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MTN-Based Adaptive Finite-Time Tracking Control for Switched Non-Linear Systems With Time-Varying State Function Constraints

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ABSTRACT

This paper studies the finite-time control problem of a class of switched non-linear systems under time-varying state constraints and proposes an adaptive finite-time controller based on a multi-dimensional Taylor network (MTN). Firstly, the time-varying tangent barrier Lyapunov functions (TBLFs) are constructed to ensure that all system states are constrained within a certain range. Secondly, MTNs are used to estimate the unknown non-linear functions during the controller design process. The proposed control scheme ensures that the tracking error of the system can converge to a small domain of the origin in a finite-time. At the same time, all the signals of the closed-loop system are semi-global practical finite-time stable (SGPFS) and all states satisfy the defined time-varying state constraints. Finally, the effectiveness of the control strategy is verified through numerical simulation examples and practical simulation examples.

1 | Introduction

Switched systems, which constitute a unique class of hybrid systems, are comprised of finite subsystems and switching signals, which are widely used in automobile test-driving systems [1], circuit systems [2] and aircraft engine systems [3]. In recent years, scholars have thoroughly studied the stability problems of various types of switched systems [4–6] by utilizing backstepping techniques and adaptive control methods. Meanwhile, taking into account the unknown non-linearities inherent in the system modeling process, numerous research results on switched non-linear systems by combining with intelligent control and adaptive control methods have been presented, such as switched non-linear systems [7], stochastic switched non-linear systems [8] and switched multi-agent systems [9]. However, the aforementioned achievements have neglected the control challenges encountered by systems in real-world environments under state-constrained conditions. Furthermore, the majority of these focus on the steady-state performance of the system, disregarding its dynamic performance.

In engineering applications, it is often encountered that the actual model is state-limited. The state-constrained region can be regarded as both the set of safe states to ensure the stable operation of the system and the optimal working interval of the state in the industrial system. In order to deal with this kind

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of system with constraints, the introduction of the barrier Lyapunov functions (BLFs) in the controller design process is an effective solution. This technique was first proposed in [10] to design an asymptotic tracking approach for output-constrained non-linear systems. Since then, a series of barrier functions, such as integral-type [11], log-type [12] and tan-type [13] have been proposed. Among them, the BLFs have been applied by many scholars to non-linear systems with output constraints [14, 15], and full state constraints [16–19]. The aforementioned results concentrate on scenarios where the system states are constrained by constant values rather than time-varying constraints. In order to solve this problem, the authors in [20] innovatively proposed an adaptive neural network control strategy for non-linear systems considering time-varying state function constraints. In addition, the authors in [21] focused on multi-input multi-output non-linear systems and designed an adaptive fuzzy self-triggered tracking control method with time-varying full state constraints. The authors in [22] further proposed a novel adaptive fuzzy control strategy for non-strict feedback non-linear systems containing input saturation and time-varying full-state constraints. The authors in [23] explored the output feedback tracking control problem for non-linear systems with asymmetric time-varying state constraints. Although the above methods have shown remarkable results in dealing with the control of non-linear systems with state constraints, it is still a major challenge to achieve effective control under arbitrary switching in the field of non-linear switching systems. In addition, the application of these research results in engineering practice is limited because they often fail to meet the stringent requirements for finite convergence time in the engineering field.

Finite-time control research has gained widespread attention due to its ability to enhance system stability, increase steady-state accuracy, and ensure rapid system response within a finite-time to achieve desired states or performance indices. For example, the authors in [24] proposed the finite-time Lyapunov stability theorem for the first time, which laid the fundamental theory of finite-time control. Subsequently, the authors in [25] provided sufficient conditions for finite-time stability. So far, finite-time control has been successfully applied to solve control design problems for a large number of systems, such as large-scale non-linear systems with output constraints [26], stochastic non-linear systems with time-varying full-state constraints [27, 28], multi-agent non-linear systems with quantized input [29]. In recent years, finite-time control has also been applied to the field of switched systems, including based on state-dependent switching methods switched non-linear systems [30], switched non-linear systems with dynamic uncertainties [31]. However, in non-linear systems capable of arbitrary switching, the application of MTN technology to solve the finite-time control problem under time-varying state constraints not only holds significant value in practical applications but also, due to the continuous evolution of system adaptability and adjustment requirements, heralds the need for further exploration and research in this field.

With the above analysis, the purpose of this paper is to solve the finite-time control problem for a class of switched non-linear systems with time-varying state constraints. The proposed control scheme ensures that the tracking error of the system converges to a small domain at the origin in a finite-time, all signals of the closed-loop system are semi-global practical finite-time stable (SGPFS) and all states satisfy the defined time-varying state constraints. Compared with the existing works, the main contributions of this paper are as follows:

1. A novel MTN-based adaptive finite-time tracking control approach for switched non-linear systems with time-varying state constraints is proposed. The utilization of MTNs instead of traditional NNs can simplify the computational complexity of the controller. Although the MTN technology has been extensively studied in switched non-linear systems [32–34], the issue of finite-time control was ignored. In contrast to [26, 27, 35], which are limited to addressing a single issue within the realm of finite-time control and time-varying full-state constraints, this paper concurrently tackles both aforementioned challenges for switched non-linear systems under arbitrary switching conditions.
2. Different from the finite-time schemes proposed in literature [31, 35, 36], a new adaptive MTN scheme is proposed to achieve the tracking control of the switched non-linear system, which has the advantages of wide applicability and simple structure. In addition, by utilizing TBLFs, the control method proposed in [20, 37] can only handle the problem of non-linear systems subject to time-varying constraints. In contrast, by applying the finite-time control method, the designed approach can be applied to switched non-linear systems with time-varying state function constraints. Therefore, the conclusions of this paper are more representative and meaningful in practical applications.

2 | Formulation of the Problem and Preliminary Preparation

2.1 | Problem Description

The following switched non-linear system is considered

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i^{\sigma(t)}\left(\overline{\boldsymbol{x}}_i\right) + \Delta_i(t) \\ 1 \leq i \leq n-1 \\ \dot{x}_n = u + f_n^{\sigma(t)}\left(\overline{\boldsymbol{x}}_n\right) + \Delta_n(t) \\ y = x_1 \end{cases} \tag{1}$$

where $\overline{\boldsymbol{x}}_i = \left[x_1, \ldots, x_i\right]^{\mathrm{T}} \in R^i (i = 1, 2, \ldots, n)$ represents the state variable. $u \in R$ and $y \in R$ are the system input and system output, respectively. $\sigma(t) : [0, \infty) \to M = \{1, 2, 3, \ldots, m\}$ is the switching signal, which represents that the subsystem of order i is active. $f_i^k\left(\overline{x}_i\right)$ represents the unknown smooth non-linear function and satisfies $f_i^k(0) = 0$. For $i = 1, 2, \ldots, n$, $\Delta_i(t)$ is the time-varying disturbance, which satisfies $\left|\Delta_i(t)\right| \leq \Lambda_i$ with $\Lambda_i \geq 0$ being constant. In addition, all state variables should be satisfied $-k_{c_i}(t) \leq x_i(t) \leq k_{c_i}(t)$, where $k_{c_i}(t)$ is a time-varying function of time t.

For the switched non-linear system (1), the following control objectives will be achieved in this paper

1. The tracking error of the system can converge to a small domain of the origin in a finite-time.

2. All the signals of the closed-loop system are SGPFS.
3. Each state $x_i(t)$, $i = 1, 2, \ldots, n$ belongs to its given set $\Omega_{x_i} = \left\{x_i : -k_{c_i}(t) \le x_i(t) \le k_{c_i}(t)\right\}$.

2.2 | Theoretical Preparation

To begin with, the definition of SGPFS is given as follows:

Definition 1. ([36]). The equilibrium point $\varsigma = 0$ of the non-linear system $\dot{\varsigma} = f(\varsigma)$ is SGPFS, if for all $\varsigma(t_0) = \varsigma_0$, there exists $\varepsilon > 0$ and a settling time $T(\varepsilon, \varsigma) < \infty$ to make $\|\varsigma(t)\| < \varepsilon$, for all $t \ge t_0 + T$.

Next, the following assumptions and lemmas are made to establish constraints and performance bounds:

Assumption 1. ([20]). For $\forall t \ge 0$, there exist constants $k_{c_i}^0$ and $k_{c_i}^j$ satisfy $k_{c_i}(t) \le K_{c_i}^0$ and the jth derivative of k_{c_i} satisfies $k_{c_i}^{(j)}(t) \le K_{c_i}^j$, where $i = 1, \ldots, n$ and $j = 1, \ldots, n$.

Assumption 2. ([20]). For $i = 1, \ldots, n$, it is assumed that $y_d(t)$ and the ith derivative of $y_d(t)$ are continuous and bounded. In other words, $y_d(t)$ satisfies $|y_d(t)| \le D_0(t) \le k_{c_1}(t)$, and the ith derivative of $y_d(t)$ satisfies $\left|y_d^{(i)}(t)\right| \le D_i$, where $D_0, D_1, \ldots, D_n$ are positive constants.

Lemma 1. ([38]). *For any real number $y_1, \ldots, y_n$ and any positive constant $0 < a < 1$, the next inequality holds*

$$\left(|y_1| + \cdots |y_n|\right)^a \le |y_1|^a + \cdots |y_n|^a \tag{2}$$

Lemma 2. ([36]). *The non-linear system $\dot{\varsigma} = f(\varsigma)$ is SGPFS, if there exists a smooth positive-definite function $V(\varsigma)$, constants $c > 0, 0 < \alpha < 1$, and $\tau > 0$ such that*

$$\dot{V}(\varsigma) \le -cV^{\alpha}(\varsigma) + \tau, t \ge 0 \tag{3}$$

2.3 | MTN

In this paper, MTN is used to approximate unknown non-linearities that occur in the controller design. For more details on MTN, refer to the previous works [32, 39–42].

Lemma 3. ([32]). *On a compact set Ω_χ, there exists an MTN of the form $\theta^{\mathrm{T}} S_{m_n}(\chi)$ can be used to estimate the continuous function $\Phi(\chi)$, that is to say, for any $\varepsilon > 0$, we have*

$$\Phi(\chi) = \theta^{\mathrm{T}} S_{m_n}(\chi) + \delta(\chi), |\delta(\chi)| \le \varepsilon \tag{4}$$

where $\chi \triangleq \left[\chi_1, \chi_2, \ldots, \chi_n\right]^{\mathrm{T}} \in R^n$, $S_{m_n}(\chi) \triangleq [\chi_1, \ldots, \chi_n, \chi_1^2, \ldots, \chi_n^2, \ldots, \chi_1^m, \ldots, \chi_n^m]^{\mathrm{T}} \in R^l$, and $\theta = \left[\theta_1, \theta_2, \ldots, \theta_l\right]^{\mathrm{T}} \in R^l$ denotes the input vector, the intermediate layer vector and the weight vector of MTN, respectively. $\delta(\chi)$ is the approximation error between $S_{m_n}(\chi)$ and $\Phi(\chi)$. In addition, the optimal weight vector θ is defined as $\theta := \arg\min_{\chi \in R^l}\left\{\sup_{\chi\in\Omega_\chi}\left|\Phi(\chi) - \theta^{\mathrm{T}} S_{m_n}(\chi)\right|\right\} \in R^l$.

3 | Main Results

3.1 | MTN-Based Adaptive Tracking Controller Design

First of all, define the following coordinate transformation

$$\begin{cases} z_1 = x_1 - y_d \\ z_i = x_i - \alpha_{i-1}, i = 2, \ldots, n \end{cases} \tag{5}$$

where $\alpha_{i-1}, i = 2, \ldots, n$ is the virtual control signal, which will be constructed in the backstepping.

At each step of the backstepping process, define $\theta_i = \max\left\{\|\theta_{i,k}\|^2\right\}$, $k \in M$, $i = 1, \ldots, n$, where $\theta_{i,k}$ is the weight vector of MTN. $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, $i = 1, \ldots, n$ is the estimation of θ_i.

Step 1: According to the system (1) and the coordinate transformation, choose the following time-varying TBLF candidate

$$V_1 = \frac{k_{b_1}^2(t)}{\pi}\tan\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) + \frac{1}{2}\tilde{\theta}_1^2 \tag{6}$$

where $k_{b_i}(t), i = 1, \ldots, n$ is a time-varying smooth bounded function and $|z_1| \le k_{b_1}(t)$.

Based on (1), the time derivative of $\dot{V}_1$ can be expressed as

$$\begin{aligned} \dot{V}_1 = {} & \frac{2k_{b_1}^2(t)}{\pi}\frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)}\tan\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) \\ & + z_1\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\left(\dot{z}_1 - z_1\frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)}\right) - \tilde{\theta}_1\dot{\hat{\theta}}_1 \end{aligned} \tag{7}$$

Define

$$\begin{cases} \overline{K}_1(t) = \sup\sqrt{\left(\frac{\dot{k}_{b_1}(t)}{k_{b_1}(t)}\right)^2 + \beta_1} \\ \kappa_1 = \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)z_1 \end{cases} \tag{8}$$

where $\overline{K}_1(t)$ is the time-varying gain, and $\beta_1 > 0$ is a positive parameter.

From (8), it follows that $-\frac{\kappa_1 z_1 \dot{k}_{b_1}(t)}{k_{b_1}(t)} \le \overline{K}_1\kappa_1 z_1$. Furthermore, considering $\dot{z}_1 = \dot{x}_1 - \dot{y}_d$ and $x_2 = z_2 + \alpha_1$, we have

$$\begin{aligned} \dot{V}_1 \le {} & \frac{2k_{b_1}^2(t)}{\pi}\overline{K}_1\tan\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) \\ & + z_1\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\left(\dot{z}_1 - z_1\overline{K}_1\right) - \tilde{\theta}_1\dot{\hat{\theta}}_1 \\ \le {} & z_1\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\left[\frac{2k_{b_1}^2(t)}{\pi z_1}\overline{K}_1\sin\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\right. \end{aligned}$$

$$\cos\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) + (z_2 + \alpha_1 + f_1^k(\overline{x}_1) + \Delta_1 - \dot{y}_d)\Bigg]$$

$$+ \kappa_1 z_1 \overline{K}_1 - \tilde{\theta}_1 \dot{\hat{\theta}}_1 \tag{9}$$

By using Young's inequality, the following inequalities can be obtained

$$\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 z_2 \le \frac{1}{2}\sec^4\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1^2 + \frac{1}{2} z_2^2 \tag{10}$$

$$\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 \Delta_1 \le \frac{1}{2}\sec^4\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1^2 + \frac{1}{2}\Lambda_1^2 \tag{11}$$

Substituting inequalities (10) and (11) into (9) yields

$$\dot{V}_1 \le \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 \left[\frac{2k_{b_1}^2(t)}{\pi z_1}\overline{K}_1 \sin\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\right.$$

$$\left.\cos\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) + (\alpha_1 + f_1^k(\overline{x}_1) - \dot{y}_d)\right]$$

$$+ \kappa_1 z_1 \overline{K}_1 - \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \sec^4\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2}\Lambda_1^2$$

$$\le \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1(\alpha_1 + \Phi_{1,k}) + \kappa_1 z_1 \overline{K}_1 - \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \frac{1}{2}\Lambda_1^2 + \frac{1}{2} z_2^2$$

$$- \frac{1}{2}\sec^4\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1^2 - \frac{1}{2}\eta_1^2 \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 \tag{12}$$

where $\Phi_{1,k} = \frac{2k_{b_1}^2(t)}{\pi z_1}\overline{K}_1 \sin\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\cos\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) + f_1^k(\overline{x}_1) - \dot{y}_d + \frac{3}{2}\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 + \frac{1}{2}\eta_1^2$ is an unknown non-linear function, and η_1 is a positive constant.

According to Lemma 3, for any $\varepsilon_{1,k} > 0$, unknown function $\Phi_{1,k}$ can be estimated by the MTN with the expression

$$\Phi_{1,k} = \theta_{1,k}^{\mathrm{T}} S_{m_1}(z_1) + \delta_{1,k}(z_1), \left|\delta_{1,k}(z_1)\right| \le \varepsilon_{1,k} \tag{13}$$

where $\delta_{1,k}(z_1)$ denotes the approximation error and $\boldsymbol{z}_1 = [z_1]^{\mathrm{T}}$.

With the help of Young's inequality, the following inequality can be obtained

$$\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 \delta_{1,k} \le \frac{1}{2}\sec^4\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1^2 + \frac{1}{2}\varepsilon_{1,k}^2 \tag{14}$$

Substituting (13) and (14) into (12) yields

$$\dot{V}_1 \le z_1 \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\left(\alpha_1 + \theta_{1,k}^{\mathrm{T}} S_{m_1}\right) + \kappa_1 z_1 \overline{K}_1 - \tilde{\theta}_1 \dot{\hat{\theta}}_1$$

$$+ \frac{1}{2} z_2^2 + \frac{1}{2}\Lambda_1^2 + \frac{1}{2}\varepsilon_{1,k}^2 - \frac{1}{2}\eta_1^2 \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 \tag{15}$$

According to Young's inequality, we can get

$$\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 \theta_{1,k}^{\mathrm{T}} S_{m_1}$$

$$\le z_1 \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\left(\frac{1}{2}\eta_1^2 + \frac{1}{2\eta_1^2}\|\theta_{1,k}\|^2 S_{m_1}^{\mathrm{T}} S_{m_1}\right)$$

$$\le z_1 \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\left(\frac{1}{2}\eta_1^2 + \frac{1}{2\eta_1^2}\theta_1 S_{m_1}^{\mathrm{T}} S_{m_1}\right) \tag{16}$$

To ensure the effectiveness of the proposed control strategy, the first virtual control signal α_1 is designed as follows

$$\alpha_1 = -\frac{1}{z_1}\iota_1 k_{b_1}^{2\alpha}(t)\cos^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\tan^{\alpha}\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)$$

$$- \frac{\hat{\theta}_1 S_{m_1}^{\mathrm{T}} S_{m_1}}{2\eta_1^2} - z_1 \overline{K}_1 \tag{17}$$

where ι_1 is a positive constant and $0.5 < \alpha < 1$.

Remark 1. According to L'Hospital rule, when $z_1 \to 0$, we know that

$$\lim_{z_1 \to 0} \frac{1}{z_1}\iota_1 k_{b_1}^{2\alpha}(t)\cos^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\tan^{\alpha}\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) \to 0 \tag{18}$$

Although $\frac{0}{0}$ cannot be computed, it does not actually produce any singularity. When $z_1 \to 0$, (18) is considered to be zero.

Substituting (16) and (17) into (15), we have

$$\dot{V}_1 \le z_1 \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\left[-\frac{1}{z_1}\iota_1 k_{b_1}^{2\alpha}(t)\cos^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\right.$$

$$\left.\tan^{\alpha}\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) - \frac{\hat{\theta}_1 S_{m_1}^{\mathrm{T}} S_{m_1}}{2\eta_1^2} - z_1 \overline{K}_1\right]$$

$$+ \kappa_1 z_1 \overline{K}_1 - \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \frac{1}{2} z_2^2 + \frac{1}{2}\Lambda_1^2 + \frac{1}{2}\varepsilon_{1,k}^2$$

$$+ \frac{1}{2\eta_1^2} z_1 \sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\theta_1 S_{m_1}^{\mathrm{T}} S_{m_1}$$

$$\le -\iota_1 k_{b_1}^{2\alpha}(t)\tan^{\alpha}\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) - \tilde{\theta}_1 \dot{\hat{\theta}}_1 + \frac{1}{2} z_2^2 + \frac{1}{2}\Lambda_1^2$$

$$+ \frac{1}{2}\varepsilon_{1,k}^2 + \frac{1}{2\eta_1^2}\sec^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) z_1 \tilde{\theta}_1 S_{m_1}^{\mathrm{T}} S_{m_1} \tag{19}$$

Step $i (2 \le i \le n-1)$: Similar to Step 1, select the Lyapunov candidate V_i as follows

$$V_i = V_{i-1} + \frac{k_{b_i}^2(t)}{\pi}\tan\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) + \frac{1}{2}\tilde{\theta}_i^2 \tag{20}$$

Then, the time derivative of V_i can be calculated as follows

$$\dot{V}_i = \dot{V}_{i-1} + \frac{2k_{b_i}^2(t)}{\pi}\frac{\dot{k}_{b_i}(t)}{k_{b_i}(t)}\tan\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) + \sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i\left(\dot{z}_i - z_i\frac{\dot{k}_{b_i}(t)}{k_{b_i}(t)}\right) - \tilde{\theta}_i\dot{\hat{\theta}}_i \tag{21}$$

Define

$$\begin{cases} \overline{K}_i(t) = \sup\sqrt{\left(\frac{k_i(t)}{k_{b_i}(t)}\right)^2 + \beta_i} \\ \kappa_i = \sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i \end{cases} \tag{22}$$

where $\overline{K}_i(t)$ is the time-varying gain, and $\beta_i > 0$ is a positive parameter.

From (22), it follows that $-\frac{\kappa_i z_i k_{b_i}(t)}{k_{b_i}(t)} \le \overline{K}_i\kappa_i z_i$. Consequently, the time derivative of V_i can be rewritten as

$$\begin{aligned}\dot{V}_i \le & -\sum_{j=1}^{i-1}\iota_j k_{b_j}^{2\alpha}(t)\tan^\alpha\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{i-1}\tilde{\theta}_j\dot{\hat{\theta}}_j + \frac{1}{2}z_i^2 \\ & + \sum_{j=1}^{i-1}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{i-1}\frac{1}{2}\varepsilon_{j,k}^2 + \sum_{j=1}^{i-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}}S_{m_j} \\ & + \frac{2k_{b_i}^2(t)}{\pi}\overline{K}_i\tan\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) + z_i\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) \\ & \left(x_{i+1} + f_i^k(\overline{x}_i) + \Delta_i - \dot{\alpha}_{i-1}\right) + \kappa_i z_i\overline{K}_i - \tilde{\theta}_i\dot{\hat{\theta}}_i \end{aligned} \tag{23}$$

According to Young's inequality, the following inequality holds

$$\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i z_{i+1} \le \frac{1}{2}\sec^4\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i^2 + \frac{1}{2}z_{i+1}^2 \tag{24}$$

$$\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i\,\Delta_i \le \frac{1}{2}\sec^4\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i^2 + \frac{1}{2}\Lambda_i^2 \tag{25}$$

According to $z_{i+1} = x_{i+1} - \alpha_i$ and substituting (24) and (25) into (23), we have

$$\begin{aligned}\dot{V}_i \le & -\sum_{j=1}^{i-1}\iota_j k_{b_j}^{2\alpha}(t)\tan^\alpha\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{i-1}\tilde{\theta}_j\dot{\hat{\theta}}_j + \frac{1}{2}z_i^2 + \sum_{j=1}^{i-1}\frac{1}{2}\Lambda_j^2 \\ & + \sum_{j=1}^{i-1}\frac{1}{2}\varepsilon_{j,k}^2 + \sum_{j=1}^{i-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}}S_{m_j} \\ & + \kappa_i z_i\overline{K}_i - \tilde{\theta}_i\dot{\hat{\theta}}_i \\ & + \sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i\left[\frac{2k_{b_i}^2(t)}{\pi z_i}\overline{K}_i\sin\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)\right. \\ & \left.\cos\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) + z_{i+1} + \alpha_i + f_i^k(\overline{x}_i) + \Delta_i - \dot{\alpha}_{i-1}\right] \\ \le & -\sum_{j=1}^{i-1}\iota_j k_{b_j}^{2\alpha}(t)\tan^\alpha\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{i}\tilde{\theta}_j\dot{\hat{\theta}}_j \\ & + \sum_{j=1}^{i}\frac{1}{2}\Lambda_j^2 + + \sum_{j=1}^{i-1}\frac{1}{2}\varepsilon_{j,k}^2 \\ & + \sum_{j=1}^{i-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}}S_{m_j} \\ & + \sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i(\Phi_i + \alpha_i) + \kappa_i z_i\overline{K}_i \\ & + \frac{1}{2}z_{i+1}^2 - \frac{1}{2}\sec^4\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i^2 - \frac{1}{2}\eta_i^2\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i \end{aligned} \tag{26}$$

where $\Phi_{i,k} = \frac{2k_{b_i}^2(t)}{\pi z_i}\overline{K}_i\sin\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)\cos\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) + f_i^k(\overline{x}_1) - \dot{\alpha}_{i-1} + \frac{3}{2}\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i + \cos^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)\frac{z_i}{2} + \frac{1}{2}\eta_i^2$ is an unknown function, and η_i is a positive constant.

According to the MTN technique in Lemma 3, for any $\varepsilon_{i,k} > 0$, $\Phi_{i,k}$ can be estimated by the MTN with the expression

$$\Phi_{i,k} = \theta_{i,k}^{\mathrm{T}}S_{m_i}(\boldsymbol{z}_i) + \delta_{i,k}(\boldsymbol{z}_i), \left|\delta_{i,k}(\boldsymbol{z}_i)\right| \le \varepsilon_{i,k} \tag{27}$$

where $\delta_{i,k}(\boldsymbol{z}_i)$ denotes the approximation error and $\boldsymbol{z}_i = [z_1, \dots, z_i]^{\mathrm{T}}$.

From Young's inequality, the inequality can be obtained

$$\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i\delta_{i,k} \le \frac{1}{2}\sec^4\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i^2 + \frac{1}{2}\varepsilon_{i,k}^2 \tag{28}$$

Substituting (27) and (28) into (26), the time derivative of V_i is

$$\begin{aligned}\dot{V}_i \le & -\sum_{j=1}^{i-1}\iota_j k_{b_j}^{2\alpha}(t)\tan^\alpha\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{i}\tilde{\theta}_j\dot{\hat{\theta}}_j + \sum_{j=1}^{i}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{i}\frac{1}{2}\varepsilon_{j,k}^2 \\ & + \sum_{j=1}^{i-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}}S_{m_j} \\ & + z_i\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)\left(\theta_{i,k}^{\mathrm{T}}S_{m_i} + \alpha_i\right) \\ & + \kappa_i z_i\overline{K}_i + \frac{1}{2}z_{i+1}^2 - \frac{1}{2}\eta_i^2\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i \end{aligned} \tag{29}$$

From Young's inequality, the following inequality can be obtained

$$\begin{aligned} & \sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)z_i\theta_{i,k}^{\mathrm{T}}S_{m_1} \\ & \le z_i\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)\left(\frac{1}{2}\eta_{\mathrm{i}}^2 + \frac{1}{2\eta_i^2}\|\theta_{\mathrm{i,k}}\|^2 S_{m_i}^{\mathrm{T}}S_{m_i}\right) \\ & \le z_i\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)\left(\frac{1}{2}\eta_i^2 + \frac{1}{2\eta_i^2}\theta_i S_{m_i}^{\mathrm{T}}S_{m_i}\right) \end{aligned} \tag{30}$$

To ensure the effectiveness of the proposed control strategy, design the virtual control signal α_i as follows

$$\alpha_i = -\frac{1}{z_i}\iota_i k_{b_i}^{2\alpha}(t)\cos^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)\tan^\alpha\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) - \frac{\hat{\theta}_i S_{m_i}^{\mathrm{T}} S_{m_i}}{2\eta_i^2} - z_i\overline{K}_i \tag{31}$$

where $\iota_i > 0$ is a positive constant.

Substituting (31) into (30) gives

$$\begin{aligned}
\dot{V}_i \le & -\sum_{j=1}^{i-1}\iota_j k_{b_j}^{2\alpha}(t)\tan^\alpha\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{i}\tilde{\theta}_j\dot{\hat{\theta}}_j + \sum_{j=1}^{i}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{i}\frac{1}{2}\varepsilon_{j,k}^2 \\
& + z_i\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right)\Bigg\{\theta_{i,k}^{\mathrm{T}} S_{m_i} - \frac{1}{z_i}\iota_i k_{b_i}^{2\alpha}(t)\cos^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) \\
& \tan^\alpha\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) - \frac{\hat{\theta}_i S_{\mathrm{m}_i}^{\mathrm{T}} S_{\mathrm{m}_i}}{2\eta_i^2} - z_i\overline{K}_i\Bigg\} \\
& + \kappa_i z_i\overline{K}_i + \frac{1}{2}z_{i+1}^2 - \frac{1}{2}z_i\eta_i^2\sec^2\left(\frac{\pi z_i^2}{2k_{b_i}^2(t)}\right) \\
& + \sum_{j=1}^{i-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}} S_{m_j} \\
\le & -\sum_{j=1}^{i}\iota_j k_{b_j}^{2\alpha}(t)\tan^\alpha\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{i}\tilde{\theta}_j\dot{\hat{\theta}}_j \\
& + \sum_{j=1}^{i}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{i}\frac{1}{2}\varepsilon_{j,k}^2 + \sum_{j=1}^{i}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}} S_{m_j} \\
& + \frac{1}{2}z_{i+1}^2
\end{aligned} \tag{32}$$

Step n: Consider the Lyapunov candidate function based on step i, the Lyapunov candidate function is considered

$$V_n = V_{n-1} + \frac{k_{b_n}^2(t)}{\pi}\tan\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) + \frac{1}{2}\tilde{\theta}_n^2 \tag{33}$$

Define

$$\begin{cases}
\overline{K}_n(t) = \sup\sqrt{\left(\frac{\dot{k}_n(t)}{k_{b_n}(t)}\right)^2 + \beta_n} \\
\kappa_n = \sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)z_n
\end{cases} \tag{34}$$

where $\overline{K}_n(t)$ is the time-varying gain, and $\beta_n > 0$ is a positive parameter.

From (34), it follows that $-\frac{\kappa_n z_n \dot{k}_{b_n}(t)}{k_{b_n}(t)} \le \overline{K}_n\kappa_n z_n$. Then, considering $\dot{z}_n = \dot{x}_n - \alpha_{n-1}$ and $x_n = z_n + \alpha_{n-1}$, the time derivative of V_n can be rewritten as

$$\begin{aligned}
\dot{V}_n = & \dot{V}_{n-1} + \frac{2k_{b_n}^2(t)}{\pi}\frac{\dot{k}_{b_n}(t)}{k_{b_n}(t)}\tan\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) \\
& + \sec^2\left(\frac{\pi z_n^2}{2k_n^2(t)}\right)z_n\left(\dot{z}_n - z_n\frac{\dot{k}_n(t)}{k_n(t)}\right) - \tilde{\theta}_n\dot{\hat{\theta}}_n \\
\le & -\sum_{j=1}^{n-1}\iota_j k_{b_j}^{2\alpha}(t)\tan^\alpha\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{n}\tilde{\theta}_j\dot{\hat{\theta}}_j + \frac{1}{2}z_n^2 \\
& + \sum_{j=1}^{n-1}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n-1}\frac{1}{2}\varepsilon_{j,k}^2 \\
& + \sum_{j=1}^{n-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}} S_{m_j} + \kappa_n z_n\overline{K}_n \\
& + z_n\sec^2\left(\frac{\pi z_n^2}{2k_{b_n^2}(t)}\right)\left[\frac{2k_{b_n}^2(t)}{\pi z_n}\sin\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)\right. \\
& \left.\cos\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) + u + f_n^k(\overline{x}_n) + \triangle_n - \dot{\alpha}_{n-1}\right]
\end{aligned} \tag{35}$$

By using Young's inequality, the following inequality can be obtained

$$\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)z_n\triangle_n \le \frac{1}{2}\sec^4\left(\frac{\pi z_j^2}{2k_{b_n}^2(t)}\right)z_n^2 + \frac{1}{2}\Lambda_n^2 \tag{36}$$

Substituting (36) into (35) yields

$$\begin{aligned}
\dot{V}_n \le & -\sum_{j=1}^{n-1}\iota_j k_{b_j}^{2\alpha}(t)\tan^\alpha\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{n}\tilde{\theta}_j\dot{\hat{\theta}}_j \\
& + \sum_{j=1}^{n-1}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n-1}\frac{1}{2}\varepsilon_{j,k}^2 + \sum_{j=1}^{n-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}} S_{m_j} \\
& + z_n\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)(\Phi_{n,k} + u) + \frac{1}{2}\Lambda_n^2 + \kappa_n z_n\overline{K}_n \\
& - \frac{1}{2}z_n^2\sec^4\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) - \frac{1}{2}z_n\eta_n^2\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)
\end{aligned} \tag{37}$$

where $\Phi_{n,k} = \frac{2k_{b_n}^2(t)}{\pi z_n}\overline{K}_n\sin\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)\cos\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) + f_n^k(\overline{\boldsymbol{x}}_1) - \dot{\alpha}_{n-1} + \sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)z_n + \cos^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)\frac{z_n}{2} + \frac{1}{2}\eta_n^2$ is a combination of non-linear functions and η_n is a positive constant.

According to the MTN technique in Lemma 3, for any $\varepsilon_{n,k} > 0$, $\Phi_{n,k}$ can be estimated by the MTN with the expression

$$\Phi_{n,k} = \theta_{n,k}^{\mathrm{T}} S_{m_n}(\boldsymbol{z}_n) + \delta_{n,k}(\boldsymbol{z}_n), \left|\delta_{n,k}(\boldsymbol{z}_n)\right| \le \varepsilon_{n,k} \tag{38}$$

where $\delta_{n,k}(\boldsymbol{z}_n)$ denotes the approximation error and $\boldsymbol{z}_n = [z_1, \dots, z_n]^{\mathrm{T}}$

From Young's inequality, the inequality can be obtained

$$\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)z_n\delta_{n,k} \le \frac{1}{2}\sec^4\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)z_n^2 + \frac{1}{2}\varepsilon_{n,k}^2 \tag{39}$$

Substituting (38) and (39) into (37), the time of V_n derivative is

$$\dot{V}_n \le -\sum_{j=1}^{n-1} \iota_j k_{b_j}^{2\alpha}(t)\tan^{\alpha}\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{n}\tilde{\theta}_j\dot{\hat{\theta}}_j + \sum_{j=1}^{n}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n}\frac{1}{2}\epsilon_{j,k}^2 + \sum_{j=1}^{n-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}}S_{m_j} + z_n\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)\left(\theta_{n,k}^{\mathrm{T}}S_{m_n}+u\right) + \kappa_n z_n\overline{K}_n - \frac{1}{2}z_n\eta_n^2\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) \tag{40}$$

From Young's inequality, we can get

$$\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)z_n\theta_{n,k}^{\mathrm{T}}S_{m_n} \le z_n\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)\left(\frac{1}{2}\eta_{\mathrm{n}}^2 + \frac{1}{2\eta_n^2}\left\|\theta_{n,k}\right\|^2 S_{m_n}^{\mathrm{T}}S_{m_n}\right) \le z_n\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)\left(\frac{1}{2}\eta_n^2 + \frac{1}{2\eta_n^2}\theta_n S_{m_n}^{\mathrm{T}}S_{m_n}\right) \tag{41}$$

Then, the time derivative of V_n can be rewritten as

$$\dot{V}_n \le -\sum_{j=1}^{n-1} \iota_j k_{b_j}^{2\alpha}(t)\tan^{\alpha}\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{n}\tilde{\theta}_j\dot{\hat{\theta}}_j + \sum_{j=1}^{n}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n}\frac{1}{2}\epsilon_{j,k}^2 + \sum_{j=1}^{n-1}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}}S_{m_j} + \sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)z_n u + \kappa_n z_n\overline{K}_n + \frac{1}{2\eta_n^2}\sec^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)z_n\theta_n S_{m_n}^{\mathrm{T}}S_{m_n} \tag{42}$$

According to (42), selecting the controller u as follows

$$u = -\frac{1}{z_n}\iota_n k_{b_n}^{2\alpha}(t)\cos^2\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right)\tan^{\alpha}\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) - \frac{\hat{\theta}_n S_{\mathrm{m}_n}^{\mathrm{T}}S_{\mathrm{m}_n}}{2\eta_n^2} - z_n\overline{K}_n \tag{43}$$

Substituting (43) into (42), we have

$$\dot{V}_n \le -\sum_{j=1}^{n} \iota_j k_{b_j}^{2\alpha}(t)\tan^{\alpha}\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) - \sum_{j=1}^{n}\tilde{\theta}_j\dot{\hat{\theta}}_j + \sum_{j=1}^{n}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n}\frac{1}{2}\epsilon_{j,k}^2 + \sum_{j=1}^{n}\frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j\tilde{\theta}_j S_{m_j}^{\mathrm{T}}S_{m_j} \tag{44}$$

Based on (44), the adaptive laws $\dot{\hat{\theta}}_j, (j = 1, 2, \dots, n)$ can be designed as follows

$$\dot{\hat{\theta}}_j = -P_j\hat{\theta}_j + \frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j S_{m_j}^{\mathrm{T}}S_{m_j} \tag{45}$$

where $P_j > 0$ are constant to be designed.

Substitute (45) into (44), it yields

$$\dot{V}_n \le -\sum_{j=1}^{n} \iota_j k_{b_j}^{2\alpha}(t)\tan^{\alpha}\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) + \sum_{j=1}^{n}P_j\tilde{\theta}_j\hat{\theta}_j + \sum_{j=1}^{n}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n}\frac{1}{2}\epsilon_{j,k}^2 \tag{46}$$

where $\iota_n > 0$ is a positive constant.

By Young's inequality, the inequality can be obtained

$$P_j\tilde{\theta}_j\hat{\theta}_j \le \frac{P_j\theta_j^2}{2} - \frac{P_j\tilde{\theta}_j^2}{2} = \frac{P_j\theta_j^2}{2} - \frac{P_j\tilde{\theta}_j^2}{2} - \frac{P_j\tilde{\theta}_j^{2\alpha}}{2} + \frac{P_j\tilde{\theta}_j^{2\alpha}}{2} \le \frac{P_j\theta_j^2}{2} - \frac{P_j\tilde{\theta}_j^2}{2} - \frac{P_j\tilde{\theta}_j^{2\alpha}}{2} + \frac{\alpha P_j\tilde{\theta}_j^2}{2} + \frac{(1-\alpha)P_j}{2} \le \frac{P_j\theta_j^2}{2} - \frac{(1-\alpha)P_j\tilde{\theta}_j^2}{2} - \frac{P_j\tilde{\theta}_j^{2\alpha}}{2} + \frac{(1-\alpha)P_j}{2} \le \frac{P_j\theta_j^2}{2} - \frac{P_j\tilde{\theta}_j^{2\alpha}}{2} + \frac{(1-\alpha)P_j}{2} \tag{47}$$

Correspondingly, substituting (47) into (46) yields

$$\dot{V}_n \le -\sum_{j=1}^{n} \iota_j k_{b_j}^{2\alpha}(t)\tan^{\alpha}\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right) + \sum_{j=1}^{n}\left(\frac{P_j\theta_j^2}{2} - \frac{P_j\tilde{\theta}_j^{2\alpha}}{2} + \frac{(1-\alpha)P_j}{2}\right) + \sum_{j=1}^{n}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n}\frac{1}{2}\epsilon_{j,k}^2 \tag{48}$$

Define $c = \min\left\{\iota_1\pi^{\alpha}, \dots, \iota_n\pi^{\alpha}, 2^{\alpha-1}P_1, \dots, 2^{\alpha-1}P_n\right\}$, (48) can be rewritten as

$$\dot{V}_n \le -c\sum_{j=1}^{n}\left[\frac{k_{b_n}^{2\alpha}(t)}{\pi^{\alpha}}\tan^{\alpha}\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) + \frac{1}{2^{\alpha}}\tilde{\theta}_n^{2\alpha}\right] + \sum_{j=1}^{n}\left(\frac{P_j\theta_j^2}{2} + \frac{(1-\alpha)P_j}{2}\right) + \sum_{j=1}^{n}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n}\frac{1}{2}\epsilon_{j,k}^2 \le -c\sum_{j=1}^{n}\left[\frac{k_{b_n}^{2\alpha}(t)}{\pi^{\alpha}}\tan^{\alpha}\left(\frac{\pi z_n^2}{2k_{b_n}^2(t)}\right) + \frac{1}{2^{\alpha}}\tilde{\theta}_n^{2\alpha}\right] + \sum_{j=1}^{n}\left(\frac{P_j\theta_j^2}{2} + \frac{(1-\alpha)P_j}{2}\right) + \sum_{j=1}^{n}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n}\frac{1}{2}\epsilon_{j,\max}^2 \tag{49}$$

where $\epsilon_{j,\max}^2 = \max\left\{\epsilon_{j,k}^2 | k \in Q\right\}$.

Applying Lemma 1, one has

$$\dot{V}_n \leq -cV_n^{\alpha} + \tau \tag{50}$$

where $\tau = \sum_{j=1}^{n}\left(\frac{P_j\theta_j^2}{2} + \frac{(1-\alpha)P_j}{2}\right) + \sum_{j=1}^{n}\frac{1}{2}\Lambda_j^2 + \sum_{j=1}^{n}\frac{1}{2}\varepsilon_{j,\max}^2$.

Now, the design of the controller has been completed, and the following theorem will be used to present our main result.

3.2 | Stability Analysis

Theorem 1. *Consider the switched non-linear system (1) satisfying Assumptions 1 and 2, if the virtual control signals and the actual controller are described by (17), (31) and (43), as well as the adaptive laws are described by (45), then the following results can be obtained: (i) all signals in the system are bounded under any switching; (ii) all states x_i, $i = 1, 2, \ldots, n$ satisfy time-varying state constraints; (iii) the tracking error converges to a small neighborhood of the origin in a finite-time.*

Proof. For the entire closed-loop system, considering the following Lyapunov function

$$V = V_n = \sum_{i=1}^{n}\frac{k_{b_i}^2(t)}{\pi}\tan\left(\frac{\pi z_i}{2k_{b_i}^2(t)}\right) + \sum_{i=1}^{n}\frac{1}{2}\tilde{\theta}_i^2 \tag{51}$$

From (50), we have $\dot{V}_n \leq -cV_n^{\alpha} + \tau$, therefore, V_n is bounded. According to the definite of V_n, we know that the TBLFs and $\tilde{\theta}_i$ are bounded, thus $\hat{\theta}_i = \theta_i - \tilde{\theta}_i$ must be bounded.

According to $z_1 = x_1 - y_d$ and $|y_d| \leq D_0$, it is easy to know that $|x_1| \leq |z_1| + |y_d| \leq k_{b_1}(t) + D_0$. Based on $k_{c_1}(t) = k_{b_1}(t) + D_0$, the output signal is bounded. Meanwhile, $z_2 = x_2 - \alpha_1$ and $\alpha_1 \leq \overline{\alpha}_1$ both hold, then $|x_2| \leq k_{c_2}$ hold, where $k_{c_2}(t) = k_{b_2}(t) + \overline{\alpha}_1$. Similarly, we can obtain the conclusion that $x_i \leq k_{c_i}(t), i = 3, \ldots, n$. Therefore, all states $x_i, i = 1, \ldots, n$ satisfy the time-varying state constraints.

Let $T^{\aleph} = \frac{1}{(1-\alpha)\eta c}\left[V^{1-\alpha}(z(0), W(0)) - \left(\frac{\tau}{(1-\eta)c}\right)^{(1-\alpha)/\alpha}\right]$ with $\boldsymbol{z}(0) = \left(z_1(0), \ldots, z_n(0)\right)^{\mathrm{T}}$ and $\boldsymbol{W}(0) = \left(\theta_1(0), \ldots, \theta_n(0)\right)^{\mathrm{T}}$, then according to Lemma 2, for $\forall t \geq T^{\aleph}$, $V^{\alpha}(\boldsymbol{z}, \boldsymbol{W}) \leq \frac{\tau}{(1-\eta)c}$ holds, which means that all the signals in the closed-loop system are SGPFS.

Furthermore, based on the definition of V, for $\forall t \geq T^{\aleph}$, the following inequality holds

$$|y - y_d| \leq k_{b_1}(t)\sqrt{\frac{2}{\pi}\arctan\left[\frac{\pi}{k_{b_1}^2(t)}\left(\frac{\pi}{(1-\beta)c}\right)^{\frac{1}{\beta}}\right]} \tag{52}$$

Thus, the tracking error converges to a small neighborhood around the origin after the finite-time $T^{\aleph}$. □

4 | Simulation Results

In this section, three examples are used to verify the effectiveness of the control scheme proposed in this paper.

Example 1. Consider a mathematical example where the switched non-linear system model can be described in the following form

$$\begin{cases} \dot{x}_1 = x_2 + f_1^{\sigma(t)}(\overline{x}_1) + 0.01\sin(t) \\ \dot{x}_2 = u + f_2^{\sigma(t)}(\overline{x}_2) + 0.1\sin(t) \\ y = x_1 \end{cases} \tag{53}$$

where the switching signal $\sigma(t)$ is defined as $\sigma(t) : [0, \infty) \to M = \{1, 2\}$, the initial state is $\boldsymbol{x}(0) = \left[x_1(0), x_2(0)\right]^{\mathrm{T}}$, the continuously differentiable functions are taken as $f_{1,1} = -x_1\sin(-0.5x_1)$, $f_{1,2} = -2\sin(x_1)\sin(-0.5x_1)$, $f_{2,1} = -3x_1\sin(x_2^2)$, $f_{2,2} = -x_1\sin(x_2^2)$, and the desired signal $y_d(t)$ is chosen as $y_d(t) = 0.32\sin(0.5t)$.

According to Theorem 1, the control structure of the system (53) can be designed as follows

$$\alpha_1 = -\frac{1}{z_1}\iota_1 k_{b_1}^{2\alpha}(t)\cos^2\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right)\tan^{\alpha}\left(\frac{\pi z_1^2}{2k_{b_1}^2(t)}\right) - \frac{\hat{\theta}_1 S_{m_1}^{\mathrm{T}} S_{m_1}}{2\eta_1^2} - z_1\overline{K}_1$$

$$\dot{\hat{\theta}}_j = -P_j\hat{\theta}_j + \frac{1}{2\eta_j^2}\sec^2\left(\frac{\pi z_j^2}{2k_{b_j}^2(t)}\right)z_j S_{m_j}^{\mathrm{T}} S_{m_j}, \; P_j > 0, \; j = 1, 2$$

$$u = -\frac{1}{z_2}\iota_2 k_{b_2}^{2\alpha}(t)\cos^2\left(\frac{\pi z_2^2}{2k_{b_2}^2(t)}\right)\tan^{\alpha}\left(\frac{\pi z_2^2}{2k_{b_2}^2(t)}\right) - \frac{\hat{\theta}_2 S_{m_2}^{\mathrm{T}} S_{m_2}}{2\eta_2^2} - z_2\overline{K}_2$$

In the simulation, the controller parameters are designed as $\eta_1 = 10$, $\eta_2 = 200$, $\iota_1 = 13.6$, $\iota_2 = 1$, and the parameters of the adaptive law are designed as $P_1 = 1.3$, $P_2 = 4$. The state constraints satisfy $|x_1| \leq 0.35\sin t + 1$ and $|x_2| \leq 0.35\sin 0.5t + 1.5$.

The simulation results are shown in Figures 1–5. Figure 1 illustrates the trajectories of y and y_d, which shows that the system output $y = x_1$ satisfying the constraint $|x_1| \leq k_{c_1}(t)$. Figure 2 shows the state of x_2, constrained within a specific time-varying constraint range. Figures 3–5 display the trajectories of the tracking error e_1, the switching signal $\sigma(t)$ and the control input $u(t)$, respectively. From the simulation results, it is evident that the controller proposed based on MTN achieves good tracking performance, and all states in the closed-loop system converge to a small time-varying region within a finite-time.

Example 2. To further verify the effectiveness of the control method proposed in this paper, consider a class of RLC circuit systems. According to [43], its dynamic system can be represented as follows

$$\begin{cases} \Phi_L = L\frac{dq_c}{dt} \\ u = \frac{R}{L}\Phi_L + \frac{q_c}{C_{\sigma(t)}} + \dot{\Phi}_L \end{cases} \tag{54}$$

where q_c represents the charge in the capacitor, u represents the input voltage source, R represents the resistance, $C_{\sigma(t)}$ represents

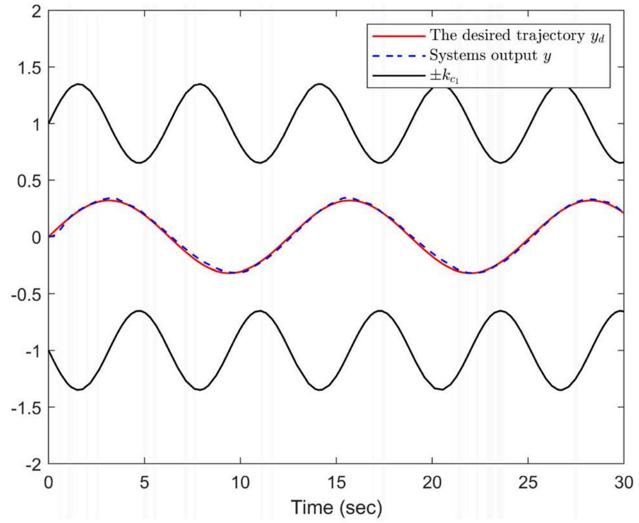

FIGURE 1 | The trajectories of y and y_d with time-varying constraints in Example 1.

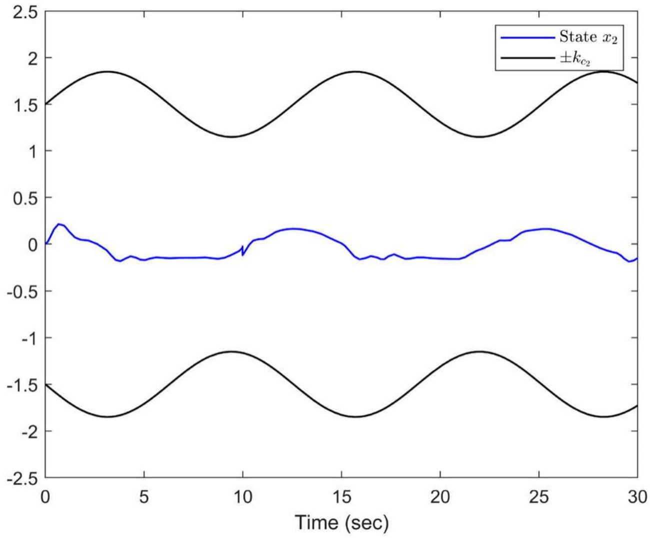

FIGURE 2 | The trajectories of x_2 with time-varying constraints in Example 1.

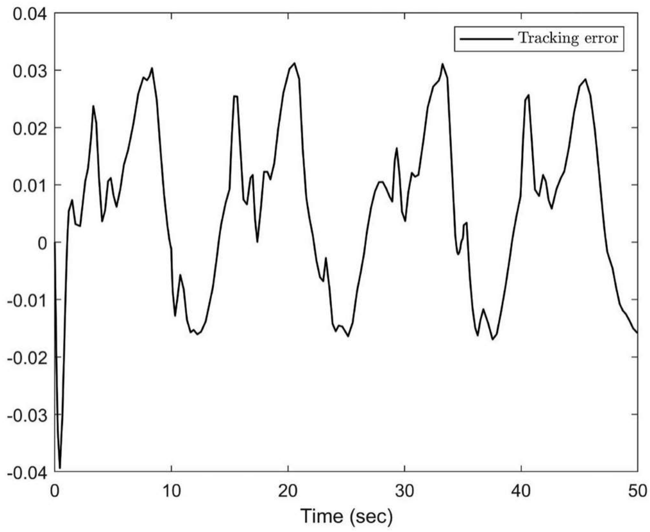

FIGURE 3 | The trajectory of tracking error in Example 1.

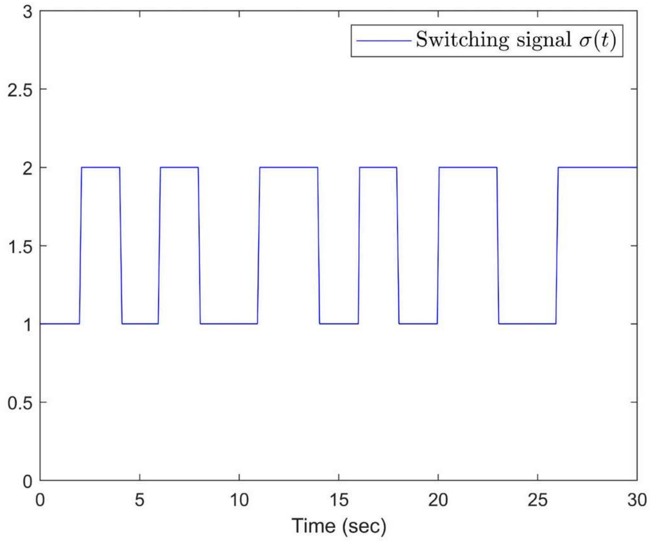

FIGURE 4 | The trajectory of the switching signal $\sigma(t)$ in Example 1.

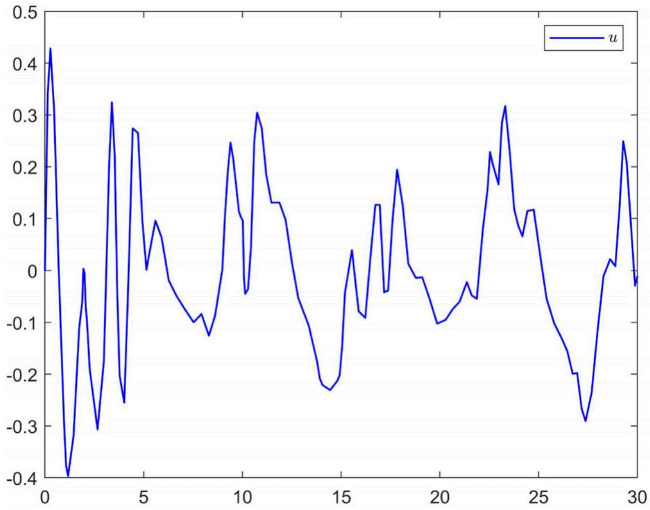

FIGURE 5 | Trajectory of u in Example 1.

the capacitance of the capacitor, Φ_L represents the magnetic flux of the inductor, and L represents the inductance of the inductor.

Let $x_1 = q_c$, $x_2 = \Phi_L$, then the system can be rewritten as follows

$$\begin{cases} \dot{x}_i = \frac{1}{L}x_2 \\ \dot{x}_2 = u - \frac{x_1}{C_{\sigma(t)}} - \frac{R}{L}x_2 \\ y = x_1 \end{cases} \tag{55}$$

Based on the Zhang, Li and Xiang [43], the parameters of (55) are chosen as $L = 1H$, $C_1 = 2F$, $C_2 = 5F$ and $R = 2\Omega$.

The control structure of the system (55) is the same as given in Example 1. In the simulation, the desired signal is selected as $y_d(t) = 0.32\sin(0.5t)$. The controller parameters are designed as $\eta_1 = 10$, $\eta_2 = 200$, $\iota_1 = 13.6$, $\iota_2 = 1$, and the parameters of the adaptive law are designed as $P_1 = 31$, $P_2 = 4$. The state constraints satisfy $|x_1| \leq 0.35\sin t + 1$ and $|x_2| \leq 0.35\sin 0.5t + 1.5$.

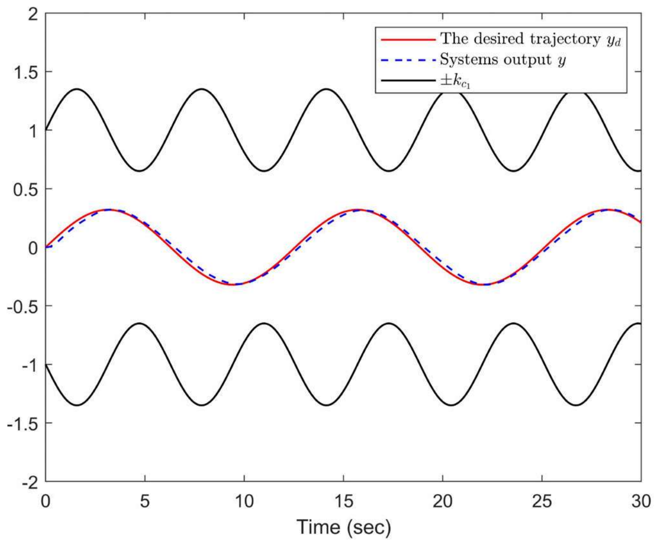

FIGURE 6 | The trajectories of y and y_d with time-varying constraints in Example 2.

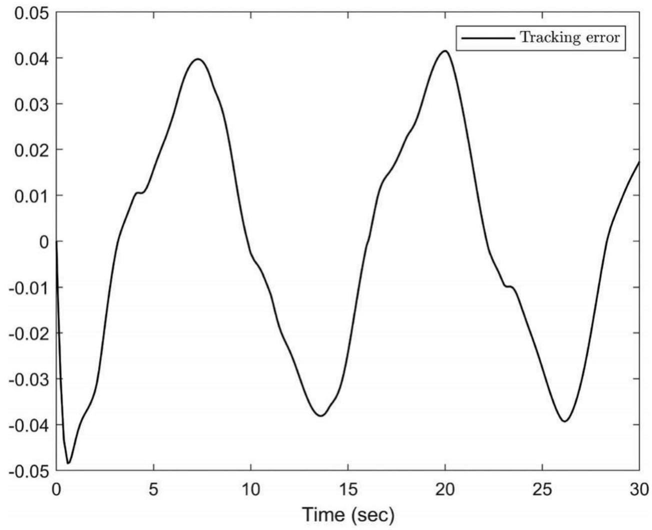

FIGURE 8 | The trajectory of tracking error in Example 2.

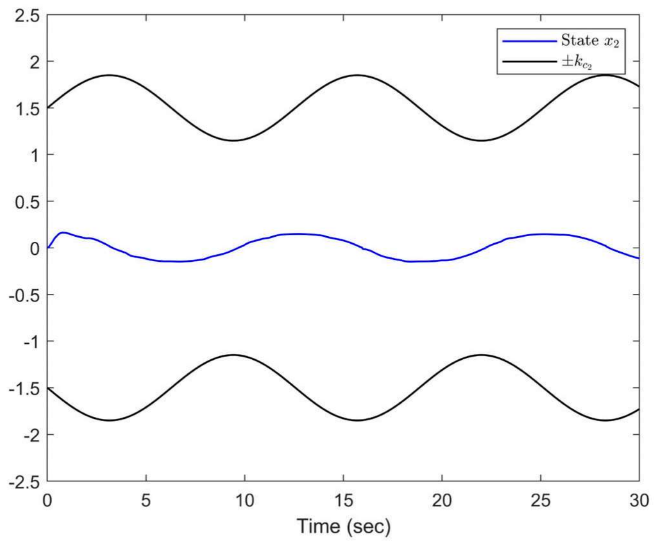

FIGURE 7 | The trajectories of x_2 with time-varying constraints in Example 2.

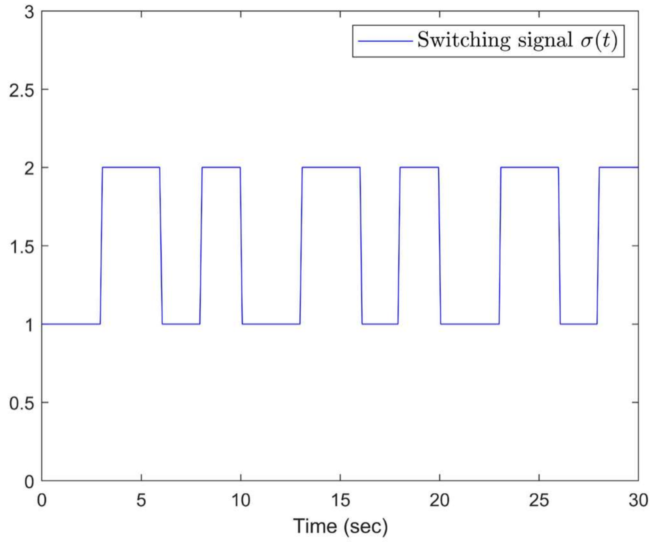

FIGURE 9 | The trajectory of the switching signal $\sigma(t)$ in Example 2.

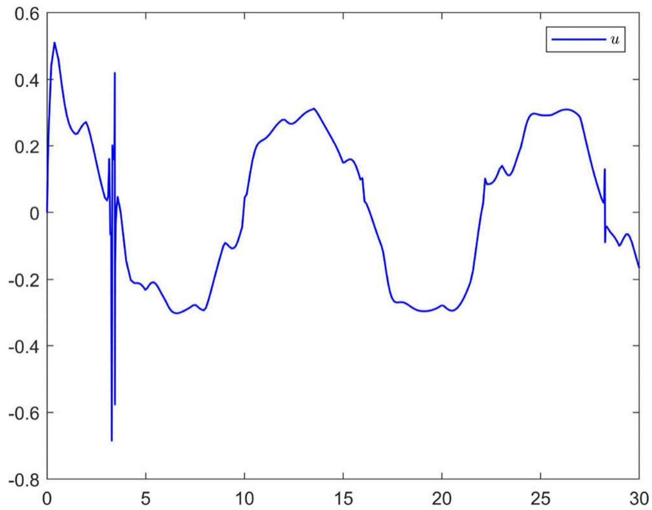

FIGURE 10 | The trajectory of u in Example 2.

The simulation results are shown in Figures 6–10. From Figure 6, we can see that the system output y can track the desired signal y_d, and $y = x_1$ satisfying the constraint $|x_1| \leq k_{c_1}(t)$. Figure 7 shows the state of x_2 constrained within a specific time-varying range. Figures 8–10 displays the trajectories of the tracking error e_1, the switching signal $\sigma(t)$ and the control input u, respectively. Figures 6–10, it can be observed that the proposed controller in this paper exhibits excellent tracking performance for real systems, further validating the effectiveness of the proposed control method.

Example 3. (Comparative example). In order to verify the effectiveness of the control approach proposed in this paper in more depth, a comparative experiment between the adaptive MTN and RBFNN control methods is given on the basis of Example 1. The simulation comparison result is shown in Figure 11.

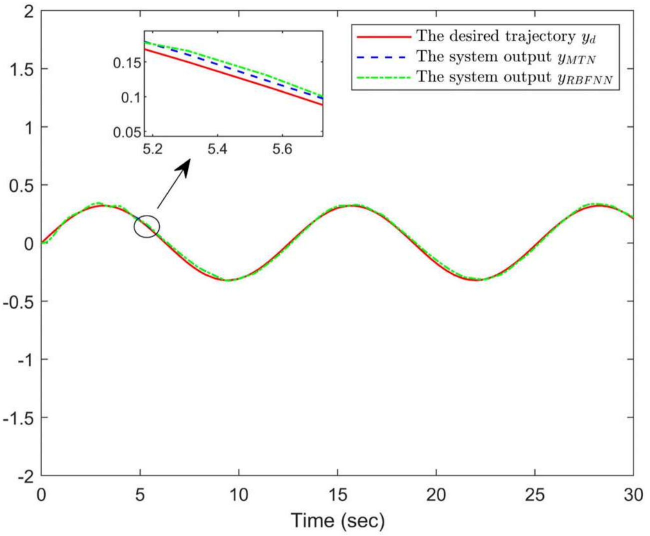

FIGURE 11 | The tracking performance comparison of MTN and RBFNN.

The simulation results clearly reveal that both MTN-based and RBFNN-based controllers can achieve satisfactory tracking control effectiveness. However, it is worth noting that the control strategy proposed in this paper demonstrates a lower computational cost while reaching comparable control effectiveness with RBFNN.

5 | Conclusion

In this paper, an adaptive finite-time control method based on MTN is proposed for a class of switched non-linear systems with full-state constraints. Firstly, the backstepping technique, grounded in the TBLFs, is introduced to preclude the violation of time-varying boundary conditions by the state constraints. Meanwhile, the MTNs are utilized to approximate the non-linear functions. The proposed control strategy guarantees that the output follows the reference signal within a finite-time. Furthermore, it ensures that all the signals in the closed-loop system are SGPFS and that all states satisfy the defined time-varying state constraints. Finally, three simulation examples fully verify the effectiveness of the proposed strategy.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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