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Research article

Adaptive finite-time control for switched nonlinear systems subject to multiple objective constraints via multi-dimensional Taylor network approach[☆]

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ABSTRACT

The finite-time control of switched nonlinear systems subject to multiple objective constraints is investigated in this article. Firstly, with the aim of dealing with the major challenge brought by multiple objective constraints, the time-varying and asymmetric barrier function is designed, which transforms multiple objective constrained systems into unconstrained systems. Secondly, the dynamic surface control technique is introduced into the backstepping design process, and the error generated in the filtering process is reduced by constructing the error compensation systems. Then, an adaptive finite-time controller based on multi-dimensional Taylor network (MTN) is proposed. The controller proposed in this article can avoid the "singularity" problem and ensure that the objective functions never violate constraints. Finally, the effectiveness of the finite-time control strategy proposed in this article is verified by the aircraft system simulation.

1. Introduction

As is known to all, the parameters or structures of practical industrial systems are likely to jump due to various factors. Therefore, the concept of hybrid systems was introduced [1]. As one of the most essential branches of hybrid systems, switched systems have been made a profound study by scholars because of its simple mathematical description [2–4]. Meanwhile, the tracking control is the key issue of switched nonlinear systems research, which is also the emphasis and difficulty of research. In an effort to solve the tracking control problem of switched nonlinear systems, H∞ control [5,6], robust control [7], adaptive control [8], sliding mode control [9] and backstepping technique [10] have been adopted. Significantly, as an effective control method, the combination of backstepping technique and adaptive control was extensively used to cope with the control problems of various switched nonlinear systems [11–13]. What is more, as a fundamental issue in the study of switched nonlinear systems, stability analysis has received focused attention. With the aim of ensuring the stability of the controlled systems in case of arbitrary switching, the common Lyapunov function (CLF) was introduced into the control process [14]. For uncertain switched nonlinear systems, an adaptive CLF-based control strategy was developed in [15], which guaranteed the stability of the controlled systems under arbitrary switching. Up to now, the CLF-based adaptive backstepping control has acquired quite a few meaningful achievements on switched nonlinear systems [16–18]. On the other hand, there exist complex uncertainty and nonlinearity in the practical systems due to the modeling accuracy or parameter variation, which is a necessary issue in the controller design and stability analysis. However, the above research results cannot cope with the control and stability problems of switched nonlinear systems with complex and uncertain nonlinear structures.

As two kinds of estimators, neural networks (NNs) and fuzzy logic systems (FLSs) can effectively approximate complex and uncertain nonlinear structures in the systems. Up to now, a host of adaptive control strategies based on NNs or FLSs have been reported [19–23]. In recent years, as a kind of NN with special structures, multi-dimensional Taylor network (MTN) paves a new approach for the control issues of various systems, such as nonlinear systems [24,25], large-scale nonlinear systems [26,27], stochastic nonlinear systems [28,29], discrete-time nonlinear systems [30,31] and switched nonlinear systems [32]. Until now, however, few results have been devoted to MTN control achievements of switched nonlinear systems, which need to be further explored. In particular, it should be noted that the aforementioned researches are unacceptable in engineering applications, which require limited convergence time. Finite-time control can

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not only accelerate the convergence speed, but also reduce the convergence time, and even improve the convergence accuracy, and also has strong anti disturbance ability. Therefore, the finite-time control has been successfully applied to solve the control design issues of a sea of systems, such as nonlinear systems [33], switched nonlinear systems [34], large-scale nonlinear systems [35] and stochastic nonlinear systems [36,37]. Nevertheless, according to the current knowledge, there are few works on the finite-time control of switched nonlinear systems via MTN technique.

In practical industrial production, the controlled systems need to be constrained for the purpose of ensuring efficiency and security. For the control problem of constrained systems, a large number of control approaches were put forward, such as reference governor approach [38], extremum-seeking control approach [39], prescribed performance control approach [40], the data-driven model approach [41] and barrier Lyapunov function (BLF) approach [42]. Among them, BLF-based control is the most widely accepted method for switched constrained systems [43–45]. For switched nonlinear systems, most of the existing BLF control methods focus on working out the issues of input constraints, state constraints or output constraints, rather than multiple objective constraints. For most process and manufacturing systems, multiple objective constraints are unavoidable, such as lowest risk and highest return in venture capital projects [46], lowest transportation cost and shortest distance in vehicle driving [47], lowest energy consumption and highest productivity in manufacturing systems [48], speed and position of joints and end-effectors in the robot manipulators [49]. Therefore, it is of important practical significance to realize the control of multiple objective constrained systems. Significantly, the control issue of nonlinear systems with multiple objective constraints was discussed in [50], and an adaptive NN-based finite-time control strategy was proposed. However, in the case of arbitrary switching, there are almost no researches on switched nonlinear systems with multiple objective constraints. Therefore, it can be concluded that the finite-time control of switched nonlinear systems subject to multiple objective constraints is confronted with great challenge, which is a meaningful topic.

In summary, the finite-time control problem of switched nonlinear systems with multiple objective constraints is discussed in this article. Firstly, in order to cope with the multiple objective constraints problem, the time-varying and asymmetric barrier function is introduced. Secondly, the dynamic surface control (DSC) technique is led into the backstepping control process to avoid the "differential explosion" problem and "singularity" problem. Thirdly, the error compensation systems are constructed to reduce the error caused by the filter. Finally, based on the MTN estimation technique, an adaptive finite-time control strategy is proposed. By comparing with the existing literatures, the main contributions of this article are as follows:

(1) This article is the first remarkable work to investigate finite-time tracking control of switched systems using MTN technique. Although the MTN technique has been extended to switched nonlinear systems by [32,51], finite-time control cannot be realized. Unlike the existing MTN-based finite-time control results, such as [35,37], the controlled object of this article is switched nonlinear systems under arbitrary switching. Even though the authors of [52,53] discussed the finite-time control of switched nonlinear systems, MTN technique and multiple objective constraints were not considered.

(2) In order to serve the aim of transforming the multiple objective constrained systems into the unconstrained systems, a novel time-varying and asymmetric barrier function is designed. The barrier function constructed in [54] to cope with output constraints is extended to multiple objective constraints, which allows this article to generalize and extend the existing achievements based on special cases. Different from [55], a new DSC-based finite-time control framework is provided, which introduces the error compensation systems to reduce the error caused by the filter.

(3) This article appears to be the first work dedicated to the finite-time-based constrained control of switched nonlinear systems, which gives a new way to balance multiple objective constraints and the system performance. In each step of backstepping design process, all unknown nonlinearities are composed into an unknown nonlinear function, which is approximated by one MTN. Moreover, based on DSC, the proposed controller does not require the derivative knowledge of the virtual control signals, which makes its implementation much easier.

Notation: R^n indicates n-dimensional real space. For a vector or matrix $\boldsymbol{x}$, $\boldsymbol{x}^{\mathrm{T}}$ defines its transpose, $|\boldsymbol{x}|$ means its absolute value, and $\|\boldsymbol{x}\|$ denotes its 2-norm. For the sake of brevity, the variables in the function will be omitted, such as $\Lambda_{i,k}(t,\bar{\boldsymbol{x}}_i)$ is simplified as $\Lambda_{i,k}$.

2. Problem formulation and preliminaries

The switched nonlinear system is considered, whose mathematical model is shown below

$$\begin{cases} \dot{x}_i = x_{i+1} + l_{i,\sigma(t)}(\bar{\boldsymbol{x}}_i) + \Lambda_{i,\sigma(t)}(t,\bar{\boldsymbol{x}}_i) \\ \dot{x}_n = u + l_{n,\sigma(t)}(\bar{\boldsymbol{x}}_n) + \Lambda_{n,\sigma(t)}(t,\bar{\boldsymbol{x}}_n) \\ y = x_1 \end{cases} \tag{1}$$

where $i = 1,2,\ldots,n-1$. y and u represent the output and control input of the controlled system, respectively. $\sigma(t): R^+ \to Q = \{1,2,\ldots,q\}$ means the switching signal and q expresses the number of subsystem. For $i = 1,2,\ldots,n$, $k \in Q$, $\bar{\boldsymbol{x}}_i = [x_1,\ldots,x_i]^{\mathrm{T}} \in R^i$ denotes the state variable in the system, $l_{i,k}(\bar{\boldsymbol{x}}_i)$ indicates unknown and smooth nonlinear function, $\Lambda_{i,k}(t,\bar{\boldsymbol{x}}_i)$ is defined as the unknown time-varying disturbance.

For the switched nonlinear system (1), the control objectives of this article can be described as the following three points:

(1) All signals in the controlled system are bounded;

(2) It is expected that the desired signal y_d can be well tracked by the system output y, and the tracking error $y - y_d$ can be adjusted to a small neighborhood of origin;

(3) Multiple objective constraints, i.e. $-b_{v1}(t) < E_i(x_1) < b_{v2}(t), i = 1,\ldots,m$, are not violated. $b_{v1}(t), b_{v2}(t)$ indicate asymmetric and time-varying constraint functions, and satisfy $-b_{v1}(t) < b_{v2}(t)$. $E_i = q_1x_1 + q_2x_1^2 + \cdots + q_ix_1^i$ means ith objective function. q_i denotes the weighting coefficient. The selection of q_i needs to ensure that the objective functions are within the constraint range.

Assumption 1 (*[32]*)**.** When $t \geq 0$, for positive constants $Y_0^1, Y_0^2, Y_1,\ldots,Y_n$, the desired signal y_d and its time derivative $y_d^{(i)}, i = 1,2,\ldots,n$ satisfy the conditions as $-Y_0^1 \leq y_d \leq Y_0^2, |\dot{y}_d| < Y_1,\ldots,\left|y_d^{(n)}\right| < Y_n$.

Assumption 2 (*[56]*)**.** The time-varying disturbance $\Lambda_{i,k}(t,\bar{\boldsymbol{x}}_i)$ in system (1) satisfies the following condition

$$\Lambda_{i,k}(t,\bar{\boldsymbol{x}}_i) \leq \phi_{i,k}(\bar{\boldsymbol{x}}_i) + d_{i,k} \tag{2}$$

where $i = 1, 2, \ldots, n$, $k \in Q$, $\phi_{i,k}(\bar{\mathbf{x}}_i)$ indicates the unknown and continuous nonlinear function, and $d_{i,k}$ denotes the unknown constant.

Lemma 1 ([53]). *On the basis of full consideration of the nonlinear system $\dot{x} = f(x, u)$, if $x(0) \in x_0$, there exist a positive constant σ and a settling time $T(\sigma, x_0) < \infty$, such that $\|x(t)\| \leq \sigma$, for $\forall t \in T$. Meanwhile, there exist $0 < s < 1$, $h > 0$, $f > 0$, $0 < g < \infty$, the time derivative of the continuous Lyapunov function V satisfies the following condition*

$$\dot{V} \leq -hV - fV^s + g \tag{3}$$

Then, the trajectory of the nonlinear system $\dot{x} = f(x, u)$ is practical finite-time stable.

Lemma 2 ([26]). *For any $e > 0$, there exists one MTN of the form $\boldsymbol{\theta}^{\mathrm{T}} P_{m_n}$ that can be adopted to estimate the continuous function $L(\boldsymbol{\omega})$ on a compact set Ω_ω, whose expression is given below*

$$L(\boldsymbol{\omega}) = \boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(\boldsymbol{\omega}) + \varepsilon(\boldsymbol{\omega}), |\varepsilon(\boldsymbol{\omega})| \leq e \tag{4}$$

where $P_{m_n}(\boldsymbol{\omega}) \triangleq \left[\omega_1, \ldots, \omega_n, \omega_1^2, \ldots, \omega_n^2, \omega_1^m, \ldots, \omega_n^m\right]^{\mathrm{T}} \in R^l$ means the middle layer vector of MTN. $\boldsymbol{\omega} \triangleq [\omega_1, \omega_2, \ldots, \omega_n]^{\mathrm{T}} \in R^n$ indicates the input vector of MTN. $\varepsilon(\boldsymbol{\omega})$ is the approximate error between $\boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(\boldsymbol{\omega})$ and $L(\boldsymbol{\omega})$. $\boldsymbol{\theta}$ is the weight vector of MTN, $\boldsymbol{\theta} := \arg\min_{\omega \in R^l}\left\{\sup_{\omega \in \Omega_\omega}\left|L(\boldsymbol{\omega}) - \boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(\boldsymbol{\omega})\right|\right\} \in R^l$.

Remark 1. As a network structure similar to radial basis function neural network (RBFNN), MTN is composed of three layers: input layer, middle layer and output layer. Unlike RBFNN, in the middle layer of MTN, polynomials are adopted instead of radial basis functions. In other words, MTN can be considered as the RBFNN with special structures.

3. Adaptive MTN tracking controller design

Firstly, with the aim of achieving multiple objective constraints, the time-varying and asymmetric barrier function is introduced, and its expression takes the form

$$\xi = \frac{E + b_{c1}}{E + b_{v1}(t)} + \frac{E - b_{c2}}{b_{v2}(t) - E} \tag{5}$$

where b_{c1}, b_{c2} are designed constants, which satisfy the conditions of $b_{c1} < b_{v1}(t)$ and $b_{c2} < b_{v2}(t)$, respectively. E is a function of x_1, whose initial state $E(0)$ is in its open region. If $E \to (-b_{v1})^+$ or $E \to b_{v2}^-$, the barrier function ξ approaches infinity. When the barrier function ξ is determined to be bounded, E also satisfies the constraints. Therefore, the problem of multiple objective functions satisfying constraints is transformed into the problem of guaranteeing the boundedness of ξ.

Remark 2. The multiple objective constraints considered in this article is to ensure that the objective functions E about x_1 evolve within the prescribed boundary. Significantly, the output constraint is a special case of multiple objective constraints, namely $E = x_1$, which is commonly found in existing constraint works. The barrier function considered in this article is a composite function of the objective functions and the constraint functions. Through mathematical derivation, it can be seen that if the boundedness of the barrier function is guaranteed, then multiple objective functions satisfy the constraint conditions. With full consideration of the influence of multiple possible conflicting objectives, it is a challenging research topic to transform the multiple objective constrained systems into the unconstrained systems.

Remark 3. Compared with [35,37,44,54], the novel asymmetric and time-varying barrier function considered in this article can deal with not only the issue of asymmetric output constraints, but also the issue of multiple objective constraints. In addition, it is worth noting that positive and negative constraint functions can be handled. Therefore, the proposed approach has a wider application scope for constrained systems.

The time derivative of the barrier function ξ can be written as

$$\dot{\xi} = \kappa_1 \dot{E} + \kappa_2 = \kappa_{11} \dot{x}_1 + \kappa_2 \tag{6}$$

where $\kappa_1 = \frac{b_{v1}(t) - b_{c1}}{[E + b_{v1}(t)]^2} + \frac{b_{v2}(t) - b_{c2}}{[b_{v2}(t) - E]^2}$, $\kappa_2 = -\frac{(E + b_{c1})\dot{b}_{v1}(t)}{[E + b_{v1}(t)]^2} - \frac{(E - b_{c2})\dot{b}_{v2}(t)}{[b_{v2}(t) - E]^2}$, $\kappa_{11} = \kappa_1 \frac{\partial E}{\partial x_1}$ and $\frac{\partial E}{\partial x_1} \neq 0$.

Secondly, the coordinate transformation is proposed

$$\begin{cases} z_1 = \xi - y_{d,c} \\ z_i = x_i - x_{i,c}, i = 2, \ldots, n \end{cases} \tag{7}$$

where $y_{d,c} = \frac{y_d + b_{c1}}{y_d + b_{v1}(t)} + \frac{y_d - b_{c2}}{b_{v2}(t) - y_d}$, $x_{i,c}$ means the output of the first-order command filter, which associates with the virtual control signal α_{i-1}.

As the order of the command filter increases, the filtering error will increase. Therefore, the composite tracking error is introduced as follows

$$w_i = z_i - \zeta_i, i = 1, 2, \ldots, n \tag{8}$$

The command filter has a certain impact on the tracking performance, so the composite signal ζ_i is defined as

$$\begin{cases} \dot{\zeta}_1 = \kappa_{11}\left(-m_1\zeta_1 + x_{2,c} - \alpha_1 + \zeta_2\right) - \lambda_1 sign(\zeta_1) \\ \dot{\zeta}_2 = -m_2\zeta_2 + x_{3,c} - \alpha_2 + \zeta_3 - \lambda_2 sign(\zeta_2) \\ \dot{\zeta}_i = -m_i\zeta_i + x_{i+1,c} - \alpha_i - \zeta_{i-1} + \zeta_{i+1} - \lambda_i sign(\zeta_i) \\ \dot{\zeta}_n = -m_n\zeta_n - \zeta_{n-1} - \lambda_n sign(\zeta_n) \end{cases} \tag{9}$$

where $i = 3, \ldots, n-1$. For $j = 1, 2, \ldots, n$, $m_j > 0$ indicates the constant, λ_j is the design parameter.

Assumption 3 ([50]). $\left|x_{i+1,c} - \alpha_i\right| \leq \beta_i$ can be achieved in the fixed time T, where $\beta_i > 0$ is the known constant.

Remark 4. With the purpose of solving the finite-time tracking control issue of the switched nonlinear system (1), Assumptions 1 and 3 are commonly adopted in the existing works [19,33,34,50]. In order to deal with unknown and discontinuous time-varying disturbance, a more general approach is introduced by Assumption 2. Significantly, Assumptions 1–3 are the basic assumptions of backstepping-based adaptive finite-time control design and the necessary conditions for controllability of the switched nonlinear system (1).

Combining with the above mentioned research content, the time derivative of the composite tracking error w_i can be indicated as

$$\begin{cases} \dot{w}_1 = \kappa_{11}\left(w_2 + l_{1,k} + \Lambda_{1,k} + m_1\zeta_1 + \alpha_1\right) + \kappa_2 - \iota_1 \dot{y}_d \\ \qquad + \iota_2 + \lambda_1 sign(\zeta_1) \\ \dot{w}_2 = w_3 + l_{2,k} + \Lambda_{2,k} + m_2\zeta_2 + \alpha_2 - \dot{x}_{2,c} + \lambda_2 sign(\zeta_2) \\ \dot{w}_i = w_{i+1} + l_{i,k} + \Lambda_{i,k} + m_i\zeta_i + \alpha_i - \dot{x}_{i,c} + \zeta_{i-1} + \lambda_i sign(\zeta_i) \\ \dot{w}_n = u + l_{n,k} + \Lambda_{n,k} + m_n\zeta_n - \dot{x}_{n,c} + \zeta_{n-1} + \lambda_n sign(\zeta_n) \end{cases} \tag{10}$$

where $i = 3, \ldots, n-1$, $\iota_1 = \frac{b_{v1}(t)-b_{c1}}{[y_d+b_{v1}(t)]^2} + \frac{b_{v2}(t)-b_{c2}}{[b_{v2}(t)-y_d]^2}$, $\iota_2 = \frac{(y_d+b_{c1})\dot{b}_{v1}(t)}{[y_d+b_{v1}(t)]^2} + \frac{(y_d-b_{c2})\dot{b}_{v2}(t)}{[b_{v2}(t)-y_d]^2}$.

Thirdly, for $k \in Q$, $i = 1, 2, \ldots, n$, $\theta_i = max\left\{\left\|\boldsymbol{\theta}_{i,k}\right\|^2\right\}$ denotes the unknown constant. $\boldsymbol{\theta}_{i,k}$ defines the weight vector of MTN. $\hat{\theta}_i$ means the estimated value of θ_i. $\tilde{\theta}_i$ represents the estimated error of θ_i, and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

Step 1: The candidate Lyapunov function is designed as

$$V_1 = \frac{1}{2}w_1^2 + \frac{1}{2}\tilde{\theta}_1^2 \tag{11}$$

In combination with the above equation, the time derivative of Lyapunov function V_1 is indicated as

$$\begin{aligned}\dot{V}_1 =&\kappa_{11}w_1\left(w_2 + l_{1,k} + \Lambda_{1,k} + m_1\zeta_1 + \alpha_1\right)\\ &+ w_1\left[\kappa_2 - \iota_1\dot{y}_d + \iota_2 + \lambda_1 sign\left(\zeta_1\right)\right] - \tilde{\theta}_1\dot{\hat{\theta}}_1\end{aligned} \tag{12}$$

On the basis of Assumption 2 and Young's inequality, the following inequalities are true

$$\begin{aligned}&\kappa_{11}w_1\Lambda_{1,k} \le \kappa_{11}w_1\left(\phi_{1,k} + d_{1,k}\right) \le \frac{1}{2}a_1^2\\ &+ \frac{1}{2a_1^2}\kappa_{11}^2 w_1^2\left(\phi_{1,k} + d_{1,k}\right)^2\end{aligned} \tag{13}$$

$$w_1\lambda_1 sign\left(\zeta_1\right) \le \frac{1}{2}w_1^2 + \frac{1}{2}\lambda_1^2 \tag{14}$$

where $a_1 > 0$ is a constant.

Substituting (13) and (14) into (12), a new form of the time derivative of Lyapunov function V_1 can be expressed as

$$\begin{aligned}&\dot{V}_1 \le \kappa_{11}w_1\left(w_2 + m_1\zeta_1 + \alpha_1\right) + w_1 L_{1,k} - \tilde{\theta}_1\dot{\hat{\theta}}_1\\ &+ \frac{1}{2}a_1^2 + \frac{1}{2}\lambda_1^2 - \frac{1}{2}w_1^2\end{aligned} \tag{15}$$

where $L_{1,k} = \frac{1}{2a_1^2}\kappa_{11}^2 w_1\left(\phi_{1,k} + d_{1,k}\right)^2 + \kappa_{11}l_{1,k} + \kappa_2 - \iota_1\dot{y}_d + \iota_2 + w_1$ is a combination of nonlinear functions. According to the MTN technique in Lemma 2, for any $e_{1,k} > 0$, $L_{1,k}$ can be estimated by a MTN, and its expression is indicated as

$$L_{1,k} = \boldsymbol{\theta}_{1,k}^{\mathrm{T}} P_{m_1} + \varepsilon_{1,k},\ \left|\varepsilon_{1,k}\right| \le e_{1,k} \tag{16}$$

where $\varepsilon_{1,k}$ represents the error between the combination of nonlinear functions $L_{1,k}$ and the term $\boldsymbol{\theta}_{1,k}^{\mathrm{T}} P_{m_1}$.

According to the form (16) of the nonlinear function estimated by MTN and Young's inequality, the following inequality can be obtained

$$\begin{aligned}w_1 L_{1,k} &\le \frac{1}{2}\eta_1^2 + \frac{1}{2\eta_1^2}w_1^2\left\|\boldsymbol{\theta}_{1,k}\right\|^2 P_{m_1}^{\mathrm{T}} P_{m_1} + \frac{1}{2}w_1^2 + \frac{1}{2}e_{1,k}^2\\ &\le \frac{1}{2}\eta_1^2 + \frac{1}{2\eta_1^2}w_1^2\theta_1 P_{m_1}^{\mathrm{T}} P_{m_1} + \frac{1}{2}w_1^2 + \frac{1}{2}e_{1,k}^2\end{aligned} \tag{17}$$

where $\eta_1 > 0$ is a constant.

In an effort to ensure the effectiveness of the proposed control strategy, the virtual control signal α_1 is designed in the following form

$$\alpha_1 = -m_1 z_1 - \frac{1}{2\kappa_{11}\eta_1^2}w_1\hat{\theta}_1 P_{m_1}^{\mathrm{T}} P_{m_1} - r_1 w_1^{2s-1} \tag{18}$$

where $r_1 > 0$ is a constant and $0 < s < 1$.

With the help of the virtual control signal α_1 and (17), the time derivative of V_1 can be described as

$$\begin{aligned}\dot{V}_1 \le&\kappa_{11}w_1 w_2 - \kappa_{11}m_1 w_1^2 - \kappa_{11}r_1 w_1^{2s} + \frac{1}{2}\eta_1^2 + \frac{1}{2}a_1^2\\ &+ \frac{1}{2}\lambda_1^2 + \frac{1}{2}e_{1,k}^2 + \tilde{\theta}_1\left(\frac{1}{2\eta_1^2}w_1^2 P_{m_1}^{\mathrm{T}} P_{m_1} - \dot{\hat{\theta}}_1\right)\end{aligned} \tag{19}$$

Step 2: On the basis of the Step 1, the candidate Lyapunov function is designed as

$$V_2 = \frac{1}{2}w_2^2 + \frac{1}{2}\tilde{\theta}_2^2 + V_1 \tag{20}$$

With the help of (20), the time derivative of the Lyapunov function V_2 can be further obtained in the form shown below

$$\begin{aligned}\dot{V}_2 =&w_2\left(w_3 + l_{2,k} + \Lambda_{2,k} + m_2\zeta_2 + \alpha_2 - \dot{x}_{2,c}\right)\\ &+ w_2\lambda_2 sign\left(\zeta_2\right) - \tilde{\theta}_2\dot{\hat{\theta}}_2 + \dot{V}_1\end{aligned} \tag{21}$$

In the light of Assumption 2 and Young's inequality, the following inequalities are correct

$$w_2\Lambda_{2,k} \le w_2\left(\phi_{2,k} + d_{2,k}\right) \le \frac{1}{2}a_2^2 + \frac{1}{2a_2^2}w_2^2\left(\phi_{2,k} + d_{2,k}\right)^2 \tag{22}$$

$$w_2\lambda_2 sign\left(\zeta_2\right) \le \frac{1}{2}w_2^2 + \frac{1}{2}\lambda_2^2 \tag{23}$$

where $a_2 > 0$ is a constant.

According to (21), (22) and (23), a new form of the time derivative of Lyapunov function V_2 can be indicated as

$$\begin{aligned}\dot{V}_2 \le&w_2\left(w_3 + m_2\zeta_2 + \alpha_2\right) + w_2 L_{2,k} - \tilde{\theta}_2\dot{\hat{\theta}}_2\\ &+ \frac{1}{2}a_2^2 + \frac{1}{2}\lambda_2^2 - \kappa_{11}w_1 w_2 - \frac{1}{2}w_2^2 + \dot{V}_1\end{aligned} \tag{24}$$

where $L_{2,k} = l_{2,k} - \dot{x}_{2,c} + \frac{1}{2a_2^2}w_2\left(\phi_{2,k} + d_{2,k}\right)^2 + \kappa_{11}w_1 + w_2$ is a combination of nonlinear functions. With the help of the MTN technique in Lemma 2, for $e_{2,k} > 0$, $L_{2,k}$ can be estimated by MTN, and its expression as follows

$$L_{2,k} = \boldsymbol{\theta}_{2,k}^{\mathrm{T}} P_{m_2} + \varepsilon_{2,k},\ \left|\varepsilon_{2,k}\right| \le e_{2,k} \tag{25}$$

where $\varepsilon_{2,k}$ indicates the error between the combination of nonlinear functions $L_{2,k}$ and the term $\boldsymbol{\theta}_{2,k}^{\mathrm{T}} P_{m_2}$.

According to the form (25) of the nonlinear function estimated by MTN and Young's inequality, the following inequality can be acquired

$$\begin{aligned}w_2 L_{2,k} &\le \frac{1}{2}\eta_2^2 + \frac{1}{2\eta_2^2}w_2^2\left\|\boldsymbol{\theta}_{2,k}\right\|^2 P_{m_2}^{\mathrm{T}} P_{m_2} + \frac{1}{2}w_2^2 + \frac{1}{2}e_{2,k}^2\\ &\le \frac{1}{2}\eta_2^2 + \frac{1}{2\eta_2^2}w_2^2\theta_2 P_{m_2}^{\mathrm{T}} P_{m_2} + \frac{1}{2}w_2^2 + \frac{1}{2}e_{2,k}^2\end{aligned} \tag{26}$$

where $\eta_2 > 0$ is a constant.

In order to ensure that the proposed control scheme is effective, the virtual control signal α_2 is designed in the following form

$$\alpha_2 = -m_2 z_2 - \frac{1}{2\eta_2^2}w_2\hat{\theta}_2 P_{m_2}^{\mathrm{T}} P_{m_2} - r_2 w_2^{2s-1} \tag{27}$$

where $r_2 > 0$ is a constant.

On the basis of (19), (26) and the virtual control signal α_2, the time derivative of V_2 can be expressed as

$$\begin{aligned}\dot{V}_2 \le& -\kappa_{11}m_1 w_1^2 - m_2 w_2^2 - \kappa_{11}r_1 w_1^{2s} - r_2 w_2^{2s} + w_2 w_3\\ &+ \sum_{j=1}^{2}\tilde{\theta}_j\left(\frac{1}{2\eta_j^2}w_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} - \dot{\hat{\theta}}_j\right) + \frac{1}{2}\sum_{j=1}^{2}\left(a_j^2 + \lambda_j^2 + \eta_j^2 + e_{j,k}^2\right)\end{aligned} \tag{28}$$

Step $i\,(3 \le i \le n-1)$: With the help of the above research content and mathematical derivation, the candidate Lyapunov function is designed as

$$V_i = \frac{1}{2}w_i^2 + \frac{1}{2}\tilde{\theta}_i^2 + V_{i-1} \tag{29}$$

With the help of (29), the time derivative of the Lyapunov function V_i can be further attained in the form shown below

$$\dot{V}_i = w_i\left(w_{i+1} + l_{i,k} + \Lambda_{i,k} + m_i\zeta_i + \alpha_i - \dot{x}_{i,c} + \zeta_{i-1}\right) + w_i\lambda_i sign\left(\zeta_i\right) - \tilde{\theta}_i\dot{\hat{\theta}}_i + \dot{V}_{i-1} \tag{30}$$

In the light of Young's inequality and Assumption 2, the following inequalities hold

$$w_i\Lambda_{i,k} \le w_i\left(\phi_{i,k} + d_{i,k}\right) \le \frac{1}{2}a_i^2 + \frac{1}{2a_i^2}w_i^2\left(\phi_{i,k} + d_{i,k}\right)^2 \tag{31}$$

$$w_i\lambda_i sign\left(\zeta_i\right) \le \frac{1}{2}w_i^2 + \frac{1}{2}\lambda_i^2 \tag{32}$$

where $a_i > 0$ is the constant.

With the help of (31) and (32), a new form of the time derivative of the Lyapunov function V_i can be indicated as

$$\dot{V}_i \le w_i\left(w_{i+1} - w_{i-1} + m_i\zeta_i + \alpha_i\right) + w_iL_{i,k} - \tilde{\theta}_i\dot{\hat{\theta}}_i + \frac{1}{2}a_i^2 + \frac{1}{2}\lambda_i^2 - \frac{1}{2}w_i^2 + \dot{V}_{i-1} \tag{33}$$

where $L_{i,k} = l_{i,k} - \dot{x}_{i,c} + \frac{1}{2a_i^2}w_i\left(\phi_{i,k} + d_{i,k}\right)^2 + w_i + z_{i-1}$ is the combination of nonlinear functions. In the light of the MTN technique in Lemma 2, for any $e_{i,k} > 0$, $L_{i,k}$ can be estimated by MTN, and its expression as follows

$$L_{i,k} = \boldsymbol{\theta}_{i,k}^{\mathrm{T}}P_{m_i} + \varepsilon_{i,k},\ \left|\varepsilon_{i,k}\right| \le e_{i,k} \tag{34}$$

where $\varepsilon_{i,k}$ means the error between the combination of nonlinear functions $L_{i,k}$ and the term $\boldsymbol{\theta}_{i,k}^{\mathrm{T}}P_{m_i}$.

According to the form (34) of the nonlinear function estimated by MTN and Young's inequality, the following inequality can be gained

$$\begin{aligned} w_iL_{i,k} &\le \frac{1}{2}\eta_i^2 + \frac{1}{2\eta_i^2}w_i^2\left\|\boldsymbol{\theta}_{i,k}\right\|^2P_{m_i}^{\mathrm{T}}P_{m_i} + \frac{1}{2}w_i^2 + \frac{1}{2}e_{i,k}^2 \\ &\le \frac{1}{2}\eta_i^2 + \frac{1}{2\eta_i^2}w_i^2\theta_iP_{m_i}^{\mathrm{T}}P_{m_i} + \frac{1}{2}w_i^2 + \frac{1}{2}e_{i,k}^2 \end{aligned} \tag{35}$$

where $\eta_i > 0$ is the constant.

So as to ensure that the proposed control scheme is effective, the virtual control signal α_i can be designed in the following form

$$\alpha_i = -m_iz_i - \frac{1}{2\eta_i^2}w_i\hat{\theta}_iP_{m_i}^{\mathrm{T}}P_{m_i} - r_iw_i^{2s-1} \tag{36}$$

where $r_i > 0$ is the constant.

In the light of mathematical derivation, (28), (35), the virtual control signal α_i and Young's inequality, the time derivative of V_i can be expressed as

$$\begin{aligned} \dot{V}_i \le &-\kappa_{11}m_1w_1^2 - \sum_{j=2}^{i}m_jw_j^2 - \kappa_{11}r_1w_1^{2s} - \sum_{j=2}^{i}r_jw_j^{2s} + w_iw_{i+1} \\ &+ \frac{1}{2}\sum_{j=1}^{i}\left(a_j^2 + \lambda_j^2 + \eta_j^2 + e_{j,k}^2\right) + \sum_{j=1}^{i}\tilde{\theta}_j\left(\frac{1}{2\eta_j^2}w_j^2P_{m_j}^{\mathrm{T}}P_{m_j} - \dot{\hat{\theta}}_j\right) \end{aligned} \tag{37}$$

Step n: On the basis of Step i, the candidate Lyapunov function is designed as

$$V_n = \frac{1}{2}w_n^2 + \frac{1}{2}\tilde{\theta}_n^2 + V_{n-1} \tag{38}$$

With the help of (38), the time derivative of the Lyapunov function V_n can be further attained in the form shown below

$$\dot{V}_n = w_n\left(u + l_{n,k} + \Lambda_{n,k} + m_n\zeta_n - \dot{x}_{n,c} + \zeta_{n-1}\right) + w_n\lambda_n sign\left(\zeta_n\right) - \tilde{\theta}_n\dot{\hat{\theta}}_n + \dot{V}_{n-1} \tag{39}$$

Combining Young's inequality with Assumption 2, the following inequalities can be gained

$$w_n\Lambda_{n,k} \le w_n\left(\phi_{n,k} + d_{n,k}\right) \le \frac{1}{2}a_n^2 + \frac{1}{2a_n^2}w_n^2\left(\phi_{n,k} + d_{n,k}\right)^2 \tag{40}$$

$$w_n\lambda_n sign\left(\zeta_n\right) \le \frac{1}{2}w_n^2 + \frac{1}{2}\lambda_n^2 \tag{41}$$

where $a_n > 0$ is a constant.

With the help of (40) and (41), a new form of the time derivative of Lyapunov function V_n can be expressed as

$$\dot{V}_n \le w_n\left(u + m_n\zeta_n - w_{n-1}\right) + w_nL_{n,k} + \frac{1}{2}a_n^2 + \frac{1}{2}\lambda_n^2 - \tilde{\theta}_n\dot{\hat{\theta}}_n - \frac{1}{2}w_n^2 + \dot{V}_{n-1} \tag{42}$$

where $L_{n,k} = l_{n,k} - \dot{x}_{n,c} + \frac{1}{2a_n^2}w_n\left(\phi_{n,k} + d_{n,k}\right)^2 + w_n + z_{n-1}$ is the combination of nonlinear functions. On the basis of the MTN technique in Lemma 2, for any $e_{n,k} > 0$, $L_{n,k}$ can be estimated by MTN, and its expression as follows

$$L_{n,k} = \boldsymbol{\theta}_{n,k}^{\mathrm{T}}P_{m_n} + \varepsilon_{n,k},\ \left|\varepsilon_{n,k}\right| \le e_{n,k} \tag{43}$$

where $\varepsilon_{n,k}$ means the error between the combination of nonlinear functions $L_{n,k}$ and the term $\boldsymbol{\theta}_{n,k}^{\mathrm{T}}P_{m_n}$.

According to the form (43) of the nonlinear function estimated by MTN and Young's inequality, the following inequality can be attained

$$\begin{aligned} w_nL_{n,k} &\le \frac{1}{2}\eta_n^2 + \frac{1}{2\eta_n^2}w_n^2\left\|\boldsymbol{\theta}_{n,k}\right\|^2P_{m_n}^{\mathrm{T}}P_{m_n} + \frac{1}{2}w_n^2 + \frac{1}{2}e_{n,k}^2 \\ &\le \frac{1}{2}\eta_n^2 + \frac{1}{2\eta_n^2}w_n^2\theta_nP_{m_n}^{\mathrm{T}}P_{m_n} + \frac{1}{2}w_n^2 + \frac{1}{2}e_{n,k}^2 \end{aligned} \tag{44}$$

where $\eta_n > 0$ is a constant.

In an effort to ensure that the proposed control scheme is effective, the actual control u can be designed in the following form

$$u = -m_nz_n - \frac{1}{2\eta_n^2}w_n\hat{\theta}_nP_{m_n}^{\mathrm{T}}P_{m_n} - r_nw_n^{2s-1} \tag{45}$$

where $r_n > 0$ is a constant.

According to (37), (44), the actual control u and Young's inequality, the time derivative of V_n can be expressed as

$$\begin{aligned} \dot{V}_n \le &-\kappa_{11}m_1w_1^2 - \sum_{j=2}^{n}m_jw_j^2 - \kappa_{11}r_1w_1^{2s} - \sum_{j=2}^{n}r_jw_j^{2s} \\ &+ \sum_{j=1}^{n}\tilde{\theta}_j\left(\frac{1}{2\eta_j^2}w_j^2P_{m_j}^{\mathrm{T}}P_{m_j} - \dot{\hat{\theta}}_j\right) + \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \lambda_j^2 + \eta_j^2 + e_{j,k}^2\right) \end{aligned} \tag{46}$$

Remark 5. The backstepping control method can not only realize the systematic and structured controller design process, but also design the control strategy for the nonlinear systems with relative order n. According to the adaptive backstepping technique, it can be ensured that all subsystems have a CLF through a series of mathematical derivations.

Remark 6. In the backstepping design process, if $2s - 1 < 0$, the "singularity" problem may arise from the term $r_iw_i^{2s-1}$ due to repeated differentiation of the virtual control signal α_i. With the aim of addressing this challenge, DSC technique is employed, which utilizes the command filter to approximate the derivative of α_i. Therefore, the calculation of the first derivative of α_i is unnecessary. As a result, the issues of "singularity" is avoided.

Remark 7. It should be noted that the MTN-based finite-time control issue of switched nonlinear systems subject to multiple objective constraints is studied for the first time. Although the tracking control of nonlinear systems with multiple objective constraints has been studied in [50], unknown time-varying disturbance is not considered, and the controlled objective is not the switched nonlinear systems. In addition, different from [50], although more complex systems are considered in this article, the proposed controller structure is simpler.

4. Stability analysis

Theorem 1. *With regard to the switched nonlinear system (1), if the virtual control signals are designed in the forms of (18), (27), (36), and the actual control input is constructed in the form of (45), and the adaptive law is fully considered and its form can be expressed as*

$$\dot{\hat{\theta}}_j = -p_j\hat{\theta}_j + \frac{1}{2\eta_j^2} w_j^2 P_{m_j}^{\mathrm{T}} P_{m_j} \tag{47}$$

where $j = 1, 2, \ldots, n$, $p_j > 0$ is the constant. Then, the control objectives of this article can be achieved.

Proof. With the help of the above analysis, the Lyapunov function is constructed in the following form

$$V = V_n = \frac{1}{2}\sum_{j=1}^{n} w_j^2 + \frac{1}{2}\sum_{j=1}^{n}\tilde{\theta}_j^2 \tag{48}$$

In the light of (46) and adaptive law (47), the time derivative of Lyapunov function V can be acquired as follows

$$\begin{aligned}\dot{V} \le & \sum_{j=1}^{n} p_j\tilde{\theta}_j\hat{\theta}_j - \kappa_{11}m_1w_1^2 - \sum_{j=2}^{n} m_jw_j^2 - \kappa_{11}r_1w_1^{2s} - \sum_{j=2}^{n} r_jw_j^{2s} \\ & + \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \lambda_j^2 + \eta_j^2 + e_{j,k}^2\right)\end{aligned} \tag{49}$$

According to Young's inequality, the term $\sum_{j=1}^{n} p_j\tilde{\theta}_j\hat{\theta}_j$ in (49) can be rewritten as

$$\sum_{j=1}^{n} p_j\tilde{\theta}_j\hat{\theta}_j \le -\frac{3}{4}p\sum_{j=1}^{n}\tilde{\theta}_j^2 + \sum_{j=1}^{n} p_j\theta_j^2 \tag{50}$$

where $p = min\left\{p_j | j = 1, 2, \ldots, n\right\}$.

On the basis of fully considering (50), a new form of time derivative of Lyapunov function V is obtained

$$\begin{aligned}\dot{V} \le & -\kappa_{11}m_1w_1^2 - \sum_{j=2}^{n} m_jw_j^2 - \kappa_{11}r_1w_1^{2s} - \sum_{j=2}^{n} r_jw_j^{2s} - \frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2 \\ & - \frac{1}{4}p\sum_{j=1}^{n}\tilde{\theta}_j^2 + \sum_{j=1}^{n} p_j\theta_j^2 - \left(\frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2\right)^s + \left(\frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2\right)^s \\ & + \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \lambda_j^2 + \eta_j^2 + e_{j,max}^2\right)\end{aligned} \tag{51}$$

where $e_{j,\max}^2 = \max\left\{e_{j,k}^2 | k \in Q\right\}$.

Based on $0 < s < 1$, for the term $\left(\frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2\right)^s - \frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2 + \sum_{j=1}^{n} p_j\theta_j^2$, considering the following two cases:

Case 1: On the basis of full consideration of $\frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2 \ge 1$, it can be achieved that $\left(\frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2\right)^s - \frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2 + \sum_{j=1}^{n} p_j\theta_j^2 \le \sum_{j=1}^{n} p_j\theta_j^2$;

Case 2: When $0 \le \frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2 < 1$ is considered, then it follows that $\left(\frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2\right)^s - \frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2 + \sum_{j=1}^{n} p_j\theta_j^2 < 1 - \frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2 + \sum_{j=1}^{n} p_j\theta_j^2$.

Consequently, through the analysis and synthesis of the above two cases, the following inequality is true in all cases

$$\left(\frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2\right)^s - \frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2 + \sum_{j=1}^{n} p_j\theta_j^2 < 1 + \sum_{j=1}^{n} p_j\theta_j^2 \tag{52}$$

In summary, based on the expression form of (52), (51) can be indicated in the following form

$$\begin{aligned}\dot{V} \le & -\kappa_{11}m_1w_1^2 - \sum_{j=2}^{n} m_jw_j^2 - \kappa_{11}r_1w_1^{2s} - \sum_{j=2}^{n} r_jw_j^{2s} - \frac{1}{4}p\sum_{j=1}^{n}\tilde{\theta}_j^2 \\ & - \left(\frac{1}{2}p\sum_{j=1}^{n}\tilde{\theta}_j^2\right)^s + 1 + \sum_{j=1}^{n} p_j\theta_j^2 \\ & + \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \lambda_j^2 + \eta_j^2 + e_{j,max}^2\right) \\ \le & -hV - fV^s + g\end{aligned} \tag{53}$$

where $h = \min\left\{2\kappa_{11}m_1, 2m_2, \ldots, 2m_n, \frac{1}{2}p\right\}$, $f = \min\{2^s\kappa_{11}r_1, 2^sr_2, \ldots, 2^sr_n, p\}$, $g = 1 + \sum_{j=1}^{n} p_j\theta_j^2 + \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \lambda_j^2 + \eta_j^2 + e_{j,\max}^2\right)$.

Combining Lemma 1 with the analysis in the [50], it can be obtained that $w_i, i = 1, 2, \ldots, n$ are bounded. $V^s \le \frac{g}{(1-\varphi)f}$ can be derived by introducing a new scale φ. Then, at a finite time $T \le \frac{1}{h(1-s)}\ln\left(\frac{hV^{1-s}(X_0)+\varphi f}{\varphi f}\right)$, $|w_i| \le \sqrt{2}\left[\frac{g}{(1-\varphi)f}\right]^{\frac{1}{2s}}$ holds.

With the aim of proving that z_i is practical finite-time stable, it is necessary to ensure that ζ_i is convergent. First of all, a Lyapunov function is constructed as

$$V_\zeta = \frac{1}{2}\sum_{j=1}^{n}\zeta_j^2 \tag{54}$$

Through mathematical knowledge and Assumption 3, the time derivative of the above Lyapunov function V_ζ can be obtained as

$$\begin{aligned}\dot{V}_\zeta \le & \kappa_{11}\beta_1\zeta_1 - \kappa_{11}m_1\zeta_1^2 + \kappa_{11}\zeta_1\zeta_2 - \sum_{j=2}^{n} m_j\zeta_j^2 \\ & + \sum_{j=2}^{n-1}\beta_j\zeta_j - \sum_{j=1}^{n}\lambda_j\left|\zeta_j\right| \\ \le & -(\lambda_1 - \kappa_{11}\beta_1)\left|\zeta_1\right| - \sum_{j=2}^{n}\left(\lambda_j - \beta_j\right)\left|\zeta_j\right| - \left(m_1 - \frac{1}{2}\right)\kappa_{11}\zeta_1^2 \\ & - \left(m_2 - \frac{1}{2}\kappa_{11}\right)\zeta_2^2 - \sum_{j=3}^{n} m_j\zeta_j^2 \\ \le & -HV_\zeta - FV_\zeta^{\frac{1}{2}}\end{aligned} \tag{55}$$

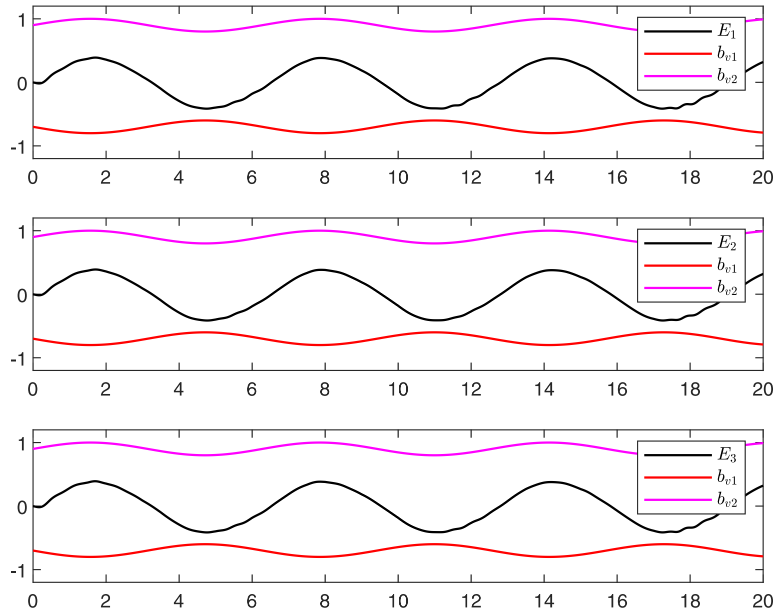

Fig. 1. Trajectories of objective functions and constraint functions.

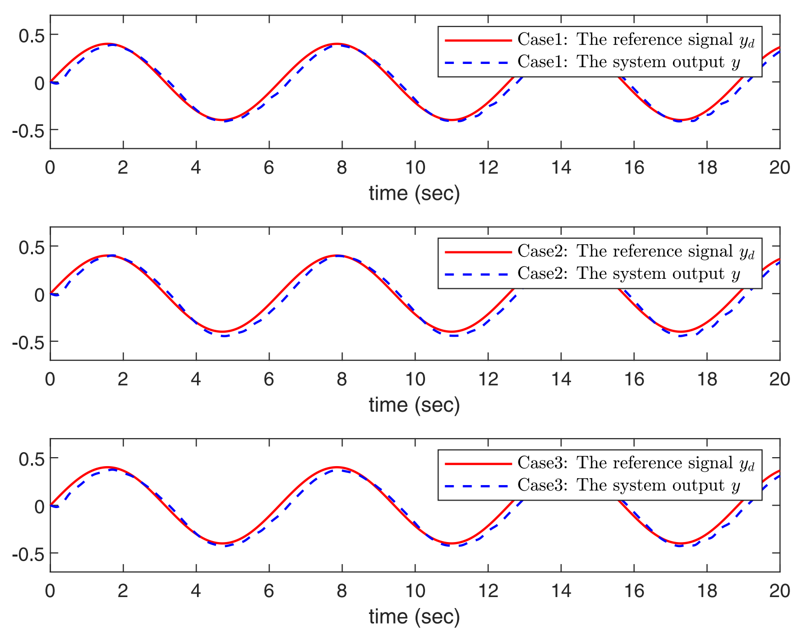

Fig. 2. System output y and reference signal y_d in three cases.

where $H = 2\min\left\{M_1, M_2, m_j | j = 3, \ldots, n\right\}$, $M_1 = \left(m_1 - \frac{1}{2}\right)\kappa_{11}$, $M_2 = m_2 - \frac{\kappa_{11}}{2}$, $F = \sqrt{2}\min\left\{\lambda_1 - \kappa_{11}\beta_1, \lambda_j - \beta_j | j = 2, \ldots, n\right\}$.

Combining with Lemma 1, it can be obtained that $\zeta_i (i = 1, \ldots, n)$ can converge to the origin in a finite time. Based on the above analysis, for $i = 1, \ldots, n$ and $j = 1, \ldots, n-1$, it can be concluded that ξ, E, x_i, z_i, w_i, $\hat{\dot{\theta}}_i$, u, α_j are bounded. Therefore, we can draw a conclusion that all signals in the closed-loop system are bounded.

That completes the proof of Theorem 1.

5. Simulation result

On the basis of the above theoretical analysis, the effectiveness of the proposed controller is further ensured through simulation. The simplified longitudinal model of the aircraft can be described in the following form

$$\begin{cases} \dot{x}_1 = \dfrac{L_\alpha x_2}{m_{\sigma(t)} V_{T_{\sigma(t)}}} - \dfrac{g\cos x_1}{V_{T_{\sigma(t)}}} + \dfrac{L_o}{m_{\sigma(t)} V_{T_{\sigma(t)}}} + \Lambda_{1,\sigma(t)}\left(t, \bar{\boldsymbol{x}}_1\right) \\ \dot{x}_2 = x_3 + \dfrac{g\cos x_1}{V_{T_{\sigma(t)}}} - \dfrac{L_o}{m_{\sigma(t)} V_{T_{\sigma(t)}}} - \dfrac{L_\alpha x_2}{m_{\sigma(t)} V_{T_{\sigma(t)}}} + \Lambda_{2,\sigma(t)}\left(t, \bar{\boldsymbol{x}}_2\right) \\ \dot{x}_3 = M_\delta u + M_\alpha x_2 + M_q x_3 + \Lambda_{3,\sigma(t)}\left(t, \bar{\boldsymbol{x}}_3\right) \\ y = x_1 \end{cases} \tag{56}$$

with $\boldsymbol{x}(\mathbf{0}) = \left[x_1(0), x_2(0), x_3(0)\right]^{\mathrm{T}} = [0, 0, 0]^{\mathrm{T}}$, $\sigma(t) : [0, \infty] \to Q = \{1, 2\}$. $M_\delta = 1$, $M_\alpha = 0.1$, $M_q = -0.02$, $\frac{L_\alpha}{m_1} = 200$,

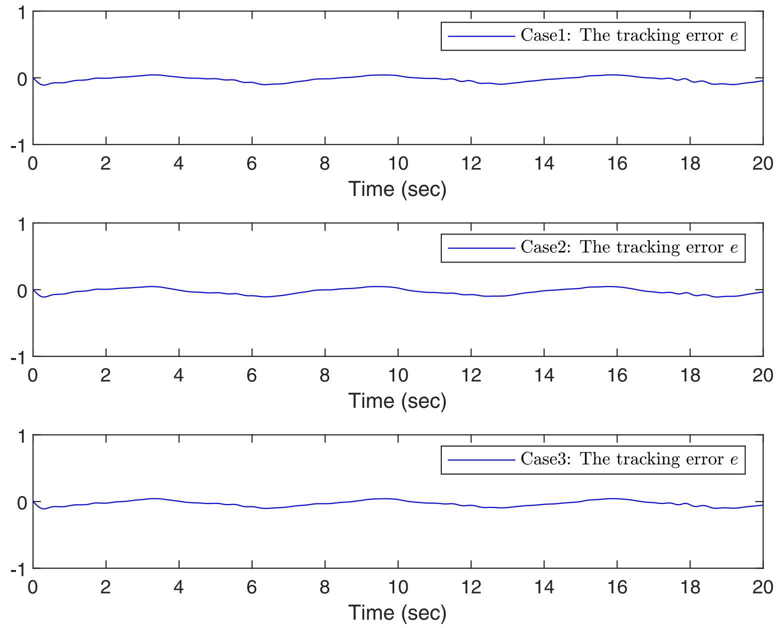

Fig. 3. The tracking error in three cases.

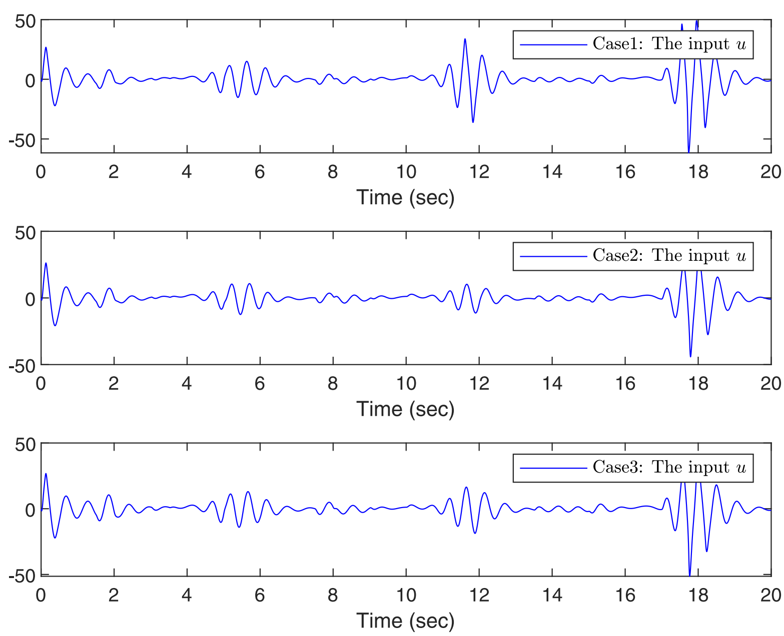

Fig. 4. System control input in three cases.

$\frac{L_\alpha}{m_2} = 100$, $\frac{L_o}{m_1} = -20$ and $\frac{L_o}{m_2} = -10$ indicate physical parameters. $g = 9.8$ m/s^2 means gravitational acceleration. $V_{T_1} = 200$ m/s and $V_{T_2} = 100$ m/s express speed of aircraft. The time-varying disturbance are described as $\Lambda_{1,1} = 0.01\sin 2t$, $\Lambda_{2,1} = 0.1\cos 2t$, $\Lambda_{3,1} = 0.05\sin t\cos 2t$, $\Lambda_{2,1} = 0.02\sin 2t$, $\Lambda_{2,2} = 0.2\cos 2t$, $\Lambda_{3,2} = 0.1\sin t\cos 2t$. The desired signal is designed as $y_d = 0.4\sin t$. The constraint functions are chosen as $b_{v1}(t) = 0.7 + 0.1\sin t$, $b_{v2}(t) = 0.9 + 0.1\sin t$. Three objective functions bounded on $E \in \Omega := \{E \in R : -b_{v1}(t) < E(x_1) < b_{v2}(t)\}$ are constructed, which are described as the following three cases:

Case 1: $E_1 = x_1$;
Case 2: $E_2 = 0.95x_1 + 0.05x_1^2$;
Case 3: $E_3 = x_1 + 0.1x_1^2 + 0.05x_1^3$.

The initial state of the error compensation system is selected as $\zeta(\mathbf{0}) = [\zeta_1(0), \zeta_2(0), \zeta_3(0)]^{\mathrm{T}} = [0.01, 0.01, 0.01]^{\mathrm{T}}$. The parameters of the controller proposed in this article are designed as $s = 0.5$, $m_1 = 8$, $m_2 = 12$, $m_3 = 18$, $r_1 = 0.01$, $r_2 = r_3 = 0.1$, $p_1 = p_2 = p_3 = \eta_1 = \eta_2 = \eta_3 = 1$, $\lambda_1 = \lambda_2 = \lambda_3 = 0.5$. The simulation results obtained in these three cases are displayed in Figs. 1–7.

The trajectories of the three objective functions and the asymmetric together with time-varying constraint functions are displayed in Fig. 1. In the three cases, Fig. 2 gives the trajectories of the system output y and the desired signal y_d. Figs. 3–6 reveal the tracking error, control input, state variable x_2 and state variable x_3 of the system (56), respectively. Fig. 7 depicts the switching signal designed in this article. The following four points can be concluded through Figs. 1–7:

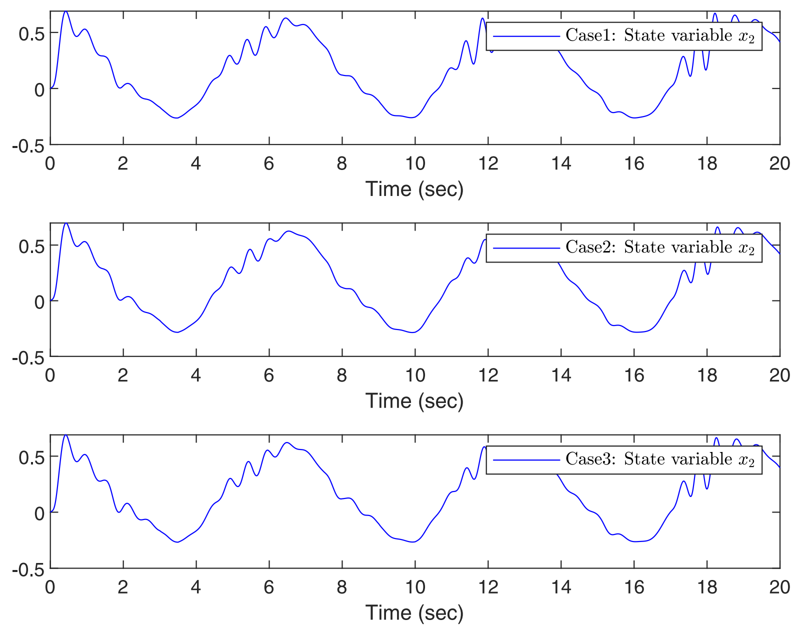

Fig. 5. State variable x_2 in three cases.

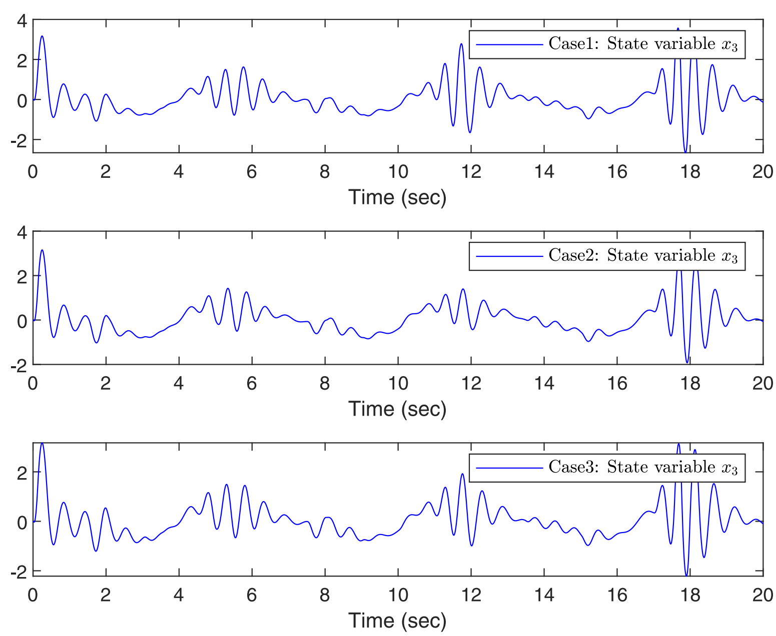

Fig. 6. State variable x_3 in three cases.

(1) The controller proposed in this article can ensure that the multiple objective constraints are satisfied;

(2) All states in the system are bounded;

(3) The desired signal can be well tracked by the system output;

(4) The tracking error can be adjusted small enough.

In a word, the finite-time control strategy proposed in this article has the advantages of real-time tracking and fast convergence in finite time, which can acquire satisfactory results.

6. Conclusion

This article discusses the finite-time control problem of switched nonlinear systems subject to multiple objective constraints. First of all, multiple objective constrained systems are transformed into unconstrained systems by designing a time-varying and asymmetric barrier function. Next, DSC technique is introduced into the backstepping design process. Moreover, the error compensation systems are constructed to mitigate the effects of the filter. Then, an adaptive control strategy is proposed by combining finite-time control with MTN technique. Finally, a practical simulation is used to demonstrate that the control strategy proposed in this article is available.

The future research direction is to consider a global control scheme for switched nonlinear systems. With the premise of overcoming the limitation of CLF, an adaptive MTN controller is designed based on the multiple Lyapunov function (MLF) method. It is a challenging problem to investigate an adaptive MLF-based global control strategy.

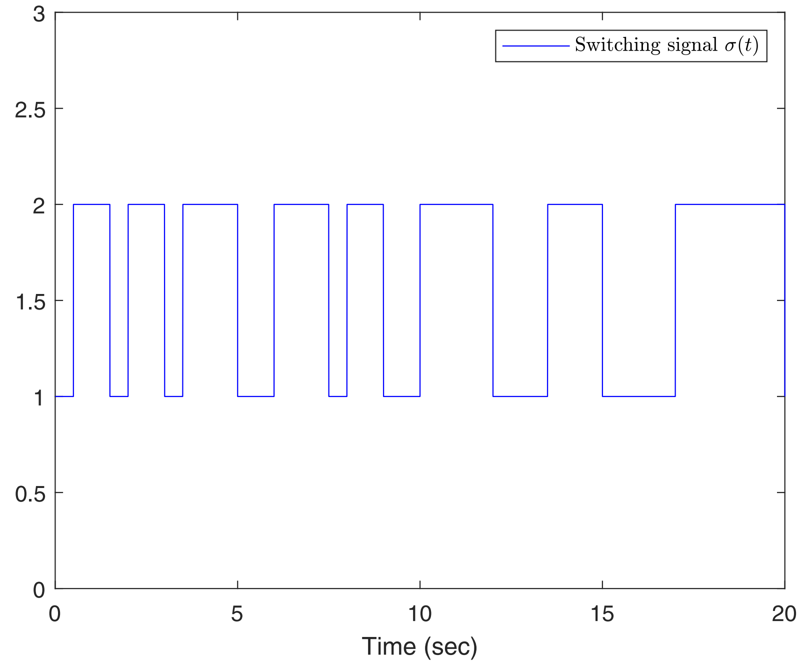

Fig. 7. Trajectory of switching signal.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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