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Adaptive Controller Design for Switched Stochastic Nonlinear Systems Subject to Unknown Dead-zone Input via New Type of Network Approach

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Abstract: In this article, adaptive tracking control for a class of switched stochastic nonlinear systems subject to unknown dead-zone input using multi-dimensional Taylor network (MTN) is studied. Firstly, the characteristic function is introduced to convert the nonlinearity of the input dead-zone into a linear model. Secondly, a novel adaptive control method based on the backstepping recursive design technique is proposed, which combines MTN and common Lyapunov functions (CLFs). Significantly, a method to reduce the computational complexity of switched stochastic nonlinear systems is proposed for the first time, which introduces characteristic function and MTN technology. The result makes clear that the proposed controller can ensure all signals of the closed-loop system are bounded in probability, and the output of the system can track reference signal well. Finally, the effectiveness of proposed control method is verified by simulation results.

Keywords: Adaptive control, dead-zone input, multi-dimensional Taylor network, stochastic nonlinear systems, switched nonlinear systems.

1. INTRODUCTION

As a kind of multiple-mode hybrid systems, switched systems can be used for mathematical modeling of many practical systems [1-3]. Therefore, it is highly concerned about the research on control design and stability analysis of switched systems [4,5]. Up to now, depending on the difference in selective Lyapunov functions, two methods can be used to cope with stability problem of switched systems. These two methods are common Lyapunov function (CLF) method [6] and multiple Lyapunov function (MLF) method [7,8]. In the case of complex switching law structure, CLF method can reduce the difficulty of controller design and stability analysis. Therefore, using CLF method to cope with the stability of switched systems has received extensive attention [9,10].

In a good many practical systems, there seems no possibility that the influence of stochastic perturbation can be averted. Therefore, quite a lot of attention has been drawn into the study of stochastic systems [11-13]. Many control methods for deterministic systems were extended to stochastic systems, among which adaptive control method was proved to be an effective method to solve the control problems of uncertain systems and complex systems [14-16]. So, the research of stochastic switched systems has also attracted much attention via adaptive control [17,18]. However, the problem of unknown functions was not considered by the above research. In fact, estimation technique is an effective method to cope with the unknown nonlinear functions in the system, including neural networks (NNs) [19-21], fuzzy logic systems (FLSs) [22-24] and multi-dimensional Taylor networks (MTNs) [25-27]. Among them, MTN, as a new type of NN, has the characteristics of simple structure and real time. For nonlinear systems, backstepping-based adaptive control strategies were proposed in [28-31] using MTN approach. The results make it clear that MTN has preferable real-time performance and robustness. However, MTN is mostly used in nonlinear systems or stochastic systems, and rarely used in switched stochastic nonlinear systems.

On the other hand, in the actual industrial control process, control systems are affected not only by stochastic perturbation, but also by the input nonlinearity. Among them, dead-zone, as one of the important types of input nonlinearity, becomes a source of system instability and even deteriorates the system performance. Therefore, it is very significant to study nonlinear systems with dead-zone [32,33]. So far, the solutions of nonlinear systems with dead-zone mainly can be summarized into two categories. One is to construct the dead-zone inverse [34].

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The other one is to transform the dead-zone into a linear model, which consists of linear input and disturbance by introducing the characteristic function [35]. Compared with the former, the latter can simplify the design of the controller, for this reason, the latter got much more attention [36,37]. Consequently, it is of great significance to extend the above method to switched stochastic nonlinear systems with dead-zone input.

Inspired by the above results, this article studies the adaptive tracking problem for the switched stochastic nonlinear systems with unknown input dead-zone. Firstly, the nonlinear problem caused by dead-zone is solved by introducing the characteristic function. Secondly, a novel MTN adaptive tracking control method is proposed. Finally, the results indicate that the control strategy is valid. The main innovations of this article are as follows:

i) It is the first time that MTN is applied to switched stochastic systems subjected to dead-zone. MTN is rarely applied in switched systems, and only [6,38] were reported so far. Even though the authors of [39,40] studied similar problem with this article, the control structure proposed in this article has the advantages of simple structure and less computational cost.

ii) The method of dealing with dead-zone input in [36] is extended to switched stochastic systems. By introducing a characteristic function, the dead-zone input can be expressed as a linear model. Different from the method of dealing with dead-zone in [34], it is unnecessary to construct the inverse of dead-zone for the method in this article, so the calculation is relatively uncomplicated.

iii) To the best of our knowledge, switched stochastic systems, dead-zone input and MTN have never before been featured in the same framework. Although the control problem of the switched systems with input dead zone was studied by the authors of [37], the stochastic case was not considered. The authors of [35] proposed a control strategy for stochastic systems with input dead zone, but this strategy cannot be applied to switched systems. Although the authors of [18] researched switched stochastic systems, dead-zone was not considered. Although the authors of [41] discussed dead zone problem of switched stochastic systems, the problem of input dead zone was not solved.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem description

Consider the following switched stochastic nonlinear system

$$\begin{cases} dx_i = \left(g_{i,\sigma(t)}\left(\bar{\boldsymbol{x}}_i\right)x_{i+1} + \ell_{i,\sigma(t)}\left(\bar{\boldsymbol{x}}_i\right)\right)dt \\ \qquad + h^{\mathrm{T}}_{i,\sigma(t)}\left(\bar{\boldsymbol{x}}_i\right)d\omega, \\ dx_n = \left(g_{n,\sigma(t)}\left(\bar{\boldsymbol{x}}_n\right)u + \ell_{n,\sigma(t)}\left(\bar{\boldsymbol{x}}_n\right)\right)dt \\ \qquad + h^{\mathrm{T}}_{n,\sigma(t)}\left(\bar{\boldsymbol{x}}_n\right)d\omega, \\ y = x_1, \end{cases} \tag{1}$$

where in system (1), $i = 1, 2, \cdots, n-1$. $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the system state vector, $\bar{\boldsymbol{x}}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}} \in \mathbb{R}^i$. $y \in \mathbb{R}$ is the system output. $\sigma(t) : [0,\infty) \to M = \{1, 2, \cdots, m\}$ stands for a piecewise continuous switching signal, and $\sigma(t) = j$, $j \in M$ implies that the j-th subsystem is active. $\ell_{i,\sigma(t)}(\cdot) : \mathbb{R}^i \to \mathbb{R}$, $g_{i,\sigma(t)}(\cdot) : \mathbb{R}^i \to \mathbb{R}$ and $h_{i,\sigma(t)}(\cdot) : \mathbb{R}^i \to \mathbb{R}^r$ are unknown smooth nonlinear functions. ω represents an independent r-dimension standard Brownian motion defined on the complete probability space $(\mathcal{W}, \mathcal{F}, \mathcal{P})$ with $\mathcal{W}$ being a sample space, $\mathcal{F}$ being a σ-field and $\mathcal{P}$ being a probability measure. $u \in \mathbb{R}$ represents the output of the dead-zone, which can be expressed as the following form.

$$u = D(\upsilon) = \begin{cases} g_r(\upsilon), & \upsilon \geq q_r, \\ 0, & q_l < \upsilon < q_r, \\ g_l(\upsilon), & \upsilon \leq q_l, \end{cases} \tag{2}$$

with υ denotes input of dead-zone. q_r and q_l are constants.

The control objective is to design an adaptive MTN controller such that all the signals in the closed-loop system remain bounded in probability, and the system output y follows the given reference signal y_d.

To facilitate control system design, we need the following assumptions.

Assumption 1: All the state variables of system (1) are assumed to be available and measurable.

Assumption 2 [42]**:** For $\forall t \geq 0$ and $\underline{Y_0}, \bar{Y}_0, Y_1, \cdots, Y_n > 0$ are constants, the reference signal y_d and up to the n-th order time derivative satisfy $-\underline{Y_0} \leq y_d(t) \leq \bar{Y}_0$, $|\dot{y}_d(t)| < Y_1$, $\cdots$, $\left|y_d^{(n)}(t)\right| < Y_n$.

Assumption 3: The dead-zone output u is not available.

Assumption 4: The dead-zone parameters $q_r > 0$ and $q_l < 0$ are unknown bounded constants.

Assumption 5 [36]**:** The functions $g_r(\upsilon)$ and $g_l(\upsilon)$ are smooth, there exist unknown positive constants $k_{l,0}$, $k_{l,1}$, $k_{r,0}$ and $k_{r,1}$ such that

$$\begin{cases} 0 < k_{l,0} \leq g'_l(\upsilon) \leq k_{l,1}, & \forall \upsilon \in (-\infty, \ q_l], \\ 0 < k_{r,0} \leq g'_r(\upsilon) \leq k_{r,1}, & \forall \upsilon \in [q_r, \ +\infty), \end{cases} \tag{3}$$

where $g'_l(\upsilon) = \left.\frac{dg_l(z)}{dz}\right|_{z=\upsilon}$, $g'_r(\upsilon) = \left.\frac{dg_r(z)}{dz}\right|_{z=\upsilon}$. $\beta_0 \leq \min\{k_{l,0}, k_{r,0}\}$ is a known positive constant.

Assumption 6: For $i = 1, 2, \cdots, n$ and $j \in M$, the sign of function $g_{i,j}$ does not change, there exist constants q_m and q_M such that $0 < q_m \leq |g_{i,j}(\bar{\boldsymbol{x}}_i)| \leq q_M < \infty$.

Based on Assumption 5 and [36], the dead-zone (2) can be rewritten as

$$u = D(\upsilon) = \Psi^{\mathrm{T}}(t)\lambda(t)\upsilon + d(\upsilon), \tag{4}$$

where $\lambda(t) = [\varphi_r(t),\ \varphi_r(t)]^{\mathrm{T}}$, $\Psi(t) = [\Psi_r(\upsilon(t)),\ \Psi_l(\upsilon(t))]^{\mathrm{T}}$. With $\varphi_r = 1$, $\Psi_r(\upsilon(t)) = g_r'(\xi_r(\upsilon))$ for $\upsilon > q_l$, and $\varphi_r = 0$, $\Psi_r(\upsilon(t)) = 0$ for $\upsilon \le q_l$. $\varphi_l = 0$, $\Psi_r(\upsilon(t)) = 0$ for $\upsilon \ge q_r$, and $\varphi_l = 1$, $\Psi_l(\upsilon(t)) = g_l'(\xi_l(\upsilon))$ for $\upsilon < q_r$. Moreover, $d(\upsilon) = -g_r'(\xi_r(\upsilon))q_r$, $\upsilon \ge q_r$, and $d(\upsilon) = -[g_l'(\xi_l(\upsilon)) + g_r'(\xi_r(\upsilon))]\upsilon$, $q_l < \upsilon < q_r$, and $d(\upsilon) = -g_l'(\xi_l(\upsilon))q_l$, $\upsilon \le q_l$, and $|d(\upsilon)| \le \bar{d}$, $\bar{d}$ is an unknown positive constant and $\bar{d} = (k_{l,1} + k_{r,1})\max\{q_r,\ -q_l\}$. $\xi_r(\upsilon) \in \begin{cases} (q_r, \upsilon), & \upsilon > q_r, \\ (\upsilon, q_r), & q_l < \upsilon \le q_r, \end{cases}$, $\xi_l(\upsilon) \in \begin{cases} (q_l, \upsilon), & q_l \le \upsilon < q_r, \\ (\upsilon, q_l), & \upsilon < q_l. \end{cases}$

Remark 1: According to (3) and (4), we can easily get $\beta_0 \le \Psi^{\mathrm{T}}(t)\lambda(t) \le k_{l,1} + k_{r,1} < \infty$.

2.2. Stochastic stability

Consider the following the stochastic system

$$d\boldsymbol{x} = f(\boldsymbol{x})\,dt + g(\boldsymbol{x})\,d\omega, \tag{5}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the system state. ω is an r-dimensional standard Brownian motion. $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}^r$ are the continuous functions that satisfy $f(\boldsymbol{0}) = 0$ and $g(\boldsymbol{0}) = 0$.

Definition 1 [28]**:** For any given $V(\boldsymbol{x}) \in \mathcal{C}^2$, which is associated with the stochastic system (5), the differential operator $\mathcal{L}$ is defined as follows:

$$\mathcal{L}V(\boldsymbol{x}) = \frac{\partial V(\boldsymbol{x})}{\partial \boldsymbol{x}} f + \frac{1}{2}\mathrm{Tr}\left\{ g^{\mathrm{T}} \frac{\partial^2 V(\boldsymbol{x})}{\partial \boldsymbol{x}^2} g \right\}. \tag{6}$$

Definition 2 [43]**:** The trajectory $\boldsymbol{x}$ of system (5) is said to be semi-globally uniformly ultimately bounded in p-th moment, if for some compact set $\Omega \in \mathbb{R}^n$ and any initial state $\boldsymbol{x}_0 = \boldsymbol{x}(t_0)$, there exist constant $\varepsilon > 0$ and time constant $T = T(\varepsilon, \boldsymbol{x}_0)$, such that $E[||x(t)||^p] < \varepsilon$ for all $t > t_0 + T$. In particular, when $p = 2$, it is usually called semi-globally uniformly ultimately bounded in mean square.

Definition 3 [39]**:** Consider the stochastic system (5), if there exists a function $V(\boldsymbol{x}) \in \mathcal{C}^2$, $\mu_1 \in \kappa_\infty$, $\mu_2 \in \kappa_\infty$, and constants $a_0 > 0$ and $b_0 > 0$, such that

$$\begin{aligned} &\mu_1(\boldsymbol{x}) \le V(\boldsymbol{x}) \le \mu_2(\boldsymbol{x}), \\ &\mathcal{L}V(\boldsymbol{x}) \le -a_0 V(\boldsymbol{x}) + b_0. \end{aligned} \tag{7}$$

Then, the system (5) is stochastically stable. And for each $\boldsymbol{x}_0 \in \mathbb{R}^n$, and it satisfies $E[V(\boldsymbol{x})] \le V(\boldsymbol{x}_0)e^{-a_0 t} + \frac{b_0}{a_0}$, $\forall t > t_0$.

2.3. Multi-dimensional Taylor network

MTN is a feedforward network composed of the input layer, the intermediate layer and the output layer. In this article, an unknown smooth nonlinear function will be approximated on a compact set Ω_z by MTN. Then, the following lemma is stated for function approximation.

Lemma 1 [28]**:** Assume that $f(z)$ is a continuous function defined on a compact set Ω_z , then, for $\forall\varepsilon > 0$, there exists a MTN $\boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(z)$, such that $f(z) = \boldsymbol{\theta}^{*\mathrm{T}} P_{m_n}(z) + \delta(z)$, where $\delta(z)$ denotes the approximation error and $|\delta(z)| \le \varepsilon$. $P_{m_n}(\boldsymbol{z}) \triangleq [z_1,\ \cdots,\ z_n,\ z_1^2,\ \cdots,\ z_n^2,\ \cdots,\ z_1^m,\ \cdots,\ z_n^m]^{\mathrm{T}} \in \mathbb{R}^l$ is the middle layer vector of MTN. $\boldsymbol{z} = [z_1,\ z_2,\ \cdots,\ z_n]^{\mathrm{T}} \in \Omega_z \subset \mathbb{R}^n$ and $\boldsymbol{\theta}^* = [\theta_1^*,\ \cdots,\ \theta_l^*]^{\mathrm{T}} \in \mathbb{R}^l$ are the input vector and the ideal weight vector of MTN. And the ideal weight vector $\boldsymbol{\theta}^* = [\theta_1,\ \cdots,\ \theta_l]^{\mathrm{T}}$ is defined as $\boldsymbol{\theta}^* := \arg\min_{\boldsymbol{\theta}\in\mathbb{R}} \left\{ \sup_{z\in\Omega_z} \left| f(z) - \boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(z) \right| \right\}$.

Remark 2: The structure of MTN is similar to the structure of radial basis function neural network (RBFNN) and MTN can be regarded as a RBFNN with special structure. However, different from RBFNN, MTN adopts polynomials instead of basis functions to approximate nonlinearity, so the computation complexity is greatly reduced.

3. MAIN RESULTS

In this section, control design and stability analysis procedure of the system will be presented by using adaptive backstepping technique.

Now, introduce the coordinate transformation described by $z_i = x_i - \alpha_{i-1}$, $i = 1, \cdots, n$, where $\alpha_0 = y_d$ and α_{i-1} are the intermediate virtual control signals, which will be designed later.

Then, we can get the following system

$$\begin{cases} dz_i = \left(g_{i,\sigma(t)}(\bar{\boldsymbol{x}}_i)x_{i+1} + \ell_{i,\sigma(t)}(\bar{\boldsymbol{x}}_i) - \nabla_{\alpha_{i-1}}\right)dt \\ \qquad + \bar{h}_{i,\sigma(t)}^{\mathrm{T}} d\omega,\ 1 \le i \le n-1, \\ dz_n = \left(g_{n,\sigma(t)}(\bar{\boldsymbol{x}}_n)\left(\Psi^{\mathrm{T}}(t)\lambda(t)\upsilon + d(\upsilon)\right)\right. \\ \qquad \left. + \ell_{n,\sigma(t)}(\bar{\boldsymbol{x}}_n) - \nabla_{\alpha_{n-1}}\right)dt + \bar{h}_{n,\sigma(t)}^{\mathrm{T}} d\omega, \end{cases} \tag{8}$$

where $\nabla_{\alpha_0} = \dot{y}_d$, $\bar{h}_{i,\sigma(t)}^{\mathrm{T}} = \left(h_{i,\sigma(t)}(\bar{\boldsymbol{x}}_i) - \sum_{j=1}^{i-1} \frac{\partial\alpha_{i-1}}{\partial x_j} h_{j,\sigma(t)}(\bar{\boldsymbol{x}}_i)\right)^{\mathrm{T}}$, $\nabla_{\alpha_{i-1}} = \sum_{j=1}^{i-1} \frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(\ell_{j,\sigma(t)}(\bar{\boldsymbol{x}}_j) + g_{j,\sigma(t)}(\bar{\boldsymbol{x}}_j)x_{j+1}\right) + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p \partial x_q} h_{p,\sigma(t)}^{\mathrm{T}} h_{q,\sigma(t)}$.

3.1. Adaptive MTN control design

Step 1: Consider the Lyapunov function as follows:

$$V_1 = \frac{1}{4}z_1^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{1,k}, \tag{9}$$

where $k \in M$ and $\tilde{\boldsymbol{\theta}}_{1,k} = \boldsymbol{\theta}_{1,k} - \hat{\boldsymbol{\theta}}_{1,k}$ is the parameter error.

According to Definition 1 and (9), we have

$$\mathcal{L}V_1 \leq z_1^3\left(g_{1,k}x_2+\bar{\ell}_{1,k}\right)-\frac{3}{4}z_1^4+\frac{3}{4}l_1^2-\tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{1,k}, \quad (10)$$

where $l_1>0$ is a design constant and $\bar{\ell}_{1,k}=\ell_{1,k}-\dot{y}_d+\frac{3}{4}z_1+\frac{3}{4l_1^2}z_1\left\|\bar{h}_{1,k}\right\|^4$.

On the other hand, by virtue of Lemma 1, a MTN can be employed to estimate the unknown function $\bar{\ell}_{1,k}$, that is to say, for any given $\varepsilon_{1,k}>0$, there exists a MTN as $\boldsymbol{\theta}_{1,k}^{\mathrm{T}}P_{m_1}(\boldsymbol{z}_1)$, such that

$$\bar{\ell}_{1,k}=\boldsymbol{\theta}_{1,k}^{\mathrm{T}}P_{m_1}(\boldsymbol{z}_1)+\delta_{1,k}(\boldsymbol{z}_1),\ |\delta_{1,k}(\boldsymbol{z}_1)|\leq\varepsilon_{1,k}, \quad (11)$$

where $\delta_{1,k}(\boldsymbol{z}_1)$ is the approximation error and $\boldsymbol{z}_1=[z_1]^{\mathrm{T}}$.

According to Young's Inequality, we have

$$z_1^3\bar{\ell}_{1,k}\leq z_1^3\boldsymbol{\theta}_{1,k}^{\mathrm{T}}P_{m_1}+\frac{3}{4}z_1^4+\frac{1}{4}\varepsilon_{1,max}^4, \quad (12)$$

where $\varepsilon_{1,\max}=\max\{\varepsilon_{1,k};\,k\in M\}$.

Considering that $x_2=z_2+\alpha_1$ and substituting (12) into (10) gives

$$\begin{aligned}\mathcal{L}V_1 \leq\ & z_1^3(g_{1,k}z_2+g_{1,k}\alpha_1)+z_1^3\hat{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}P_{m_1}\\ &+\frac{1}{4}\varepsilon_{1,\max}^4+\frac{3}{4}l_1^2+\tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}\left(z_1^3P_{m_1}-\dot{\hat{\boldsymbol{\theta}}}_{1,k}\right). \quad (13)\end{aligned}$$

Choosing the intermediate virtual control signal α_1 as

$$\alpha_1=-\frac{1}{q_m}\left(r_1|z_1|+\left|\hat{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}P_{m_1}\right|\right)\operatorname{sgn}(z_1), \quad (14)$$

where $r_1>0$ is a constant.

The following inequalities are true by making use of Young's Inequality

$$z_1^3g_{1,k}z_2\leq\frac{3}{4}g_{1,k}z_1^4+\frac{1}{4}g_{1,k}z_2^4. \quad (15)$$

Substituting (14) and (15) into (13) gives

$$\begin{aligned}\mathcal{L}V_1 \leq & -\left(r_1-\frac{3}{4}g_{1,k}\right)z_1^4+\frac{1}{4}g_{1,k}z_2^4+\frac{1}{4}\varepsilon_{1,\max}^4\\ &+\frac{3}{4}l_1^2+\tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}\left(z_1^3P_{m_1}-\dot{\hat{\boldsymbol{\theta}}}_{1,k}\right). \quad (16)\end{aligned}$$

Step i ($2\leq i\leq n-1$)**:** Consider the Lyapunov function as follows:

$$V_i=V_{i-1}+\frac{1}{4}z_i^4+\frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,k}, \quad (17)$$

where $\tilde{\boldsymbol{\theta}}_{i,k}=\boldsymbol{\theta}_{i,k}-\hat{\boldsymbol{\theta}}_{i,k}$ are the parameter error.

By Definition 1 and Young's Inequality, we have

$$\begin{aligned}\mathcal{L}V_i \leq\ & \mathcal{L}V_{i-1}+z_i^3\left(g_{i,k}x_{i+1}+\bar{\ell}_{i,k}\right)-\frac{3}{4}z_i^4+\frac{3}{4}l_i^2\\ &-\tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{i,k}, \quad (18)\end{aligned}$$

where $l_i>0$ are design constants and $\bar{\ell}_{i,k}=\ell_{i,k}-\nabla_{\alpha_{i-1}}+\frac{3}{4}z_i+\frac{3}{4l_i^2}z_i||\bar{h}_{i,k}||^4$.

Similarly, by virtue of Lemma 1, for any given $\varepsilon_{i,k}>0$, there exists a MTN as $\boldsymbol{\theta}_{i,k}^{\mathrm{T}}P_{m_i}$ such that

$$\bar{\ell}_{i,k}=\boldsymbol{\theta}_{i,k}^{\mathrm{T}}P_{m_i}(\boldsymbol{z}_i)+\delta_{i,k}(\boldsymbol{z}_i),\ |\delta_{i,k}(\boldsymbol{z}_i)|\leq\varepsilon_{i,k}, \quad (19)$$

where $\delta_{i,k}(\boldsymbol{z}_i)$ are the approximation error and $\boldsymbol{z}_i=[z_1,\ldots,z_i]^{\mathrm{T}}$.

According to Young's Inequality, we have

$$z_i^3\bar{\ell}_{i,k}\leq z_i^3\boldsymbol{\theta}_{i,k}^{\mathrm{T}}P_{m_i}+\frac{3}{4}z_i^4+\frac{1}{4}\varepsilon_{i,max}^4, \quad (20)$$

where $\varepsilon_{i,\max}=\max\{\varepsilon_{i,k};\,k\in M\}$.

Considering that $x_{i+1}=z_{i+1}+\alpha_i$ and substituting (20) into (18) give

$$\begin{aligned}\mathcal{L}V_i \leq\ & \mathcal{L}V_{i-1}+z_i^3(g_{i,k}z_{i+1}+g_{i,k}\alpha_i)+z_i^3\hat{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}P_{m_i}\\ &+\frac{1}{4}\varepsilon_{i,max}^4+\frac{3}{4}l_i^2+\tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\left(z_i^3P_{m_i}-\dot{\hat{\boldsymbol{\theta}}}_{i,k}\right). \quad (21)\end{aligned}$$

Choosing the intermediate virtual control law α_i as

$$\alpha_i=-\frac{1}{q_m}\left(r_i|z_i|+\left|\hat{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}P_{m_i}\right|\right)\operatorname{sgn}(z_i), \quad (22)$$

where $r_i>0$ are constants.

By Young's inequality, the following inequalities hold

$$z_i^3g_{i,k}z_{i+1}\leq\frac{3}{4}g_{i,k}z_i^4+\frac{1}{4}g_{i,k}z_{i+1}^4. \quad (23)$$

According to Math Induction, substituting (22) and (23) into (21) give

$$\begin{aligned}\mathcal{L}V_i \leq & -\sum_{j=1}^{i}\left(r_j-\frac{3}{4}g_{j,k}\right)z_j^4+\frac{1}{4}\sum_{j=1}^{i}g_{j,k}z_{j+1}^4\\ &+\sum_{j=1}^{i}\tilde{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}}\left(z_j^3P_{m_j}-\dot{\hat{\boldsymbol{\theta}}}_{j,k}\right)+\frac{1}{4}\sum_{j=1}^{i}\varepsilon_{j,\max}^4\\ &+\frac{3}{4}\sum_{j=1}^{i}l_j^2. \quad (24)\end{aligned}$$

Step n**:** Consider the Lyapunov function as follows:

$$V_n=V_{n-1}+\frac{1}{4}z_n^4+\frac{1}{2}\tilde{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{n,k}, \quad (25)$$

where $\tilde{\boldsymbol{\theta}}_{n,k}=\boldsymbol{\theta}_{n,k}-\hat{\boldsymbol{\theta}}_{n,k}$ is the parameter error.

By Definition 1 and Young's Inequality, we have

$$\begin{aligned}\mathcal{L}V_n \leq & \mathcal{L}V_{n-1}+z_n^3\left(g_{n,k}\left(\Psi^{\mathrm{T}}\lambda\upsilon+d(\upsilon)\right)+\bar{\ell}_{n,k}\right)\\ &-\frac{3}{4}z_n^4+\frac{3}{4}l_n^2-\tilde{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{n,k}, \quad (26)\end{aligned}$$

where $l_n>0$ is a design constant and $\bar{\ell}_{n,k}=\ell_{n,k}-\nabla_{\alpha_{n-1}}+\frac{3}{4}z_n+\frac{3}{4l_n^2}z_n||\bar{h}_{n,k}||^4$.

Similarly, by virtue of Lemma 1, for any given $\varepsilon_{n,k} > 0$, there exists a MTN as $\boldsymbol{\theta}_{n,k}^{\mathrm{T}} P_{m_n}$, such that

$$\bar{\ell}_{n,k} = \boldsymbol{\theta}_{n,k}^{\mathrm{T}} P_{m_n}(z_n) + \delta_{n,k}(z_n), \ |\delta_{n,k}(z_n)| \leq \varepsilon_{n,k}, \quad (27)$$

where $\delta_{n,k}(z_n)$ is the approximation error and $z_n = [z_1, \ldots, z_n]^{\mathrm{T}}$.

By Young's Inequality and (27), we have

$$z_n^3 \bar{\ell}_{n,k} \leq z_n^3 \boldsymbol{\theta}_{n,k}^{\mathrm{T}} P_{m_n} + \frac{3}{4} z_n^4 + \frac{1}{4} \varepsilon_{n,\max}^4, \quad (28)$$

where $\varepsilon_{n,\max} = \max\{\varepsilon_{n,k}; k \in M\}$.

Construct the actual control input signal υ as follows:

$$\upsilon = -\frac{1}{\beta_0 q_m} \left(r_n |z_n| + \left| \hat{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}} P_{m_n} \right| \right) \operatorname{sgn}(z_n), \quad (29)$$

where $r_n > 0$ is a constant.

By Young's Inequality and (29), we have

$$z_n^3 g_{n,k} \Psi^{\mathrm{T}} \lambda \upsilon \leq -r_n z_n^4 - \left| z_n^3 \hat{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}} P_{m_n} \right|, \quad (30)$$

$$z_n^3 g_{n,k} d(\upsilon) \leq \frac{3}{4} g_{n,k} z_n^4 + \frac{1}{4} g_{n,k} \bar{d}^4. \quad (31)$$

According to Assumption 6, substituting (24), (28), (30), and (31) into (26) gives

$$\begin{aligned} \mathcal{L}V_n \leq & -\sum_{j=1}^{n} c_j z_j^4 + \sum_{j=1}^{n} \tilde{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}} \left(z_j^3 P_{m_j} - \dot{\hat{\boldsymbol{\theta}}}_{j,k} \right) \\ & + \frac{1}{4} \sum_{j=1}^{n} \varepsilon_{j,\max}^4 + \frac{3}{4} \sum_{j=1}^{n} l_j^2 + \frac{1}{4} q_M \bar{d}^4, \end{aligned} \quad (32)$$

where $c_j = r_j - q_M > 0$.

Remark 3: In the process of backstepping control, the combination of nonlinear functions appearing in each step can be solved by a MTN. Although NN and FLS can also obtain similar results, the computational complexity of the controller proposed in this article is significantly reduced due to its structural characteristics. Although control strategies based on NN and FLS [23,39] were proposed for switched stochastic systems, the MTN controller proposed in this article has a simpler structure.

3.2. Stability analysis

Theorem 1: Under Assumptions 1-6, consider the switched stochastic nonlinear system (1) with unknown dead-zone (2). If the control input υ is chosen as (29), the intermediate virtual control signals α_i are described as (14) and (22), with the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_{i,k}$ are defined as

$$\dot{\hat{\boldsymbol{\theta}}}_{i,k} = -\eta_i \hat{\boldsymbol{\theta}}_{i,k} + z_i^3 P_{m_i}, \ i = 1, \ \cdots, \ n, \quad (33)$$

where η_i are positive constants. Then, for any bounded initial conditions, the proposed adaptive MTN control scheme can guarantee that all signals in the closed-loop system are semi-globally bounded and the tracking error converges to a small neighborhood around the origin.

Proof: Consider the following Lyapunov function

$$V = V_n = \frac{1}{4} \sum_{i=1}^{n} z_i^4 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_{i,k}. \quad (34)$$

Combining (32) and (33), one has

$$\begin{aligned} \mathcal{L}V \leq & -\sum_{j=1}^{n} c_j z_j^4 + \sum_{j=1}^{n} \eta_j \tilde{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}} \hat{\boldsymbol{\theta}}_{j,k} \\ & + \frac{1}{4} \sum_{j=1}^{n} \varepsilon_{j,\max}^4 + \frac{3}{4} \sum_{j=1}^{n} l_j^2 + \frac{1}{4} q_M \bar{d}^4. \end{aligned} \quad (35)$$

For the term $\sum_{j=1}^{n} \eta_j \tilde{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}} \hat{\boldsymbol{\theta}}_{j,k}$, we have

$$\sum_{j=1}^{n} \eta_j \tilde{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}} \hat{\boldsymbol{\theta}}_{j,k} \leq -\frac{1}{2} \eta \sum_{j=1}^{n} \tilde{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_{j,k} + \frac{1}{2} \sum_{j=1}^{n} \eta_j \|\boldsymbol{\theta}_{j,k}\|^2, \quad (36)$$

where $\eta = \min\{\eta_j; \ j = 1, 2, \cdots, n\}$.

Substituting (36) into (35) gives

$$\begin{aligned} \mathcal{L}V \leq & -\sum_{j=1}^{n} c_j z_j^4 - \frac{1}{2} \eta \sum_{j=1}^{n} \tilde{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_{j,k} + \frac{1}{4} q_M \bar{d}^4 \\ & + \frac{1}{2} \sum_{j=1}^{n} \eta_j \left\| \tilde{\boldsymbol{\theta}}_{j,k} \right\|^2 + \frac{1}{4} \sum_{j=1}^{n} \varepsilon_{j,\max}^4 + \frac{3}{4} \sum_{j=1}^{n} l_j^2 \\ \leq & -a_0 V + b_0, \end{aligned} \quad (37)$$

with $a_0 = \min\{2c_j, \ \eta \mid j = 1, \ \cdots, \ n\}$ and $b_0 = \frac{1}{2} \sum_{j=1}^{n} \eta_j \left\| \tilde{\boldsymbol{\theta}}_{j,k} \right\|^2 + \frac{1}{4} \sum_{j=1}^{n} \varepsilon_{j,\max}^4 + \frac{3}{4} \sum_{j=1}^{n} l_j^2 + \frac{1}{4} q_M \bar{d}^4$.

From (37), Definition 3 and the method similar to [44], we can conclude that all signals in the closed-loop system can be guaranteed to be semi-globally bounded in probability. Moreover, $E\left[|y - y_d|^4\right] \leq 4E[V(t)] \leq \frac{8b_0}{a_0}$, therefore, the tracking error converges to a small neighborhood around the origin.

In addition, the design process of the MTN-based adaptive controller can be summarized in Fig. 1. □

4. SIMULATION RESULTS

Example 1: In this section, to demonstrate the effectiveness of the proposed approach, we consider the following switched stochastic nonlinear system

$$\begin{cases} dx_1 = \left(g_{1,\sigma(t)} x_2 + \ell_{1,\sigma(t)} \right) dt + h_{1,\sigma(t)}^{\mathrm{T}} d\omega, \\ dx_2 = \left(g_{2,\sigma(t)} x_3 + \ell_{2,\sigma(t)} \right) dt + h_{2,\sigma(t)}^{\mathrm{T}} d\omega, \\ dx_3 = \left(g_{3,\sigma(t)} u + \ell_{3,\sigma(t)} \right) dt + h_{3,\sigma(t)}^{\mathrm{T}} d\omega, \\ y = x_1, \end{cases} \quad (38)$$

with the dead-zone $D(\upsilon)$ defined as

$$u = D(\upsilon) = \begin{cases} 1.2(\upsilon - 2), & \upsilon \geq 2, \\ 0, & -0.5 < \upsilon < 2, \\ \upsilon + 0.5, & \upsilon \leq -0.5, \end{cases} \quad (39)$$

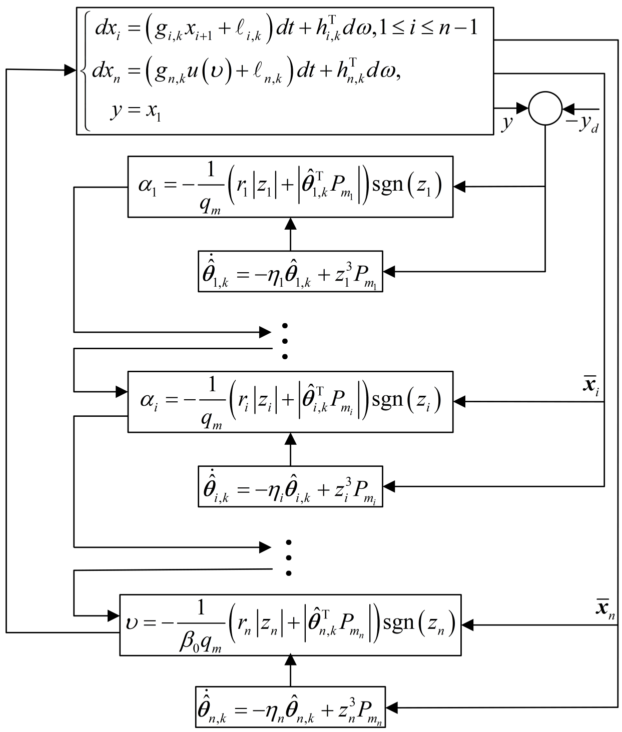

Fig. 1. The design process of the MTN-based adaptive controller.

where $\sigma(t) : [0, \infty) \to M = \{1, 2\}$ and $g_{1,1} = 1$, $g_{1,2} = 1 + \frac{x_1^2}{1+x_1^2}$, $\ell_{1,1} = -2x_1 e^{-0.5x_1}$, $\ell_{1,2} = -2\sin x_1 e^{-0.5x_1}$, $g_{2,1} = 1$, $g_{2,2} = 1 + \sin x_2^2$, $\ell_{2,1} = -3x_1 \cos x_2^2$, $\ell_{2,2} = -x_1 \sin x_2^2$, $g_{3,1} = 1$, $g_{3,2} = 1 + 0.5\sin x_1 x_2$, $\ell_{3,1} = -2x_2 x_3$, $\ell_{3,2} = -x_1 x_2 x_3$, $h_{1,1} = x_1$, $h_{2,1} = x_2 \sin x_1$, $h_{3,1} = \sin x_3$, $h_{1,2} = \sin x_1$, $h_{2,2} = x_1 \sin x_2$, $h_{3,2} = x_1 \sin x_3$. According to Theorem 1, the intermediate virtual control signals α_1, α_2, the true control law υ and the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_{1,k}$, $\dot{\hat{\boldsymbol{\theta}}}_{2,k}$, $\dot{\hat{\boldsymbol{\theta}}}_{3,k}$ are designed. The design parameters are chosen as $q_m = 0.5$, $\beta_0 = 0.2$, $r_1 = 10$, $r_2 = 4$, $r_3 = 1$, $\eta_1 = \eta_2 = \eta_3 = 1$. The simulation results are shown in Figs. 2-4. Fig. 2 shows the system output y and the reference signal y_d. From Fig. 2, it can be seen that the good tracking performance has been achieved. Fig. 3 indicates the trajectories of dead-zone output u and control input υ. Fig. 4 demonstrates that the state variables x_2, x_3 and switching signal $\sigma(t)$. It can be seen from the simulation results in Figs. 2-4 that the presented adaptive design scheme can guarantee the boundedness of the closed-loop system.

Example 2: Consider a class of continuous stirred tank reactor with two modes feed stream under multiplicative white noise [17].

$$\begin{cases} dx_1 = \left(x_2 + \ell_{1,\sigma(t)}\right) dt + \frac{1}{4}\ell_{1,\sigma(t)} d\omega, \\ dx_2 = u dt, \\ y = x_1, \end{cases} \tag{40}$$

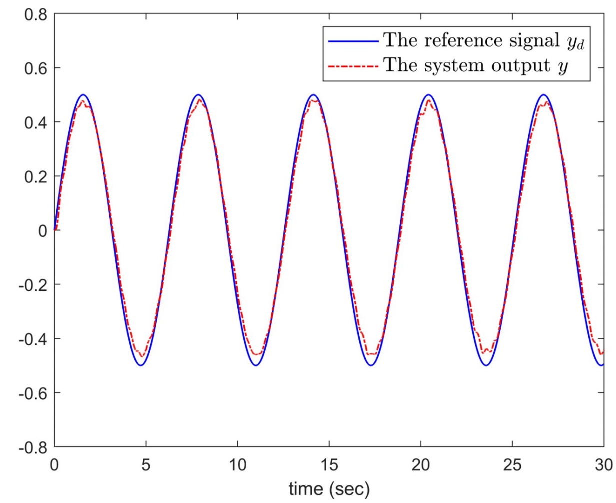

Fig. 2. The trajectories of output y and the reference signal y_d in Example 1.

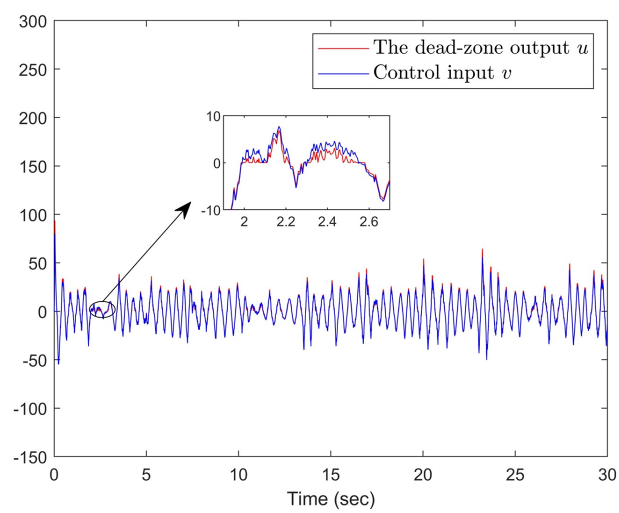

Fig. 3. The trajectories of u and υ in Example 1.

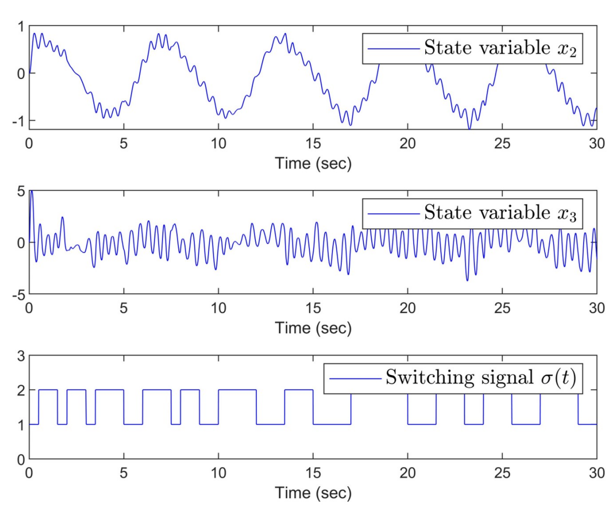

Fig. 4. The trajectories of state variables x_2, x_3 and switching signal $\sigma(t)$ in Example 1.

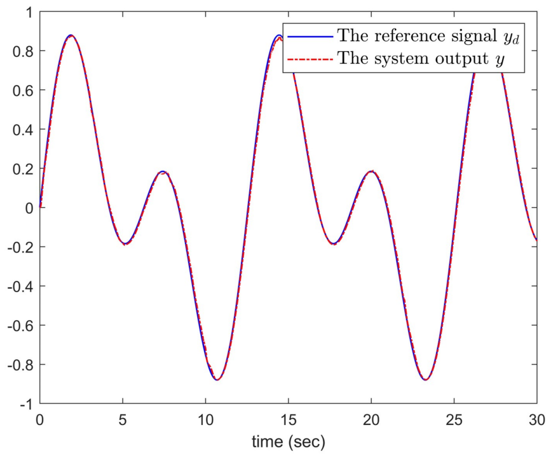

Fig. 5. The trajectories of output y and the reference signal y_d in Example 2.

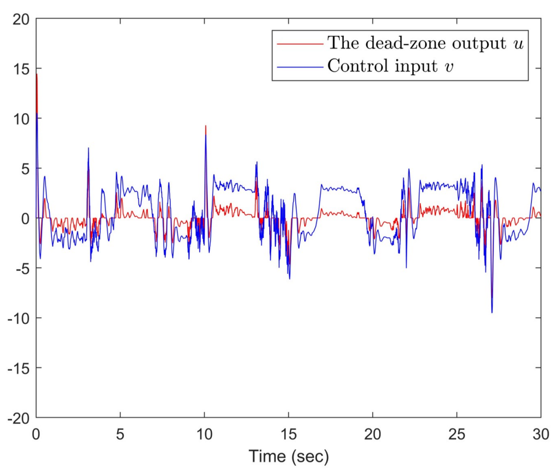

Fig. 6. The trajectories of u and υ in Example 2.

where $\sigma(t) : [0, +\infty) \to \{1, 2\}$, $\ell_{1,1} = -0.5x_1$, $\ell_{1,2} = -2x_1$. The initial condition is $[x_1(0), x_2(0)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$. The reference signal is $y_d = 0.5(\sin t + \sin(0.5t))$.

In order to give the simulation results, we assume that

$$u = \begin{cases} 0.1(\upsilon - 2.5)^2 + (\upsilon - 2.5), & \upsilon \geq 2.5, \\ 0, & -1.5 < \upsilon < 2.5, \\ (\upsilon + 1.5), & \upsilon \leq -1.5. \end{cases} \tag{41}$$

Similarly, according to Theorem 1, the intermediate virtual control signal α_1, the true control law υ and the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_{1,k}, \dot{\hat{\boldsymbol{\theta}}}_{2,k}$ are designed. The design parameters are chosen as $r_1 = 20$, $r_2 = 1$, $\eta_1 = 10$ and $\eta_2 = 10$. The simulation results are shown in Figs. 5-7. It can be seen that the tracking control performance is still fairly satisfactory, which further verify the effectiveness of the control method designed in this article.

Remark 4: In order to obtain excellent tracking performance, a set of optimized parameters is selected based on experience by continuously adjusting the parameters.

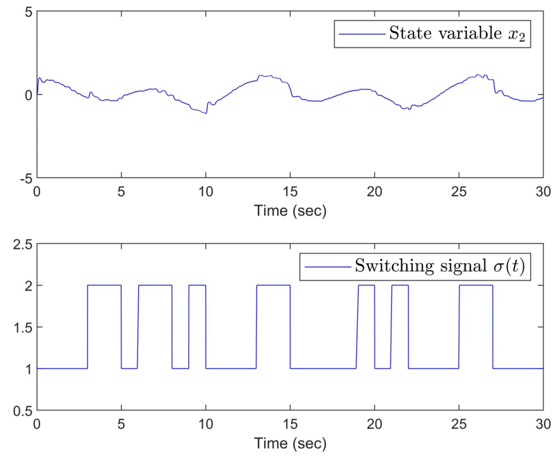

Fig. 7. The trajectories of state variables x_2 and switching signal $\sigma(t)$ in Example 2.

5. CONCLUSION

This article copes with adaptive tracking control problem for switched stochastic nonlinear systems subject to unknown input dead-zone. First, for the nonlinear problem caused by the input dead-zone, the input dead-zone is transformed into a linear model by introducing the characteristic function. Then, in the control process, a novel adaptive tracking controller is proposed based on CLF backstepping design method and MTN approximation performance. The derivation results show that all the signals of the closed loop system are bounded in probability. Significantly, a method to reduce the computational complexity of switched stochastic nonlinear systems by introducing characteristic function and MTN is proposed for the first time. Finally, two examples are given to verify the effectiveness of the proposed controller.

Based on the research results in this article, future research direction will turn to the design of adaptive MTN controller for switched stochastic nonlinear systems subject to output constraints.

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