adaptive control scheme with the help of the BLFs method. However, the ETC, the ATC, the UCDs and the full state constraints of the stochastic nonlinear systems have not yet been studied within the same theoretical framework, which was the motivation for our current research.

On the basis of the above investigation, this article dedicates to developing a novel MTN-based event-triggered adaptive ATC approach for stochastic nonlinear systems with full-state constraints, which can simultaneously overcome the multiple difficulties of implementing ATC under stochastic disturbances, state constraints, and network resources limitation. Compared to the existing results, the main innovations of this article are highlighted as follows:

(1) This article is devoted to investigating the event-triggered adaptive ATC problem for stochastic nonlinear systems with UCDs and full state constraints by using the MTN estimation method. By introducing ETC strategy, boundary estimation method and BLFs, a new event-triggered adaptive control scheme is established to compensate for the effects of UCDs, stochastic disturbances and state constraints, while ensuring the ATC performance of the system under avoiding communication overload and computational ability limitations. In addition, different from the traditional method of constructing Lyapunov functions in References 29 and 39, a novel Lyapunov function with lower bound of UCDs is constructed in this article, so that the upper bound of UCDs is no longer needed.
(2) It is worth pointing out that the event-triggered adaptive MTN asymptotic controller designed in this article has the following two advantages. First, the proposed controller based on switching threshold strategy (STS) has better asymptotic tracking performance than the fixed threshold strategy (FTS) used in Reference 21 and the RTS used in References 30-32. Second, due to the simple structure of MTN, the controller designed has lower computational complexity, and has stronger applicability especially when the computing abilities are limited.
(3) It is the first time that the adaptive ATC, the MTN estimation method and the ETC are considered for stochastic nonlinear systems with full-state constraints and UCDs. Although the event-triggered adaptive ATC strategies for different nonlinear systems with full state constraints were studied in References 39 and 41, the stochastic disturbances, the control directions, and the MTN estimation method were not considered. In addition, despite the adaptive MTN tracking control of stochastic nonlinear systems with full-state constraints have been studied in References 44 and 45, but these studies cannot be directly used for handling the ATC problem for stochastic nonlinear systems, let alone in the case where network resources are limited and the control directions are unknown. Therefore, the issue and systems studied in this article are more general.

2 | PROBLEM DESCRIPTION AND PRELIMINARY

2.1 | Problem description

Considering the stochastic nonlinear system with UCDs, which can be described as follows

$$\begin{cases} dx_i = \left(f_i\left(\overline{\boldsymbol{x}}_i\right) + g_i\left(\overline{\boldsymbol{x}}_i\right)x_{i+1}\right)dt + h_i^{\mathrm{T}}\left(\overline{\boldsymbol{x}}_i\right)d\boldsymbol{\omega} \\ \quad i = 1, 2, \ldots, n-1 \\ dx_n = \left(f_n\left(\overline{\boldsymbol{x}}_n\right) + g_n\left(\overline{\boldsymbol{x}}_n\right)u\right)dt + h_n^{\mathrm{T}}\left(\overline{\boldsymbol{x}}_n\right)d\boldsymbol{\omega} \\ \quad y = x_1, \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$ denotes the state vector of the system with $\overline{\boldsymbol{x}}_i = [x_1, x_2, \ldots, x_i]^{\mathrm{T}} \in R^i$; $u \in R$ and $y \in R$ represent the system input and output, respectively. $g_i(\cdot)$ denotes the control direction of the system, $f_i(\cdot)$ and $h_i(\cdot)$ are the unknown and smooth nonlinear functions, $\boldsymbol{\omega}$ is a standard Wiener process. All states in the system are restricted to set $\wp_{x_i} = \left\{x_i \in R \,\middle|\, |x_i| < k_{c,i}\right\}$, where $k_{c,i} > 0$ is designed as a constant.

The aim of this article is to design a BLFs-based event-triggered adaptive MTN asymptotic controller for stochastic nonlinear system (1) such that

(i) The tracking error asymptotically converges to zero while ensuring that the specified state constraints of the system are not violated.
(ii) All closed-loop signals are bounded on $[0, +\infty)$.
(iii) The controller excludes Zeno phenomenon.

In order to achieve the above purpose, the following lemmas and assumptions are needed in the control strategy process.

Assumption 1. The reference signal y_d and its ith order time derivatives $y_d^{(i)}, i = 1, \ldots, n$ satisfy $|y_d| \le \mathrm{A}_1 < k_{c,1}$ and $\left|y_d^{(i)}\right| < \mathrm{B}_i$, where A_1 and $\mathrm{B}_i, i = 1, 2, \ldots, n$ are positive constants.

Assumption 2. The sign of $g_i(\cdot)$ in system (1) is known and that there exists an unknown positive constant $\underline{g}_i$ such that $0 < \underline{g}_i < |g_i(\cdot)|$. In this article, we assume that $g_i(\cdot) > 0$ without losing its generality.

Remark 1. It must be said that the Assumption 1 is one of the necessary assumptions for solving the full state constraints problem, which also can be seen in References 36,37, and 43. In addition, compared with References 10,20, and 43, the stochastic nonlinear systems studied in this article have the characteristic of UCDs. Therefore, the controlled system of this article is more representative.

Lemma 1 (38). *For any $\vartheta \in R$ and any positive bounded and continuous function $\sigma(t)$, the following inequality holds*

$$|\vartheta| \le \frac{\vartheta^2}{\sqrt{\vartheta^2 + \sigma^2(t)}} + \sigma(t) \tag{2}$$

with $\sigma(t)$ satisfies

$$\lim_{t \to +\infty} \int_{t_0}^{t} \sigma(s)\,ds \le \overline{\sigma} < +\infty, \tag{3}$$

where $\overline{\sigma} > 0$ is a positive constant.

Lemma 2 (Young's inequality[10]). *For $\forall (x, y) \in R^2$ and $\forall \tau > 0$, the following inequality must hold*

$$xy \le \frac{\tau^{\rho}}{\rho}|x|^{\rho} + \frac{1}{\nu\tau^{\nu}}|y|^{\nu}, \tag{4}$$

where $\rho > 1$, $\nu > 1$ are constants and satisfy $(\rho - 1)(\nu - 1) = 1$.

Lemma 3 (19). *For $\forall \Upsilon \in R$ and any positive continuous function $\gamma(t)$ with $t > 0$, one can get the following inequality*

$$0 \leq |\Upsilon| - \Upsilon \tanh\left(\frac{\Upsilon}{\gamma(t)}\right) \leq 0.2785\gamma(t). \tag{5}$$

2.2 | Stochastic systems theory

For convenience, considering the following stochastic nonlinear system of general form

$$d\boldsymbol{x} = f(\boldsymbol{x})\,dt + h(\boldsymbol{x})\,d\boldsymbol{\omega}, \tag{6}$$

where $\boldsymbol{x} \in R^n$ stands for the system state vector; $f(\boldsymbol{x}) : R^n \to R^n$ and $h(\boldsymbol{x}) : R^n \to R^{n\times r}$ satisfying conditions $f(\mathbf{0}) = \mathbf{0}$ and $h(\mathbf{0}) = \mathbf{0}$ are locally Lipschitz functions; $\boldsymbol{\omega}$ is a standard Wiener process.

Definition 1 (20). For systems (6), for any second-order continuously differentiable function $V(\boldsymbol{x})$, defining the differential operator L as follows

$$LV(\boldsymbol{x}) = \frac{\partial V}{\partial \boldsymbol{x}} f + \frac{1}{2} Tr\left\{h^{\mathrm{T}} \frac{\partial^2 V}{\partial \boldsymbol{x}^2} h\right\}, \tag{7}$$

where Tr{∘} denotes the trace of ∘.

2.3 | Multi-dimensional Taylor network

In order to overcome the difficulty of adaptive controller design caused by unknown nonlinear functions in system (1), the estimation property of MTN is used to design the controller. The relevant lemma is described as follows.

Lemma 4 (44). *If $f(Z)$ is a continuous and unknown function defined on the compact set $\mathrm{M} \subset R^n$, for $\forall \varepsilon > 0$, there always exists an MTN $\boldsymbol{W}^{\mathrm{T}} S_{m_n}(\boldsymbol{Z})$ that is used to estimate $f(\boldsymbol{Z})$. The estimate rule for MTN is formulated as follows*

$$f(\boldsymbol{Z}) = \boldsymbol{W}^{\mathrm{T}} S_{m_n}(\boldsymbol{Z}) + \delta(\boldsymbol{Z}), \tag{8}$$

where $\boldsymbol{Z} = [z_1, \dots, z_n]^{\mathrm{T}} \in \mathrm{M} \subset R^n$, $\boldsymbol{W} = [W_1, \dots, W_l]^{\mathrm{T}} \in R^l$ and $S_{m_n}(\boldsymbol{Z}) = \left[z_1, \dots, z_n, z_1^2, z_1z_2, \dots, z_1z_n, z_2z_3, \dots, z_n^2, z_1^m, \dots, z_n^m\right]^{\mathrm{T}} \in R^l$ are the input vector of MTN, the weight vector of MTN and the intermediate input layer of MTN, respectively. $\delta(\boldsymbol{Z})$ represents the error generated in the estimation with $|\delta(\boldsymbol{Z})| \leq \varepsilon$.

2.4 | Event-triggered control strategy based on switching threshold

It is worth noting that the network resources in the actual systems are limited. In order to reduce the occupation of network resources, the following ETC strategy based on switching threshold is adopted in the co-design with the adaptive controller.

$$u(t) = \varpi(t_k), \forall t \in [t_k, t_{k+1}), \tag{9}$$

$$t_{k+1} = \begin{cases} \inf\{t \in R \mid |\mathrm{e}(t)| \geq M\} & |u(t)| > Q \\ \inf\{t \in R \mid |\mathrm{e}(t)| \geq \chi|u(t)| + M_1\} & |u(t)| \leq Q, \end{cases} \tag{10}$$

where Q is a known design parameter and t_k, $0 < \chi < 1$, $k \in z^+$, M, M_1 are some positive design parameters. $u(t)$ denotes a constant refers to the input of the system when the event is triggered; t_k is the event trigger moment; $\mathrm{e}(t) = \varpi(t) - u(t)$ stands for the measurement error.

In the process ETC based on STS, constructing $\vec{\mathrm{e}}$ as follows

$$\vec{\mathrm{e}} = \max\left\{\chi Q + M_1, M\right\}. \tag{11}$$

Remark 2. It can be seen from (9) that the control value u keeps the previous value $\varpi(t_k)$ unchanged during the time $t \in [t_k, t_{k+1})$. From (10), when $\mathrm{e}(t)$ satisfies the pre-given conditions, the time t will be marked as t_{k+1} and the control input value of the system will be change to $u(t_{k+1})$.

Remark 3. It is worth noting that the ETC strategy based on RTS $\begin{cases} u(t) = \varpi(t_k), \forall t \in [t_k, t_{k+1}) \\ t_{k+1} = \inf\left\{t \in R \mid |\mathrm{e}(t)| \geq \chi |u(t)| + M_1\right\} \end{cases}$ designed in References 30-32, when the value of $|u(t)|$ is too large, a great measurement error $\mathrm{e}(t)$ is inevitable. In this case, if the event is suddenly triggered, then the control signal u will suddenly change causing a large pulse to act on the system. It will lead to performance degradation of the system, especially in ATC. In order to avoid such problem, the STS designed in this article combines the advantages of the FTS $\begin{cases} u(t) = \varpi(t_k), \forall t \in [t_k, t_{k+1}) \\ t_{k+1} = \inf\left\{t \in R \mid |\mathrm{e}(t)| \geq M\right\} \end{cases}$ used in Reference 21 and the RTS. When $|u(t)|$ is too large, the FTS is adopted such that the measurement error $\mathrm{e}(t)$ keeps bounded to ensure asymptotic tracking performance of the system; otherwise when u satisfies $|u(t)| \leq Q$, the RTS is adopted so that the system can obtain more precise control.

Remark 4. Parameters M and M_1 can determine the lower bound of the time intervals to ensure that the controller excludes Zeno phenomenon, which will be demonstrated in the following theory and simulations.

3 | CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, a new BLFs-based event-triggered adaptive MTN asymptotic controller is designed by introducing boundary estimation method and ETC strategy. To implement the controller design, defining the coordinate transformation with the following form

$$\begin{cases} z_1 = x_1 - y_d \\ z_i = x_i - \alpha_{i-1}, i = 2, \dots, n, \end{cases} \tag{12}$$

where z_i is error variable and α_{i-1} is virtual control signal will be designed later. Given a compact set $\Phi_{z_i} = \left\{z_i \in R \,\middle|\, |z_i| < k_{b,i}\right\}, i = 1, 2, \dots, n$, with $k_{b,i} = k_{c,i} - \mathrm{A}_i$ is specified as positive constant in subsequent simulations.

The boundary estimation method is applied to dispose the unknown parameters designed in this article. Therefore, the following definitions are introduced

$$\lambda_i = \frac{\|\boldsymbol{W}_i\|}{\underline{g}_i}, \tau_i = \frac{\varepsilon_i + \frac{1}{2}|\varsigma_i|}{\underline{g}_i}, i = 1, 2, \dots, n, \tag{13}$$

and

$$\tilde{\lambda}_i = \lambda_i - \hat{\lambda}_i, \tilde{\tau}_i = \tau_i - \hat{\tau}_i, \tag{14}$$

where ς_i will be defined in Lemma 5, $\tilde{\lambda}_i$ and $\tilde{\tau}_i$ represent the parameter errors λ_i and τ_i, respectively.

To facilitate the subsequent ATC design process, the following Lemma 5 is given by applying Lemma 1:

Lemma 5. *For every $i = 1, 2, \dots, n$, the nonlinear functions $\varphi_i = \frac{\Xi_i S_{m_i}^{\mathrm{T}} S_{m_i}}{\sqrt{\Xi_i^2 S_{m_i}^{\mathrm{T}} S_{m_i} + \sigma_i^2}}$ and $\varsigma_i = \frac{z_i\left(3k_{b,i}^4 + z_i^4\right)^2 \Gamma_i^2}{(k_{b,i}^4 - z_i^4)^3} \Big/ \sqrt{\frac{z_i^4\left(3k_{b,i}^4 + z_i^4\right)^2 \Gamma_i^2}{(k_{b,i}^4 - z_i^4)^4} + \sigma_i^2}$ satisfy the following properties*

$$\frac{1}{\underline{g}_i}\Xi_i \boldsymbol{W}_i^{\mathrm{T}} S_{m_i} \leq \lambda_i \Xi_i \varphi_i + \lambda_i \sigma_i, \tag{15}$$

$$\frac{z_i^2\left(3k_{b,i}^4+z_i^4\right)\Gamma_i}{(k_{b,i}^4-z_i^4)^2}\leq \Xi_i\varsigma_i+\sigma_i, \tag{16}$$

where Ξ_i *and* Γ_i *will be defined later*.

Proof. See Appendix. ∎

3.1 | Event-triggered adaptive asymptotic controller design

Step 1: According to $z_1=x_1-y_d$, the derivative of z_1 with respect to t can be calculated as follows

$$dz_1=\left(g_1x_2+f_1-\dot{y}_d\right)dt+h_1^{\mathrm{T}}d\boldsymbol{\omega}. \tag{17}$$

To remove the constraint on the upper bound of the UCDs, constructing the first BLF as follows

$$V_1=\frac{1}{4\underline{g}_1}\log\frac{k_{b,1}^4}{k_{b,1}^4-z_1^4}+\frac{1}{2}\tilde{\lambda}_1^2+\frac{1}{2}\tilde{\tau}_1^2, \tag{18}$$

where $\tilde{\lambda}_i$ and $\tilde{\tau}_i$ can be referred in (14) and $k_{b,1}=k_{c,1}-\mathrm{A}_1$. In addition, V_1 is continuous on a compact set $\Phi_{z_1}=\left\{z_1\in R\,\middle|\,|z_1|<k_{b,1}\right\}$.

According to Definition 1, the following equation can be obtained as follows

$$\begin{aligned}LV_1&=\frac{1}{\underline{g}_1}\frac{z_1^3}{k_{b,1}^4-z_1^4}\left[g_1\left(z_2+\alpha_1\right)+f_1-\dot{y}_d\right]-\tilde{\lambda}_1\dot{\hat{\lambda}}_1-\tilde{\tau}_1\dot{\hat{\tau}}_1+\frac{1}{2\underline{g}_1}\frac{z_1^2\left(3k_{b,1}^4+z_1^4\right)\|h_1\|^2}{(k_{b,1}^4-z_1^4)^2}\\&\triangleq\frac{1}{\underline{g}_1}\Xi_1\left[g_1\left(z_2+\alpha_1\right)+f_1-\dot{y}_d\right]-\tilde{\lambda}_1\dot{\hat{\lambda}}_1-\tilde{\tau}_1\dot{\hat{\tau}}_1+\frac{1}{2\underline{g}_1}\frac{z_1^2\left(3k_{b,1}^4+z_1^4\right)\Gamma_1}{(k_{b,1}^4-z_1^4)^2},\end{aligned} \tag{19}$$

where $\Xi_1=\frac{z_1^3}{k_{b,1}^4-z_1^4}$ and $\Gamma_1=\|h_1\|^2$.

Then, according to Lemma 5, the following inequalities can be obtained with the help of Young's inequality

$$\frac{g_1}{\underline{g}_1}\Xi_1z_2\leq\frac{g_1}{\underline{g}_1}\frac{3}{4}\Xi_1^{\frac{4}{3}}+\frac{g_1}{\underline{g}_1}\frac{1}{4}z_2^4, \tag{20}$$

$$\frac{1}{2\underline{g}_1}\frac{z_1^2\left(3k_{b,1}^4+z_1^4\right)\Gamma_1}{(k_{b,1}^4-z_1^4)^2}\leq\frac{1}{2\underline{g}_1}\Xi_1\varsigma_1+\frac{1}{2\underline{g}_1}\sigma_1. \tag{21}$$

Substituting (20) and (21) into (19) yields

$$\begin{aligned}LV_1&\leq\frac{1}{\underline{g}_1}\Xi_1\left(g_1\alpha_1+f_1+\frac{3}{4}g_1\Xi_1^{\frac{1}{3}}+\frac{1}{2}\varsigma_1-\dot{y}_d\right)+\frac{g_1}{\underline{g}_1}\frac{1}{4}z_2^4+\frac{1}{2\underline{g}_1}\sigma_1-\tilde{\lambda}_1\dot{\hat{\lambda}}_1-\tilde{\tau}_1\dot{\hat{\tau}}_1\\&\triangleq\frac{1}{\underline{g}_1}\Xi_1\left(g_1\alpha_1+\mathrm{H}_1+\frac{1}{2}\varsigma_1\right)+\frac{g_1}{\underline{g}_1}\frac{1}{4}z_2^4+\frac{1}{2\underline{g}_1}\sigma_1-\tilde{\lambda}_1\dot{\hat{\lambda}}_1-\tilde{\tau}_1\dot{\hat{\tau}}_1,\end{aligned} \tag{22}$$

where $\mathrm{H}_1=f_1+\frac{3}{4}g_1\Xi_1^{\frac{1}{3}}-\dot{y}_d$.

Since H_1 is an unknown smooth function, by applying Lemma 4, it can be estimated by an MTN with the following form

$$\mathrm{H}_1=\boldsymbol{W}_1^{\mathrm{T}}S_{m_1}\left(\boldsymbol{Z}_1\right)+\delta\left(\boldsymbol{Z}_1\right), \tag{23}$$

where $\boldsymbol{Z}_1=[z_1]^{\mathrm{T}}$ is the input vector of MTN and $\delta_1\left(\boldsymbol{Z}_1\right)$ is the estimation error and satisfies $|\delta_1\left(\boldsymbol{Z}_1\right)|\leq\varepsilon_1$.

According to Lemma 5, the following inequality can be deduced from (13) and (23)

$$\frac{1}{\underline{g}_1}\Xi_1\left(\mathrm{H}_1+\frac{1}{2}\varsigma_1\right)=\frac{1}{\underline{g}_1}\Xi_1\left(\boldsymbol{W}_1^{\mathrm{T}}S_{m_1}(\boldsymbol{Z}_1)+\delta(\boldsymbol{Z}_1)+\frac{1}{2}\varsigma_1\right)$$
$$\leq\lambda_1\Xi_1\varphi_1+\lambda_1\sigma_1+|\Xi_1|\frac{\varepsilon_1+\frac{1}{2}|\varsigma_1|}{\underline{g}_1}$$
$$=\lambda_1\Xi_1\varphi_1+\lambda_1\sigma_1+|\Xi_1|\,\tau_1. \tag{24}$$

Then, according to Lemma 1, the following inequality holds

$$|\Xi_1|\,\tau_1\leq\frac{\tau_1\Xi_1^2}{\sqrt{\Xi_1^2+\sigma_1^2}}+\tau_1\sigma_1. \tag{25}$$

In light of (25), the (24) can be rewritten as

$$\frac{1}{\underline{g}_1}\Xi_1\left(\mathrm{H}_1+\frac{1}{2}\varsigma_1\right)\leq\lambda_1\Xi_1\varphi_1+\frac{\tau_1\Xi_1^2}{\sqrt{\Xi_1^2+\sigma_1^2}}+\lambda_1\sigma_1+\tau_1\sigma_1. \tag{26}$$

Substituting (26) into (22), we have

$$LV_1\leq\frac{g_1}{\underline{g}_1}\Xi_1\alpha_1+\hat{\lambda}_1\Xi_1\varphi_1+\frac{\hat{\tau}_1\Xi_1^2}{\sqrt{\Xi_1^2+\sigma_1^2}}-\tilde{\lambda}_1\left(\dot{\hat{\lambda}}_1-\Xi_1\varphi_1\right)$$
$$-\tilde{\tau}_1\left(\dot{\hat{\tau}}_1-\frac{\Xi_1^2}{\sqrt{\Xi_1^2+\sigma_1^2}}\right)+\sigma_1\left(\lambda_1+\tau_1+\frac{1}{2\underline{g}_1}\right)+\frac{g_1}{\underline{g}_1}\frac{1}{4}z_2^4. \tag{27}$$

Then, designing the virtual control signal α_1 and the adaptive laws $\dot{\hat{\lambda}}_1$, $\dot{\hat{\tau}}_1$ as follows

$$\alpha_1=-r_1z_1-\hat{\lambda}_1\varphi_1-\frac{\hat{\tau}_1\Xi_1}{\sqrt{\Xi_1^2+\sigma_1^2}}, \tag{28a}$$

$$\dot{\hat{\lambda}}_1=\Xi_1\varphi_1-\sigma_1\hat{\lambda}_1, \tag{28b}$$

$$\dot{\hat{\tau}}_1=\frac{\Xi_1^2}{\sqrt{\Xi_1^2+\sigma_1^2}}-\sigma_1\hat{\tau}_1, \tag{28c}$$

where r_1 is designed as a positive constant.

According to (28), the (27) can be simplified as

$$LV_1\leq-r_1\frac{z_1^4}{k_{b,1}^4-z_1^4}+\sigma_1\tilde{\lambda}_1\hat{\lambda}_1+\sigma_1\tilde{\tau}_1\hat{\tau}_1+\sigma_1\left(\lambda_1+\tau_1+\frac{1}{2\underline{g}_1}\right)+\frac{g_1}{\underline{g}_1}\frac{1}{4}z_2^4, \tag{29}$$

where the term $\frac{g_1}{\underline{g}_1}\frac{1}{4}z_2^4$ will be removed in the next step.

Step $i\,(2\leq i\leq n-1)$: For $z_i=x_i-\alpha_{i-1}$, the derivative of z_i with respect to t can be calculated as follows

$$dz_i=(g_ix_{i+1}+f_i-\Lambda\alpha_{i-1})\,dt+\tilde{h}_i^{\mathrm{T}}d\boldsymbol{\omega}, \tag{30}$$

where $\Lambda\alpha_{i-1} = \sum_{j=1}^{i-1} \frac{\partial\alpha_{i-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) + \sum_{j=0}^{i-1} \frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} + \sum_{j=1}^{i-1} \frac{\partial\alpha_{i-1}}{\hat{\lambda}_j}\dot{\hat{\lambda}}_j + \sum_{j=1}^{i-1} \frac{\partial\alpha_{i-1}}{\hat{\tau}_j}\dot{\hat{\tau}}_j + \frac{1}{2}\sum_{j,k=1}^{i-1} \frac{\partial^2\alpha_{i-1}}{\partial x_j \partial x_k} h_j^{\mathrm{T}} h_k$ and $\tilde{h}_i = h_i - \sum_{j=1}^{i-1} \frac{\partial\alpha_{i-1}}{\partial x_j} h_j$.

To remove the constraint on the upper bound of the UCDs, constructing the ith BLF as follows

$$V_i = V_{i-1} + \frac{1}{4\underline{g}_i} \log \frac{k_{b,i}^4}{k_{b,i}^4 - z_i^4} + \frac{1}{2}\tilde{\lambda}_i^2 + \frac{1}{2}\tilde{\tau}_i^2, \tag{31}$$

where $\tilde{\lambda}_i$ and $\tilde{\tau}_i$ can refer to the definitions in (14) and $k_{b,i} = k_{c,i} - \mathrm{A}_i$ with $|\alpha_{i-1}| < \mathrm{A}_i$. V_i is continuous on a compact set $\Phi_{z_i} = \left\{z_i \in R \,\middle|\, |z_i| < k_{b,i}\right\}$.

According to Definition 1, the following equation can be obtained as follows

$$LV_i = LV_{i-1} + \frac{1}{\underline{g}_i}\Xi_i\left[g_i\left(z_{i+1} + \alpha_i\right) + f_i - \Lambda\alpha_{i-1}\right] - \tilde{\lambda}_i\dot{\hat{\lambda}}_i - \tilde{\tau}_i\dot{\hat{\tau}}_i + \frac{1}{2\underline{g}_i}\frac{z_i^2\left(3k_{b,i}^4 + z_i^4\right)\Gamma_i}{(k_{b,i}^4 - z_i^4)^2}, \tag{32}$$

where $\Xi_i = \frac{z_i^3}{k_{b,i}^4 - z_i^4}$ and $\Gamma_i = \left\|\tilde{h}_i\right\|^2$.

Then, according to Lemma 5, the following inequalities can be obtained with the help of Young's inequality

$$\frac{g_i}{\underline{g}_i}\Xi_i z_{i+1} \le \frac{g_i}{\underline{g}_i}\frac{3}{4}\Xi_i^{\frac{4}{3}} + \frac{g_i}{\underline{g}_i}\frac{1}{4}z_{i+1}^4, \tag{33}$$

$$\frac{1}{2\underline{g}_i}\frac{z_i^2\left(3k_{b,i}^4 + z_i^4\right)\Gamma_i}{(k_{b,i}^4 - z_i^4)^2} \le \frac{1}{2\underline{g}_i}\Xi_i\varsigma_i + \frac{1}{2\underline{g}_i}\sigma_i. \tag{34}$$

Substituting (33) and (34) into (32) yields

$$LV_i \le LV_{i-1} + \frac{1}{\underline{g}_i}\Xi_i\left(g_i\alpha_i + \mathrm{H}_i + \frac{1}{2}\varsigma_i\right) - \frac{g_{i-1}}{\underline{g}_{i-1}}\frac{1}{4}z_i^4 + \frac{g_i}{\underline{g}_i}\frac{1}{4}z_{i+1}^4 + \frac{1}{2\underline{g}_i}\sigma_i - \tilde{\lambda}_i\dot{\hat{\lambda}}_i - \tilde{\tau}_i\dot{\hat{\tau}}_i, \tag{35}$$

where $\mathrm{H}_i = f_i + \frac{3}{4}g_i\Xi_i^{\frac{1}{3}} - \Lambda\alpha_{i-1} + \frac{\underline{g}_i g_{i-1}}{4\underline{g}_{i-1}} z_i\left(k_{b,i}^4 - z_i^4\right)$.

Similarly, since H_i is an unknown smooth function, by applying Lemma 4, it can be estimated by an MTN with the following form

$$\mathrm{H}_i = \boldsymbol{W}_i^{\mathrm{T}} S_{m_i}\left(\boldsymbol{Z}_i\right) + \delta\left(\boldsymbol{Z}_i\right), \tag{36}$$

where $\boldsymbol{Z}_i = [z_1, \ldots, z_i]^{\mathrm{T}}$ is the input vector and $\delta_i\left(\boldsymbol{Z}_i\right)$ is the estimation error and satisfies $|\delta_i\left(\boldsymbol{Z}_i\right)| \le \varepsilon_i$.

According to Lemma 5, the following inequality can be deduced from (15), (36)

$$\begin{aligned} \frac{1}{\underline{g}_i}\Xi_i\left(\mathrm{H}_i + \frac{1}{2}\varsigma_i\right) &= \frac{1}{\underline{g}_i}\Xi_i\left(\boldsymbol{W}_i^{\mathrm{T}} S_{m_i}\left(\boldsymbol{Z}_i\right) + \delta\left(\boldsymbol{Z}_i\right) + \frac{1}{2}\varsigma_i\right) \\ &\le \lambda_i\Xi_i\varphi_i + \lambda_i\sigma_i + |\Xi_i|\frac{\varepsilon_i + \frac{1}{2}|\varsigma_i|}{\underline{g}_i} \\ &= \lambda_i\Xi_i\varphi_i + \lambda_i\sigma_i + |\Xi_i|\,\tau_i. \end{aligned} \tag{37}$$

Then, according to Lemma 1, the following inequality holds

$$|\Xi_i|\,\tau_i \le \frac{\tau_i\Xi_i^2}{\sqrt{\Xi_i^2 + \sigma_i^2}} + \tau_i\sigma_i. \tag{38}$$

In light of (38), the (37) can be rewritten as

$$\frac{1}{\underline{g}_i}\Xi_i\left(\mathrm{H}_i+\frac{1}{2}\varsigma_i\right)\leq\lambda_i\Xi_i\varphi_i+\frac{\tau_i\Xi_i^2}{\sqrt{\Xi_i^2+\sigma_i^2}}+\lambda_i\sigma_i+\tau_i\sigma_i. \tag{39}$$

Substituting (39) into (37), we have

$$LV_i\leq LV_{i-1}+\frac{g_i}{\underline{g}_i}\Xi_i\alpha_i+\hat{\lambda}_i\Xi_i\varphi_i+\frac{\hat{\tau}_i\Xi_i^2}{\sqrt{\Xi_i^2+\sigma_i^2}}-\tilde{\lambda}_i\left(\dot{\hat{\lambda}}_i-\Xi_i\varphi_i\right)-\tilde{\tau}_i\left(\dot{\hat{\tau}}_i-\frac{\Xi_i^2}{\sqrt{\Xi_i^2+\sigma_i^2}}\right)-\frac{g_{i-1}}{\underline{g}_{i-1}}\frac{1}{4}z_i^4+\sigma_i\left(\lambda_i+\tau_i+\frac{1}{2\underline{g}_i}\right)+\frac{g_i}{\underline{g}_i}\frac{1}{4}z_{i+1}^4 \tag{40}$$

with

$$LV_{i-1}\leq-\sum_{j=1}^{i-1}r_j\frac{z_j^4}{k_{b,j}^4-z_j^4}+\sum_{j=1}^{i-1}\sigma_j\tilde{\lambda}_j\hat{\lambda}_j+\frac{g_{i-1}}{\underline{g}_{i-1}}\frac{1}{4}z_i^4+\sum_{j=1}^{i-1}\sigma_j\tilde{\tau}_j\hat{\tau}_j+\sum_{j=1}^{i-1}\sigma_j\left(\lambda_j+\tau_j+\frac{1}{2\underline{g}_j}\right). \tag{41}$$

Then, designing the virtual control signal α_i and the adaptive laws $\dot{\hat{\lambda}}_i$, $\dot{\hat{\tau}}_i$ as follows

$$\alpha_i=-r_iz_i-\hat{\lambda}_i\varphi_i-\frac{\hat{\tau}_i\Xi_i}{\sqrt{\Xi_i^2+\sigma_i^2}}, \tag{42a}$$

$$\dot{\hat{\lambda}}_i=\Xi_i\varphi_i-\sigma_i\hat{\lambda}_i, \tag{42b}$$

$$\dot{\hat{\tau}}_i=\frac{\Xi_i^2}{\sqrt{\Xi_i^2+\sigma_i^2}}-\sigma_i\hat{\tau}_i, \tag{42c}$$

where r_i is designed as a positive constant.

According to (42), the (40) can be simplified as

$$LV_i\leq-\sum_{j=1}^{i}r_j\frac{z_j^4}{k_{b,j}^4-z_j^4}+\sum_{j=1}^{i}\sigma_j\tilde{\lambda}_j\hat{\lambda}_j+\sum_{j=1}^{i}\sigma_j\tilde{\tau}_j\hat{\tau}_j+\sum_{j=1}^{i}\sigma_j\left(\lambda_j+\tau_j+\frac{1}{2\underline{g}_j}\right)+\frac{g_i}{\underline{g}_i}\frac{1}{4}z_{i+1}^4. \tag{43}$$

Step n: For $z_n=x_n-\alpha_{n-1}$, the derivative of z_n with respect to t can be calculated as follows

$$dz_n=(g_nu+f_n-\Lambda\alpha_{n-1})\,dt+\tilde{h}_n^{\mathrm{T}}d\boldsymbol{\omega}, \tag{44}$$

where $\Lambda\alpha_{n-1}=\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(g_jx_{j+1}+f_j\right)+\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)}+\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\hat{\lambda}_j}\dot{\hat{\lambda}}_j+\sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\hat{\tau}_j}\dot{\hat{\tau}}_j+\frac{1}{2}\sum_{j,k=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_j\partial x_k}h_j^{\mathrm{T}}h_k$ and $\tilde{h}_n=h_n-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}h_j$.

To remove the constraint on the upper bound of the UCDs, constructing the nth BLF as follows

$$V_n=V_{n-1}+\frac{1}{4\underline{g}_n}\log\frac{k_{b,n}^4}{k_{b,n}^4-z_n^4}+\frac{1}{2}\tilde{\lambda}_n^2+\frac{1}{2}\tilde{\tau}_n^2, \tag{45}$$

where $\tilde{\lambda}_n$ and $\tilde{\tau}_n$ can refer to the definitions in (14) and $k_{b,n}=k_{c,n}-\mathrm{A}_n$ with $|\alpha_{n-1}|<\mathrm{A}_n$. V_n is continuous on a compact set $\Phi_{z_n}=\left\{z_n\in R\,\middle|\,|z_n|<k_{b,n}\right\}$.

According to Definition 1,the following equation holds

$$LV_n = LV_{n-1} + \frac{1}{\underline{g}_n}\Xi_n\left(g_n u + f_n - \Lambda\alpha_{n-1}\right) - \tilde{\lambda}_n\dot{\hat{\lambda}}_n - \tilde{\tau}_n\dot{\hat{\tau}}_n + \frac{1}{2\underline{g}_n}\frac{z_n^2\left(3k_{b,n}^4 + z_n^4\right)\Gamma_n}{(k_{b,n}^4 - z_n^4)^2}, \tag{46}$$

where $\Xi_n = \frac{z_n^3}{k_{b,n}^4 - z_n^4}$ and $\Gamma_n = \left\|\tilde{h}_n\right\|^2$.

Referring to Lemma 5, the following inequality is holds

$$\frac{1}{2\underline{g}_n}\frac{z_n^2\left(3k_{b,n}^4 + z_n^4\right)\Gamma_n}{(k_{b,n}^4 - z_n^4)^2} \le \frac{1}{2\underline{g}_n}\Xi_n\varsigma_n + \frac{1}{2\underline{g}_n}\sigma_n. \tag{47}$$

Substituting (47) into (46), it is easy to get the following inequality

$$LV_n \le LV_{n-1} + \frac{1}{\underline{g}_n}\Xi_n\left(g_n u + \mathrm{H}_n + \frac{1}{2}\varsigma_n\right) - \frac{g_{n-1}}{\underline{g}_{n-1}}\frac{1}{4}z_n^4 + \frac{1}{2\underline{g}_n}\sigma_n - \tilde{\lambda}_n\dot{\hat{\lambda}}_n - \tilde{\tau}_n\dot{\hat{\tau}}_n, \tag{48}$$

where $\mathrm{H}_n = f_n - \Lambda\alpha_{n-1} + \frac{\underline{g}_n g_{n-1}}{4\underline{g}_{n-1}}z_n\left(k_{b,n}^4 - z_n^4\right)$.

Similarly, since H_n is an unknown smooth function, by applying Lemma 4, it can be estimated by an MTN with the following form

$$\mathrm{H}_n = W_n^{\mathrm{T}}S_{m_n}\left(\boldsymbol{Z}_n\right) + \delta\left(\boldsymbol{Z}_n\right), \tag{49}$$

where $\boldsymbol{Z}_n = [z_1, \ldots, z_n]^{\mathrm{T}}$ is the input vector and $\delta_n\left(\boldsymbol{Z}_n\right)$ is the estimation error and satisfies $\left|\delta_n\left(\boldsymbol{Z}_n\right)\right| \le \varepsilon_n$.

According to Lemma 5, the following inequality can be deduced from (13) and (49)

$$\begin{aligned}
\frac{1}{\underline{g}_n}\Xi_n\left(\mathrm{H}_n + \frac{1}{2}\varsigma_n\right) &= \frac{1}{\underline{g}_n}\Xi_n\left(\boldsymbol{W}_n^{\mathrm{T}}S_{m_n}\left(\boldsymbol{Z}_n\right) + \delta\left(\boldsymbol{Z}_n\right) + \frac{1}{2}\varsigma_n\right) \\
&\le \lambda_n\Xi_n\varphi_n + \lambda_n\sigma_n + \left|\Xi_n\right|\frac{\varepsilon_n + \frac{1}{2}\left|\varsigma_n\right|}{\underline{g}_n} \\
&= \lambda_n\Xi_n\varphi_n + \lambda_n\sigma_n + \left|\Xi_n\right|\tau_n.
\end{aligned} \tag{50}$$

Then, according to Lemma 1, we can get the following inequality

$$\left|\Xi_n\right|\tau_n \le \frac{\tau_n\Xi_n^2}{\sqrt{\Xi_n^2 + \sigma_n^2}} + \tau_n\sigma_n. \tag{51}$$

In light of (51), the (50) can be rewritten as

$$\frac{1}{\underline{g}_n}\Xi_n\left(\mathrm{H}_n + \frac{1}{2}\varsigma_n\right) \le \lambda_n\Xi_n\varphi_n + \frac{\tau_n\Xi_n^2}{\sqrt{\Xi_n^2 + \sigma_n^2}} + \lambda_n\sigma_n + \tau_n\sigma_n. \tag{52}$$

Substituting (52) into (48) and simplify to get

$$\begin{aligned}
LV_n \le &-\sum_{j=1}^{n-1} r_j\frac{z_j^4}{k_{b,j}^4 - z_j^4} + \sum_{j=1}^{n-1}\sigma_j\tilde{\lambda}_j\hat{\lambda}_j + \sum_{j=1}^{n-1}\sigma_j\tilde{\tau}_j\hat{\tau}_j + \sum_{j=1}^{n-1}\sigma_j\left(\lambda_j + \tau_j + \frac{1}{2\underline{g}_j}\right) \\
&+ \frac{g_n}{\underline{g}_n}\Xi_n u + \hat{\lambda}_n\Xi_n\varphi_n + \frac{\hat{\tau}_n\Xi_n^2}{\sqrt{\Xi_n^2 + \sigma_n^2}} - \tilde{\lambda}_n\left(\dot{\hat{\lambda}}_n - \Xi_n\varphi_n\right) - \tilde{\tau}_n\left(\dot{\hat{\tau}}_n - \frac{\Xi_n^2}{\sqrt{\Xi_n^2 + \sigma_n^2}}\right) + \sigma_n\left(\lambda_n + \tau_n + \frac{1}{2\underline{g}_n}\right).
\end{aligned} \tag{53}$$

Through the above analysis, designing the event-triggered controller ϖ, the virtual control signal α_n and the adaptive laws $\dot{\hat{\lambda}}_n$, $\dot{\hat{\tau}}_n$ as follows

$$\varpi(t) = \alpha_n - \overline{M}\tanh\left(\frac{\Xi_n \overline{M}}{\sigma_n}\right), \tag{54a}$$

$$\alpha_n = -r_n z_n - \hat{\lambda}_n \varphi_n - \frac{\hat{\tau}_n \Xi_n}{\sqrt{\Xi_n^2 + \sigma_n^2}}, \tag{54b}$$

$$\dot{\hat{\lambda}}_n = \Xi_n \varphi_n - \sigma_n \hat{\lambda}_n, \tag{54c}$$

$$\dot{\hat{\tau}}_n = \frac{\Xi_n^2}{\sqrt{\Xi_n^2 + \sigma_n^2}} - \sigma_n \hat{\tau}_n, \tag{54d}$$

where r_n and $\overline{M} \geq \vec{e}$ are designed as positive constants.

Remark 5. It is worth noting that the controller (54a) is designed based on ETC, which can ensure the realization of the control objectives of the system, and has the advantages of saving the communication resources and alleviating the computational burden. In addition, to ensure that the system (1) achieves ATC performance, in (54a), a positive integral function σ_n is introduced into the design of the event-triggered controller.

Remark 6. According to (10), it concluded that $|\varpi(t) - u(t)| < M$ or $|\varpi(t) - u(t)| < \chi Q + M_1$ is established in time interval $t \in [t_k, t_{k+1})$. This together with (11), it can be further obtained that $|\varpi(t) - u(t)| < \vec{e}$. Therefore, there exists a continuous time-varying parameter $\hbar(t)$, satisfying $|\hbar(t)| < 1$, $\forall t \in [t_k, t_{k+1})$, such that $u(t) = \varpi(t) - \hbar(t)\vec{e}$.

Remark 7. From (13) and (14), $\hat{\lambda}_n$ and $\hat{\tau}_n$ are estimates of λ_n and τ_n, respectively, so they are also greater than zero. And according to the definition of φ_n in Lemma 5, we can get $\Xi_n \varphi_n \geq 0$. Therefore, we can further deduce that $\frac{g_n}{\underline{g}_n}\Xi_n \alpha_n \leq 0$, and thus the inequality $\frac{g_n}{\underline{g}_n}\Xi_n \alpha_n \leq \Xi_n \alpha_n$ holds.

Then, substituting (54) into (53), the following inequality is holds

$$\begin{aligned} LV_n \leq & -\sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4} - \frac{g_n}{\underline{g}_n}\Xi_n \overline{M}\tanh\left(\frac{\Xi_n \overline{M}}{\sigma_n}\right) - \frac{g_n}{\underline{g}_n}\Xi_n \hbar(t)\vec{e} + \sum_{j=1}^{n}\sigma_j \tilde{\lambda}_j \hat{\lambda}_j + \sum_{j=1}^{n}\sigma_j \tilde{\tau}_j \hat{\tau}_j \\ & + \sum_{j=1}^{n-1}\sigma_j\left(\lambda_j + \tau_j + \frac{1}{2\underline{g}_j}\right) + \sigma_n\left(\lambda_n + \tau_n + \frac{1}{2\underline{g}_n}\right). \end{aligned} \tag{55}$$

According to applying Lemma 3, the following inequality is easily obtained

$$LV_n \leq -\sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4} + \sum_{j=1}^{n}\sigma_j \tilde{\lambda}_j \hat{\lambda}_j + \sum_{j=1}^{n}\sigma_j \tilde{\tau}_j \hat{\tau}_j + \sum_{j=1}^{n-1}\sigma_j\left(\lambda_j + \tau_j + \frac{1}{2\underline{g}_j}\right) + \sigma_n\left(\lambda_n + \tau_n + \frac{1}{2\underline{g}_n} + 0.2785\frac{g_n}{\underline{g}_n}\right). \tag{56}$$

In addition, the following two inequalities hold

$$\tilde{\lambda}_j \hat{\lambda}_j = \tilde{\lambda}_j\left(\lambda_j - \tilde{\lambda}_j\right) = -\tilde{\lambda}_j^2 + \tilde{\lambda}_j \lambda_j \leq \frac{\lambda_j^2}{4}, \tag{57}$$

$$\tilde{\tau}_j \hat{\tau}_j = \tilde{\tau}_j\left(\tau_j - \tilde{\tau}_j\right) = -\tilde{\tau}_j^2 + \tilde{\tau}_j \tau_j \leq \frac{\tau_j^2}{4}. \tag{58}$$

Substituting (57) and (58) into (56), we have

$$\begin{aligned} LV_n \leq & -\sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4} + \sum_{j=1}^{n}\frac{\lambda_j^2 + \tau_j^2}{4}\sigma_j + \sum_{j=1}^{n-1}\sigma_j\left(\lambda_j + \tau_j + \frac{1}{2\underline{g}_j}\right) + \sigma_n\left(\lambda_n + \tau_n + \frac{1}{2\underline{g}_n} + 0.2785\frac{g_n}{\underline{g}_n}\right) \\ \triangleq & -\sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4} + \sum_{j=1}^{n}\beta_j \sigma_j, \end{aligned} \tag{59}$$

where $\beta_j = \frac{\lambda_j^2 + \tau_j^2}{4} + \lambda_j + \tau_j + \frac{1}{2\underline{g}_j}, j = 2, \ldots, n-1$, $\beta_n = \frac{\lambda_n^2 + \tau_n^2}{4} + \lambda_n + \tau_n + \frac{1}{2\underline{g}_n} + 0.2785\frac{g_n}{\underline{g}_n}$.

Remark 8. It can be seen from References 12 and 13 that the MTN has the characteristics of simple structure and small calculation amount, so the event-triggered adaptive MTN asymptotic controller constructed in this article has the advantage of low computational complexity, and is more suitable for microprocessors with low computing abilities.

Remark 9. During the design process of control strategies, the $\frac{1}{4\underline{g}_i}$ term is added to construct Lyapunov function to remove the upper bound of the UCDs. By constructing smooth functions φ_i, ς_i and introducing the positive integrable function σ_i, the controller (54a) can ensure the ATC performance of stochastic nonlinear system (1). In addition, the effect of stochastic disturbances is eliminated by introducing boundary estimation method and defining adaptive variable τ_j.

3.2 | Stability analysis

Theorem 1. *Under the Assumptions 1 and 2, consider the stochastic nonlinear system (1) with state constraints. For any bounded initial condition, the proposed the BLFs-based event-triggered adaptive MTN asymptotic controller in (54a), the virtual control signals in (28a), (42a), (54b) with the adaptive laws in (28b), (28c), (42b), (42c), (54c), (54d) can ensure that*

(1) All closed-loop signals are bounded on $[0, +\infty)$.
(2) The specified state constraints of the system are not violated.
(3) The tracking error asymptotically converges to zero.
(4) The proposed controller excludes Zeno phenomenon.

Proof. According to Lemma 4 and inequality (3), integrating (59) with respect to time t from t_0 to t yields

$$\begin{aligned} 0 &\le E\left(V_n\left(x\left(o_\kappa \wedge t\right)\right)\right) \\ &\le V_n\left(x\left(t_0\right)\right) + \int_{t_0}^{o_\kappa \wedge t} \sum_{j=1}^{n} \beta_j \sigma_j(s)\, ds - E\int_{t_0}^{o_\kappa \wedge t} \sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4} ds \\ &\le V_n\left(x\left(t_0\right)\right) + \sum_{j=1}^{n} \beta_j \overline{\sigma}_j, \end{aligned} \tag{60}$$

where $o_\kappa \wedge t$ represents the minimum of o_κ and t, and $o_\kappa = \inf\{t \ge t_0, \kappa \ge 0 : |x(t)| \ge \kappa\}$.

First, from the definition of V_n in (45), we have

$$\inf_{|x| \ge R} V_n(x) \to \infty \ \ as \ \ R \to \infty. \tag{61}$$

Then, by applying Lemma 2 in Reference 46, so one can get that the system (1) has a bounded and unique solution on $[0, +\infty)$. Therefore, according to the above analysis, we know that z_j, $\tilde{\lambda}_j$ and $\tilde{\tau}_j, j = 1, 2, \dots, n$ are bounded on $[0, +\infty)$.

Second, in view of $\tilde{\lambda}_i = \lambda_i - \hat{\lambda}_i$ and $\tilde{\tau}_i = \tau_i - \hat{\tau}_i$, it can be seen that $\hat{\lambda}_i$ and $\hat{\tau}_i$ are bounded on $[0, +\infty)$ because of λ_i and τ_i are both constants. From $z_1 = x_1 - y_d$ and $|y_d| \le \mathrm{A}_1$, we have $|x_1| \le |z_1| + |y_d| < k_{c,1} - \mathrm{A}_1 + \mathrm{A}_1 = k_{c,1}$. Since $z_2 = x_2 - \alpha_1$ and $|\alpha_1| \le \mathrm{A}_2$, we have $|x_2| \le |z_2| + |\alpha_1| < k_{c,2} - \mathrm{A}_2 + \mathrm{A}_2 = k_{c,2}$. Similarly, we can further infer that $|x_i| \le |z_i| + |\alpha_{i-1}| < k_{c,i} - \mathrm{A}_i + \mathrm{A}_i = k_{c,i}, i = 3, \dots, n$. Finally, the boundedness of controller $\varpi(t)$ and system input u can be ensured are bounded on $[0, +\infty)$ through (54a) and (9). Therefore, all closed-loop signals are bounded on $[0, +\infty)$ and all the specified state constraints of the system are not violated.

Third, according to (60), the following inequality can be obtained

$$E\int_{t_0}^{o_\kappa \wedge t} \sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4} ds \le V_n\left(x\left(t_0\right)\right) + \sum_{j=1}^{n} \beta_j \overline{\sigma}_j. \tag{62}$$

Note that $\lim_{\kappa\to\infty}\lim_{t\to\infty}(o_\kappa \wedge t) = \infty$, by applying Fatou's lemma, the following inequality holds

$$E\int_{t_0}^{\infty}\sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4} ds \le V_n(x(t_0)) + \sum_{j=1}^{n}\beta_j\overline{\sigma}_j$$
$$< +\infty. \tag{63}$$

From (63), it can be seen that $\sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4}$ is continuous and bounded. Besides, since $|z_i| < k_{b,i}$, we know that $\sum_{j=1}^{n} r_j \frac{z_j^4}{k_{b,j}^4 - z_j^4} > 0$. Then, according to stochastic Barbalat lemma in Reference 46, we can get

$$\mathrm{P}\left(\lim_{t\to\infty}|z_j(t)| = 0\right) = 1. \tag{64}$$

Therefore, the tracking error asymptotically converges to zero.

Finally, we need to prove that the event-triggered controller (54a) excludes Zeno phenomenon. Therefore, we need to prove that for $\forall k \in z^+$, there exists a time constant $t^* > 0$ such that $\{t_{k+1} - t_k\} \ge t^*$. Based on $\mathrm{e}(t) = \varpi(t) - u(t)$, we can get

$$\frac{d\mathrm{e}(t)}{dt} \le |\dot{\mathrm{e}}(t)| = |\dot{\varpi}(t)|. \tag{65}$$

From (54a), $\dot{\varpi}(t)$ is a continuous function composed of α_n, z_n, and σ_n. Since α_n, z_n, and σ_n are both bounded, so there is a constant $\mu > 0$ such that $|\dot{\varpi}(t)| \le \mu$. Besides, from $\mathrm{e}(t_k) = 0$ and $\lim_{t\to t_{k+1}}\mathrm{e}(t) = M$ or $\lim_{t\to t_{k+1}}\mathrm{e}(t) = \chi|u(t)| + M_1$, thus we can further infer that the lower bound of execution intervals t^* must satisfy $t^* \ge \frac{M}{\mu}$ or $t^* \ge \frac{\chi|u(t)|+M_1}{\mu} \ge \frac{M_1}{\mu}$. Thus, the controller excludes Zeno phenomenon.

To sum up, Theorem 1 is proved. ■

Remark 10. The Zeno phenomenon refers to the infinite number of triggers occurring within a finite time in the ETC, which not only leads to the inability to guarantee convergence of the system but also defeats the purpose of reducing the number of triggers.

4 SIMULATION RESULTS

Three simulation examples of stochastic nonlinear systems are given in this section to verify the feasibility of the developed control scheme.

Example 1 (Numerical example). Consider a numerical example of a second-order stochastic nonlinear system, which is described as follows

$$\begin{cases} dx_1 = (x_2 - 0.2x_1)\,dt + 0.1x_1\cos(x_2)\,d\boldsymbol{\omega} \\ dx_2 = (u - 2x_2)\,dt + (1 - \cos(x_1))\,d\boldsymbol{\omega} \\ \quad y = x_1. \end{cases} \tag{66}$$

According to Theorem 1, the control strategy of the system (66) can be designed as follows

$$\alpha_i = -r_i z_i - \hat{\lambda}_i\varphi_i - \frac{\hat{\tau}_i\Xi_i}{\sqrt{\Xi_i^2 + \sigma_i^2}}, i = 1, 2, \tag{67}$$

$$\dot{\hat{\lambda}}_i = \Xi_i\varphi_i - \sigma_i\hat{\lambda}_i, i = 1, 2, \tag{68}$$

$$\dot{\hat{\tau}}_i = \frac{\Xi_i^2}{\sqrt{\Xi_i^2 + \sigma_i^2}} - \sigma_i\hat{\tau}_i, i = 1, 2, \tag{69}$$

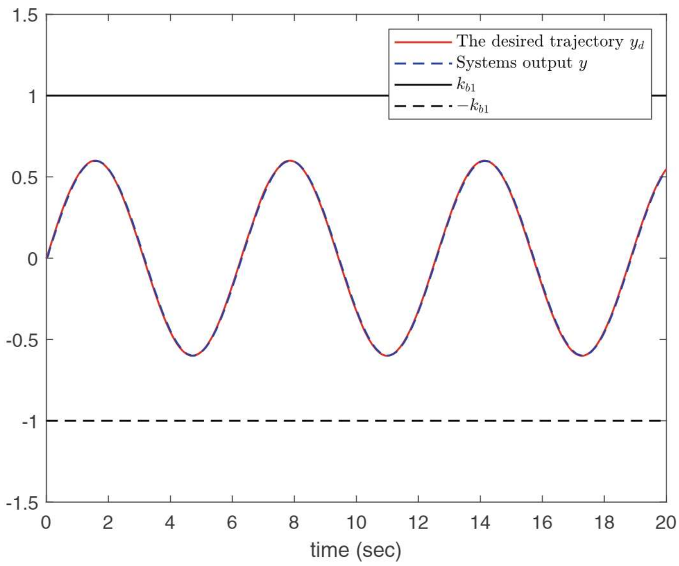

FIGURE 1 The curves of y_d and y.

$$\varpi(t) = \alpha_2 - \overline{M}\tanh\left(\frac{\Xi_2\overline{M}}{\sigma_2}\right), \tag{70}$$

where $\varphi_i = \frac{\Xi_i S_{m_i}^T S_{m_i}}{\sqrt{\Xi_i^2 S_{m_i}^T S_{m_i} + \sigma_i^2}}, i = 1, 2.$

The parameters are chosen as $r_1 = 55$, $r_2 = 50$, $k_{b,1} = 1$, $k_{b,2} = 1.9$, $\sigma_1 = 4e^{-0.2t}$, $\sigma_2 = 2e^{-0.25t}$, $\overline{M} = 13$, $[x_1(0), x_2(0)]^{\mathrm{T}} = [0.01, 0.01]^{\mathrm{T}}$, $[\lambda_1(0), \lambda_2(0)]^{\mathrm{T}} = [5, 5]^{\mathrm{T}}$, $[\tau_1(0), \tau_2(0)]^{\mathrm{T}} = [4, 4]^{\mathrm{T}}$, and $y_d = 0.6\sin(t)$. The parameters of the ETC strategy are designed as $M = 10$, $M_1 = 1$, $\chi = 0.5$, and $Q = 20$. Simulation results are shown in Figures 1–5.

Figure 1 shows the curves of the reference signal y_d and the system output y. Figure 2 depicts the state curve x_2 and the tracking error $y - y_d$. From Figures 1 and 2, it can be seen that the asymptotic tracking performance of the system (66) is achieved and the state constraints are not violated. In addition, Figure 3 describes the controller output ϖ and control input u, which demonstrates that the FTS is adopted to maintain the asymptotic tracking performance of the system when the input $u > 20$, otherwise switch to the RTS to obtain more precise control. Figure 4 displays the boundedness of adaptive parameters $\hat{\lambda}_i, i = 1, 2$ and $\hat{\tau}_i, i = 1, 2$. All the trigger moments and time intervals are shown in Figure 5, which shows that the proposed controller can avoid Zeno phenomenon. According to the above analysis of Figures 1–5, the control objectives are achieved by the controller designed in this article.

Example 2 (Practical example). A spring-mass-damper system is considered to further verify the effectiveness of the proposed control scheme, which can be governed by the following stochastic nonlinear system[47]

$$\begin{cases} dx_1 = x_2 dt + (1 - \cos(x_1))\, d\boldsymbol{\omega} \\ dx_2 = \left(\frac{1}{m_1}u - \frac{K}{m_1}x_1 - \frac{C}{m_1}x_2\right) dt + \sin(x_2)\, d\boldsymbol{\omega} \\ \quad y = x_1 \end{cases} \tag{71}$$

with the parameters of system are chosen as $m_1 = 1$ kg, $K = 0.3$ N/m, $C = 0.5$ Ns/m.

Similarly, the control strategy of the system (71) can be designed as (67)–(70).

In simulation, the parameters are chosen as $r_1 = 30$, $r_2 = 55$, $k_{b,1} = 1$, $k_{b,2} = 1.5$, $\sigma_1 = 4e^{-0.3t}$, $\sigma_2 = 2e^{-0.2t}$, $\overline{M} = 10$, $[x_1(0), x_2(0)]^{\mathrm{T}} = [0.01, 0.01]^{\mathrm{T}}$, $[\lambda_1(0), \lambda_2(0)]^{\mathrm{T}} = [6, 6]^{\mathrm{T}}$, $[\tau_1(0), \tau_2(0)]^{\mathrm{T}} = [3, 3]^{\mathrm{T}}$, and $y_d = 0.5\sin(t)$. The parameters of the ETC strategy are designed as $M = 8$, $M_1 = 1$, $\chi = 0.5$, and $Q = 10$.

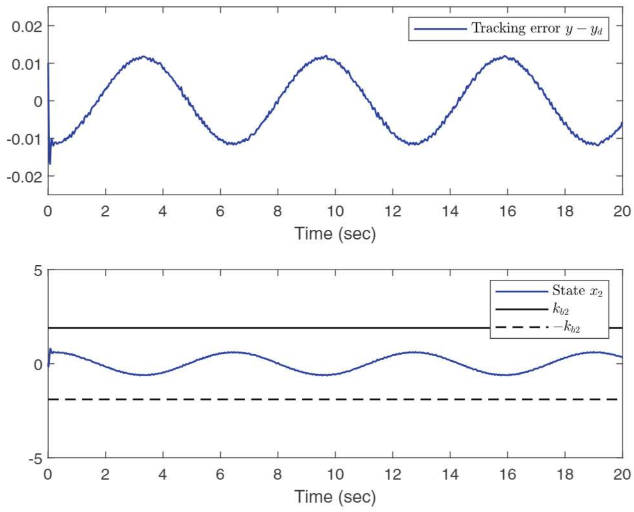

FIGURE 2 The curves tracking error $y - y_d$ and state x_2.

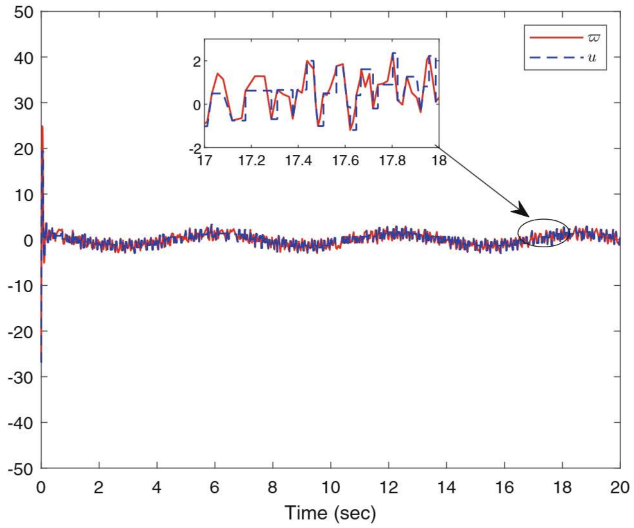

FIGURE 3 The curves of controller output ϖ and control input u.

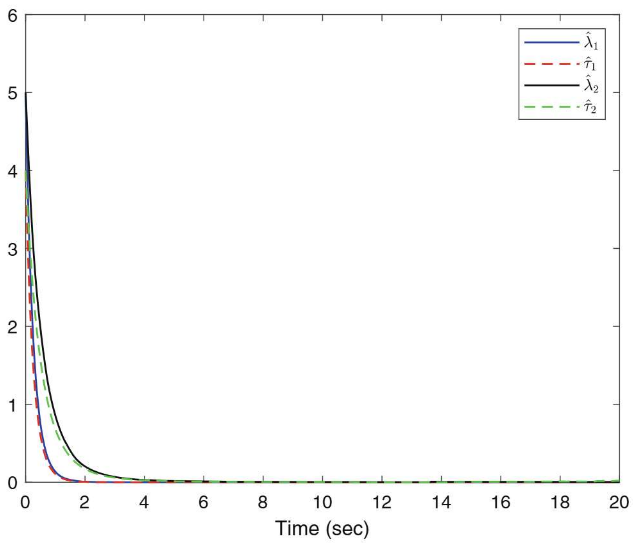

FIGURE 4 The adaptive parameters $\hat{\lambda}_i, i = 1, 2$ and $\hat{\tau}_i, i = 1, 2$.

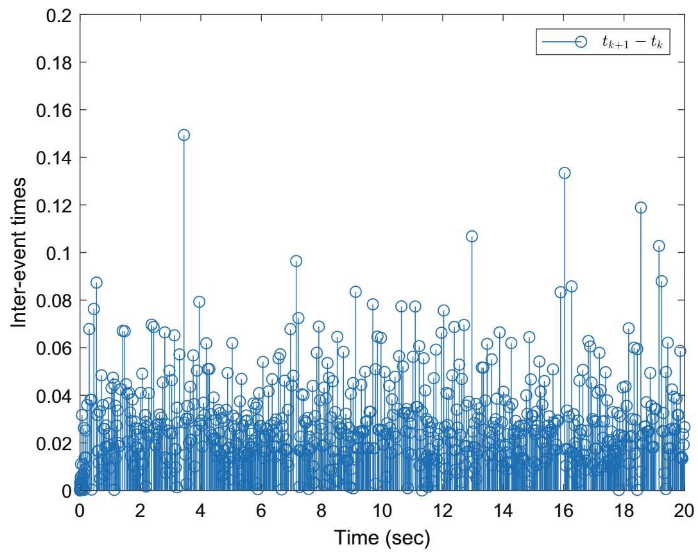

FIGURE 5 The trigger moments and time intervals.

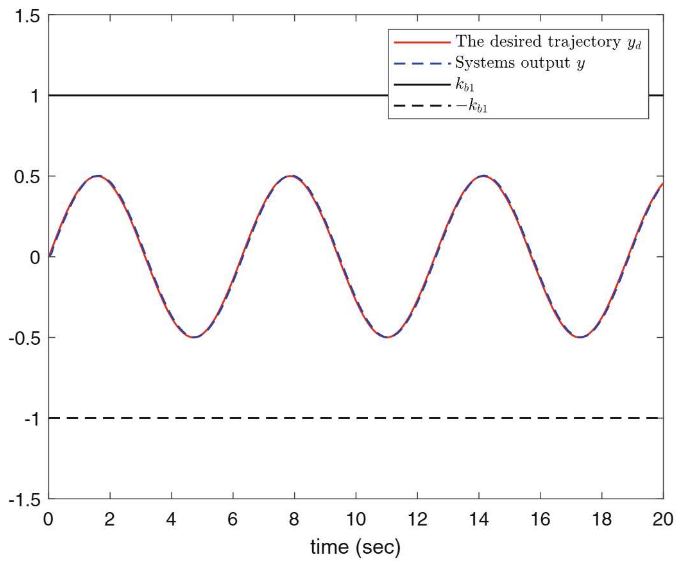

FIGURE 6 The curves of y_d and y.

The control results of the spring-mass-damper system (71) are shown in Figures 6–10, and they can prove that the controller proposed in this article not only ensures that the state constraints are not violated but also realizes ATC performance under the premise of saving communication resources. Therefore, the effectiveness of the event-triggered adaptive ATC scheme proposed in this article is further demonstrated its value in actual applications.

Remark 11. In theory, a satisfactory tracking performance can be achieved by arbitrarily increasing r_i or decreasing σ_i. In addition, it can be seen from (28a), (42a), and (54b) that increasing r_i may increase the amplitude of the control signals, which will lead to the decline of the system control performance. Therefore, the design parameters should be chosen appropriate to achieve the balance between control performance and tracking performance.

Example 3. To reveal more advantages of the proposed method, we compare it with the RTS-based ETC strategy in References 30-32 and the FTS-based ETC strategy in Reference 21 for the system (66), respectively. The comparison results are shown in Figures 11 and 12

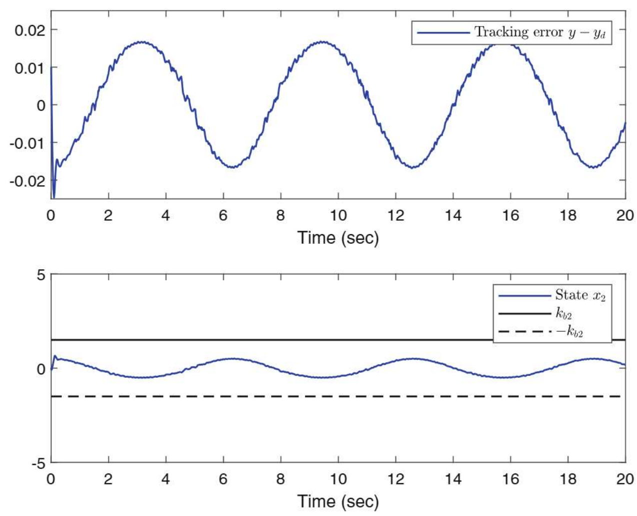

FIGURE 7 The curves tracking error $y - y_d$ and state x_2.

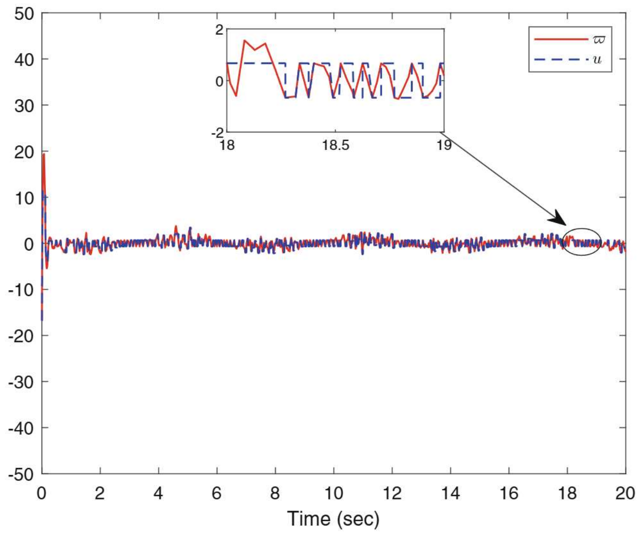

FIGURE 8 The curves of controller output ϖ and control input u.

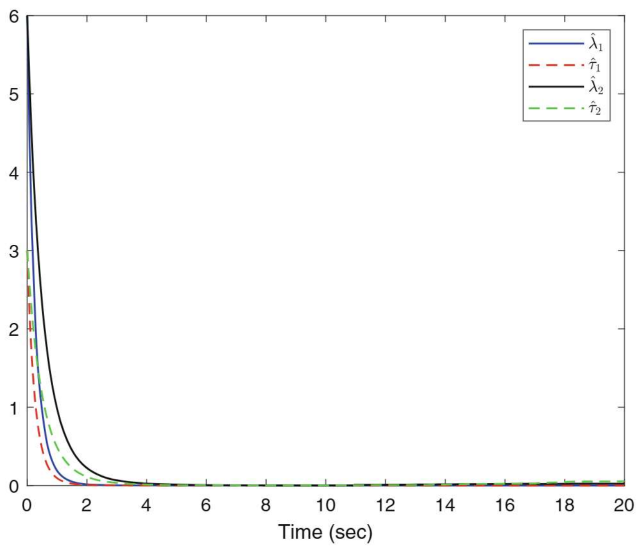

FIGURE 9 The adaptive parameters $\hat{\lambda}_i, i = 1, 2$ and $\hat{\tau}_i, i = 1, 2$.

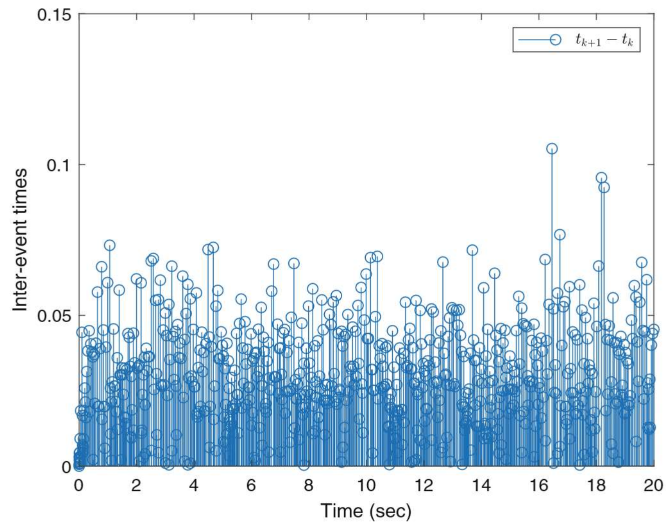

FIGURE 10 The trigger moments and time intervals.

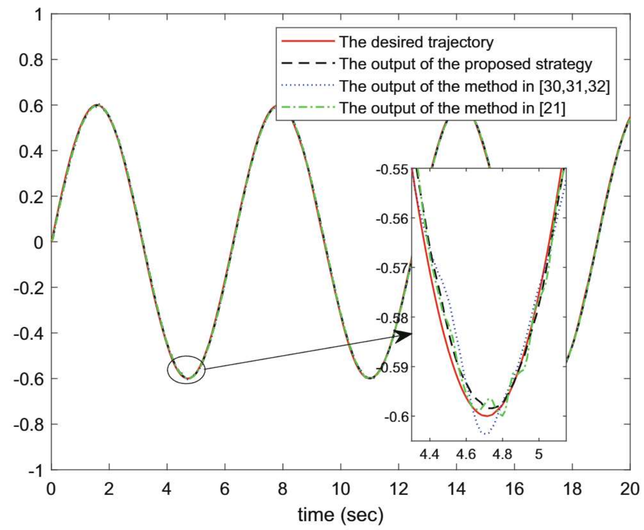

FIGURE 11 Comparisons of tracking trajectories of three ETC strategies.

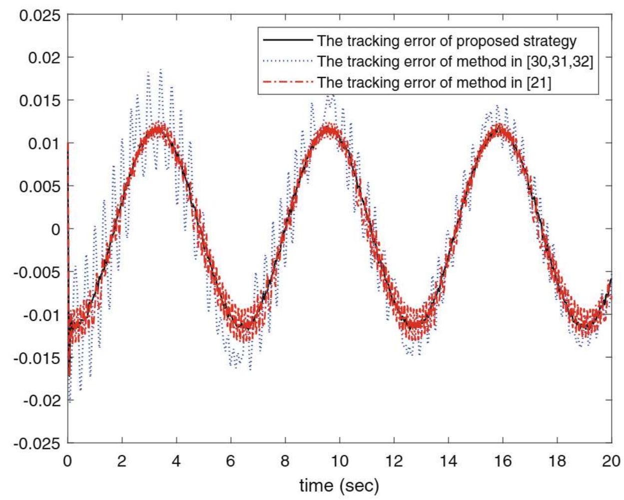

FIGURE 12 Comparisons of tracking error $y - y_d$ of three ETC strategies.

Figures 11 and 12 show the tracking performance and the tracking error of three ETC strategies, respectively. The simulation results show that the proposed ETC method achieves a relatively better asymptotic tracking performance, and the tracking error has a smaller amplitude. Therefore, we can conclude that the proposed ETC method has significant advantages in protecting the system from large pulses while improving its asymptotic tracking performance.

5 | CONCLUSION

This article is devoted to solving the event-triggered ATC problem for stochastic nonlinear systems with UCDs and full-state constraints. By constructing the new BLFs and introducing the boundary estimation method, a novel event-triggered adaptive MTN ATC scheme based on STS is proposed to handle the presence of UCDs and state constraints arising in stochastic nonlinear systems, such that the perfect asymptotic tracking performance and control performance are achieved, and the communication channel bandwidth and the computing abilities are successfully relieved. The simulation results not only prove the effectiveness of the proposed event-triggered adaptive controller based on STS, but also show the superiority of the proposed controller in asymptotic tracking performance compared with those designed based on FTS and RTS. The future work is to study the application of the proposed control scheme in stochastic nonlinear systems with unknown hysteresis to achieve ATC performance.

CONFLICT OF INTEREST STATEMENT

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

ORCID

Yang Du https://orcid.org/0000-0001-9503-6433
Lian-Lian Zhai https://orcid.org/0000-0002-9844-2506
Yu-Qun Han https://orcid.org/0000-0002-9055-2954

REFERENCES

1. Wu ZJ, Xie XJ, Zhang SY. Adaptive backstepping controller design using stochastic small-gain theorem. *Automatica*. 2007;43(4): 608-620.
2. Wang XJ, Cao Y, Niu B, Song YD. A novel bipartite consensus tracking control for multiagent systems under sensor deception attacks. *IEEE Trans Cybern*. 2022. doi:10.1109/TCYB.2022.3225361
3. Li GF, Tian Y, Chen YH. Adaptive robust control for a class of stochastic nonlinear uncertain systems. *IEEE Access*. 2020;8:51610-51620.
4. Yao LN, Li LF, Guan YC, Wang H. Fault diagnosis and fault-tolerant control for non-Gaussian nonlinear stochastic systems via entropy optimisation. *Int J Syst Sci*. 2019;50(21):1-13.
5. Qi WH, Zong GD, Karimi HR. Sliding mode control for nonlinear stochastic singular semi-Markov jump systems. *IEEE Trans Automat Contr*. 2020;65(1):361-368.
6. Wu J, Chen XM, Zhao QJ, Li J, Wu ZG. Adaptive neural dynamic surface control with prespecified tracking accuracy of uncertain stochastic nonstrict-feedback systems. *IEEE Trans Cybern*. 2022;52(5):3408-3421.
7. Yang XL, Li J, Wu SY, Li XB. Adaptive NN prescribed performance control design for uncertain switching nonlinear systems with periodically time-varying parameters. *Neural Comput Appl*. 2022;34(1):617-629.
8. Zhou B, Yang L, Wang CD, Lai GY, Chen Y. Adaptive finite-time tracking control of robot manipulators with multiple uncertainties based on a low-cost neural approximator. *J Franklin Inst*. 2022;359(10):4938-4958.
9. Wang XJ, Niu B, Wang HQ, Zhao XD, Chen WD. Prescribed performance-based finite-time consensus technology of nonlinear multiagent systems and application to FDPs. *IEEE Trans Circuits Syst II Express Briefs*. 2023;70(2):591-595.
10. You ZY, Wang F. Adaptive fast finite-time fuzzy control of stochastic nonlinear systems. *IEEE Trans Fuzzy Syst*. 2022;30(7):2279-2288.
11. Kang SJ, Liu PX, Wang HQ. Command filter-based adaptive fuzzy decentralized control for large-scale nonlinear systems. *Nonlinear Dyn*. 2021;105(4):3239-3253.
12. Han YQ, Yan HS. Adaptive multi-dimensional Taylor network tracking control for SISO uncertain stochastic non-linear systems. *IET Control Theory Appl*. 2018;12(8):1107-1115.

13. Yan HS, Han YQ. Decentralized adaptive multidimensional Taylor network tracking control for a class of large-scale stochastic nonlinear systems. *Int J Adapt Control Signal Process*. 2019;33(4):664-683.
14. Han YQ. Adaptive tracking control for a class of stochastic nonlinear systems with input saturation constraint using multi-dimensional Taylor network. *IET Control Theory Appl*. 2020;14(9):1193-1199.
15. Wang MX, Zhu SL, Liu SM, Du Y, Han YQ. Design of adaptive finite-time fault-tolerant controller for stochastic nonlinear systems with multiple faults. *IEEE Trans Autom Sci Eng*. 2022. doi:10.1109/TASE.2022.3206328
16. Zhu SL, Wang MX, Han YQ. Adaptive multi-dimensional Taylor network control for nonlinear stochastic systems with time-delay. *Proc Inst Mech Eng I*. 2022;236(3):579-591.
17. Xing LT, Wen CY, Liu ZT, Su HY, Cai JP. Event-triggered adaptive control for a class of uncertain nonlinear systems. *IEEE Trans Automat Contr*. 2017;62(4):2071-2076.
18. Xing LT, Wen CY, Liu ZT, Su HY, Cai JP. Event-triggered output feedback control for a class of uncertain nonlinear systems. *IEEE Trans Automat Contr*. 2019;64(1):290-297.
19. Wang HQ, Xu K, Qiu JB. Event-triggered adaptive fuzzy fixed-time tracking control for a class of nonstrict-feedback nonlinear systems. *IEEE Trans Circuits Syst I Regul Pap*. 2021;68(7):3058-3068.
20. Liu Z, Wang JH, Chen CLP, Zhang Y. Event trigger fuzzy adaptive compensation control of uncertain stochastic nonlinear systems with actuator failures. *IEEE Trans Fuzzy Syst*. 2018;26(6):3770-3781.
21. Tabuada P. Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Trans Automat Contr*. 2007;52(9):1680-1685.
22. Wang LQ, Dong JX. Adaptive fuzzy consensus tracking control for uncertain fractional-order multiagent systems with event-triggered input. *IEEE Trans Fuzzy Syst*. 2020;30(2):310-320.
23. Zhao GL, Wang YF, Fu XW. Hybrid event-triggered consensus tracking of multi-agent systems with discrete control update. *IEEE Trans Circuits Syst II Express Briefs*. 2022;69(4):2206-2210.
24. Zhang L, Zong GD, Zhao XD, Zhao N. Output reachable set synthesis of event-triggered control for singular Markov jump systems under multiple cyber-attacks. *IEEE/ACM Trans Netw*. 2022;30(6):2849-2857.
25. Ran GT, Shu Z, Lam HK, Liu J, Li CJ. Dissipative tracking control of nonlinear markov jump systems with incomplete transition probabilities: a multiple-event-triggered approach. *IEEE Trans Fuzzy Syst*. 2022;1-11. doi:10.1109/TFUZZ.2022.3225672
26. Xu WQ, Liu XP, Wang HQ, Zhou YC. Event-triggered adaptive NN tracking control for MIMO nonlinear discrete-time systems. *IEEE Trans Neural Netw Learn Syst*. 2022;33(12):7414-7424.
27. Pan HH, Zhang D, Sun WC, Yu XH. Event-triggered adaptive asymptotic tracking control of uncertain MIMO nonlinear systems with actuator faults. *IEEE Trans Cybern*. 2022;52(9):8655-8667.
28. Zhu CL, Liu R, Li BM, Xia JW, Zhang N. Neural network-based event-triggered adaptive asymptotic tracking control for switched nonlinear systems. *Int J Control Autom Syst*. 2022;20(6):2021-2031.
29. Wu LB, Park JH, Xie XP, Liu YJ, Yang ZC. Event-triggered adaptive asymptotic tracking control of uncertain nonlinear systems with unknown dead-zone constraints. *Appl Math Comput*. 2020;386:125528.
30. Hu XY, Li YX, Tong SC, Hou ZS. Event-triggered adaptive fuzzy asymptotic tracking control of nonlinear pure-feedback systems with prescribed performance. *IEEE Trans Cybern*. 2021;53:2380-2390. doi:10.1109/TCYB.2021.3118835
31. Hu XY, Li YX, Hou ZS, Niu B. Event-triggered prescribed performance adaptive fuzzy asymptotic tracking of nonstrict-feedback nonlinear systems. *Int J Robust Nonlinear Control*. 2021;31(12):5776-5795.
32. Li YX, Hu XY, Che WW, Hou ZS. Event-based adaptive fuzzy asymptotic tracking control of uncertain nonlinear systems. *IEEE Trans Fuzzy Syst*. 2021;29(10):3003-3013.
33. Tang ZL, Ge SS, Tee KP, He W. Robust adaptive neural tracking control for a class of perturbed uncertain nonlinear systems with state constraints. *IEEE Trans Syst Man Cybern Syst*. 2016;46(12):1618-1629.
34. Zhao W, Liu YJ, Liu L. Observer-based adaptive fuzzy tracking control using integral barrier Lyapunov functionals for a nonlinear system with full state constraints. *IEEE/CAA J Autom Sin*. 2021;8(3):617-627.
35. Wang CX, Wu YQ, Yu JB. Barrier Lyapunov functions-based dynamic surface control for pure-feedback systems with full state constraints. *IET Control Theory Appl*. 2017;11(4):524-530.
36. Liu YJ, Tong SC. Barrier Lyapunov functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints. *Automatica*. 2016;64:70-75.
37. Liu YJ, Ma L, Liu L, Tong SC, Chen CLP. Adaptive neural network learning controller design for a class of nonlinear systems with time-varying state constraints. *IEEE Trans Neural Netw Learn Syst*. 2020;31(1):66-75.
38. Li YX. Barrier Lyapunov function-based adaptive asymptotic tracking of nonlinear systems with unknown virtual control coefficients. *Automatica*. 2020;121:109181.
39. Jin X, Li YX, Tong SC. Adaptive event-triggered control design for nonlinear systems with full state constraints. *IEEE Trans Fuzzy Syst*. 2021;29(12):3803-3811.
40. Tang L, Yang MY, Liu YJ, Tong SC. Adaptive output feedback fuzzy fault-tolerant control for nonlinear full-state-constrained switched systems. *IEEE Trans Cybern*. 2021;53:2325-2334. doi:10.1109/TCYB.2021.3116950
41. Liao XM, Liu Z, Philip Chen CL, Zhang Y. Event-triggered fuzzy control for nonlinear time-delay system with full-state constraints and unknown hysteresis. *J Franklin Inst*. 2022;359(4):1582-1611.
42. Wu Y, Xie XJ, Hou ZG. Adaptive fuzzy asymptotic tracking control of state-constrained high-order nonlinear time-delay systems and its applications. *IEEE Trans Cybern*. 2022;52(3):1671-1680.

43. Gao TT, Liu YJ, Li DP, Tong SC, Li TS. Adaptive neural control using tangent time-varying BLFs for a class of uncertain stochastic nonlinear systems with full state constraints. *IEEE Trans Cybern*. 2021;51(4):1943-1953.
44. Han YQ. Adaptive control of a class of stochastic nonlinear systems with full state constraints and input saturation using multi-dimensional Taylor network. *Asian J Control*. 2022;24(4):1609-1621.
45. Li N, Han YQ, He WJ, Zhu SL. Control design for stochastic nonlinear systems with full-state constraints and input delay: a new adaptive approximation method. *Int J Control Autom Syst*. 2022;20(8):2768-2778.
46. Yu X, Xie XJ. Output feedback regulation of stochastic nonlinear systems with stochastic iISS inverse dynamics. *IEEE Trans Automat Contr*. 2010;55(2):304-320.
47. Sui S, Chen CLP, Tong SC. A novel adaptive NN prescribed performance control for stochastic nonlinear systems. *IEEE Trans Neural Netw Learn Syst*. 2021;32(7):3196-3205.

How to cite this article: Du Y, Zhu S-L, Zhai L-L, Han Y-Q. Switching threshold-based event-triggered adaptive asymptotic tracking control for stochastic nonlinear systems with full-state constraints. *Int J Robust Nonlinear Control*. 2023;33(13):7908-7928. doi: 10.1002/rnc.6803

APPENDIX

Proof: According to (13), we have

$$\begin{aligned}\frac{1}{\underline{g}_i}\Xi_i \boldsymbol{W}_i^{\mathrm{T}} S_{m_i} &\leq \frac{1}{\underline{g}_i} |\Xi_i| \left\|\boldsymbol{W}_i^{\mathrm{T}}\right\| \left\|S_{m_i}\right\| \\ &= \lambda_i |\Xi_i| \left\|S_{m_i}\right\|. \end{aligned} \tag{A1}$$

By using Lemma 1, we have

$$\lambda_i |\Xi_i| \left\|S_{m_i}\right\| \leq \frac{\lambda_i \Xi_i^2 S_{m_i}^T S_{m_i}}{\sqrt{\Xi_i^2 S_{m_i}^T S_{m_i} + \sigma_i^2}} + \lambda_i \sigma_i. \tag{A2}$$

Let $\varphi_i = \frac{\Xi_i S_{m_i}^T S_{m_i}}{\sqrt{\Xi_i^2 S_{m_i}^T S_{m_i} + \sigma_i^2}}$, then the above inequality is rewritten as

$$\lambda_i |\Xi_i| \left\|S_{m_i}\right\| \leq \lambda_i \Xi_i \varphi_i + \lambda_i \sigma_i. \tag{A3}$$

Therefore, based on the above analysis, we can infer that

$$\frac{1}{\underline{g}_i}\Xi_i \boldsymbol{W}_i^{\mathrm{T}} S_{m_i} \leq \lambda_i \Xi_i \varphi_i + \lambda_i \sigma_i. \tag{A4}$$

Similarly, we can get

$$\frac{z_i^2\left(3k_{b,i}^4 + z_i^4\right)\Gamma_i}{(k_{b,i}^4 - z_i^4)^2} \leq \Xi_i \varsigma_i + \sigma_i. \tag{A5}$$

Thus, the proof of Lemma 5 is completed.