Design of Adaptive Finite-Time Fault-Tolerant Controller for Stochastic Nonlinear Systems With Multiple Faults

Ming-Xin Wang, Shan-Liang Zhu, Si-Min Liu, Yang Du, and Yu-Qun Han

***Abstract*—In this paper, the adaptive fault-tolerant control (FTC) problem is addressed for the stochastic nonlinear systems with multiple faults. The multiple faults, including the actuator and abrupt system faults, are first discussed in the same theoretical framework. The unknown nonlinearities are approximated by multi-dimensional Taylor networks (MTNs). By taking advantage of backstepping technique and finite-time control, the actual control law and virtual control signals are constructed, and then a novel adaptive FTC scheme based on the finite-time control method is proposed. The proposed controller guarantees that the closed-loop system is semi-global finite-time stable in probability (SGFSP) and the tracking error converges to a small neighborhood around the origin in the finite-time. Lastly, three examples are given to illustrate the effectiveness of the proposed scheme.**

***Note to Practitioners*—This research is motivated by the fact that actuators faults exist widely in real applications, which often degrade the control accuracy of the system and even result in the system instability. So far, the actuator and abrupt system faults of the stochastic nonlinear system have not yet been considered under the same theoretical framework. Therefore, this study designs a new MTN-based adaptive finite-time FTC scheme, which can ensure that the closed-loop system is SGFSP. The proposed control scheme has excellent practical value.**

***Index Terms*—Adaptive control, fault-tolerant control, finite-time, stochastic nonlinear systems, MTN.**

I. INTRODUCTION

IT IS well known that actual control systems are characterized by nonlinearity, uncertainty and complexity, and also affected by external disturbances inevitably. For this reason, the research on the control of stochastic nonlinear systems has attracted great attention [1], [2], [3], [4]. So far, to deal with the unknown nonlinear functions in control systems, many approximation-based intelligent control methods have been successfully reported, such as the neural network (NN) control [5], [6], [7], [8], the fuzzy logic systems (FLSs) control [9], [10] and the multi-dimensional Taylor network (MTN) control [11], [12], [13], [14]. Among them, the MTN control method has obtained more and more attention because of its simple structure and fast function approximation ability, and many MTN-based control schemes have been gained [15], [16], [17], [18], [19], [20]. For example, authors in [15] constructed a new adaptive MTN control scheme for the stochastic nonlinear systems, which improved the real-time performance. For the stochastic nonlinear systems with input saturation, author in [17] proposed a MTN-based control strategy via the backstepping technology. However, it should be noted that most of the above results focused on asymptotic stability or exponential stability, and failed to consider the convergence performance of the systems. In recent years, the research hotspot has been transferred to the finite-time control due to the fact that the finite-time control has the good quality of finite-time convergence [21], [22], [23], [24].

In view of practical application value, the control schemes based on the finite-time stability theory have advantages of high accuracy, fast convergence and good robustness [25], [26], [27]. Therefore, the finite-time control problems of stochastic systems have attracted more and more attention. For instance, authors in [28] and [29] respectively proposed a novel stability theory and a controller design algorithm for the above problems. So far, many schemes based on the finite-time control have been gained for different stochastic systems, such as stochastic nonlinear systems [30], [31], switched stochastic nonlinear systems [32], high-order stochastic nonlinear systems [33], [34], non-triangular stochastic nonlinear systems [35], full state constraints stochastic nonlinear systems [36], quantized stochastic nonlinear systems [37] and stochastic nonlinear systems with input quantization [38]. Although a great of finite-time control schemes have been proposed for stochastic systems, the research results on fault tolerant control (FTC) based on finite-time stability theory are relatively infrequent, and many valuable control problems have not been investigated.

In fact, with the rapid development of modern industry, the structure of industry systems is more and more complex, and the function is more and more abundant. Due to the high complexity of the systems, the probability of failure of system components is also greatly increased [39], [40]. The occurrence of faults may degrade the control accuracy of the system and even result in the system instability [41], [42]. Therefore, the FTC problem has gained a wide concern, and a lot of meaningful schemes have been proposed for different systems with actuators faults, for instance, nonlinear systems [43], multi-input multi-output nonlinear systems [44], [45], and stochastic nonlinear systems [46], [47]. Specially, due to the

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The authors are with the School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao 266061, China (e-mail: yuqunhan@163.com).

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importance of application background, the FTC problems of stochastic nonlinear systems have become a research focus, and many meaningful achievements have been obtained [48], [49], [50], [51], [52], [53]. However, the finite-time control, the actuator faults and the abrupt system faults of the stochastic nonlinear systems have not yet been considered under the same theoretical framework, which have an important role in promoting our current research.

Motivated by the above discussion, for the stochastic nonlinear systems with actuator and abrupt system faults, the adaptive MTN finite-time FTC problem is considered in this study. By using the finite-time stability theory and the adaptive MTN method, a new adaptive finite-time MTN FTC scheme is designed. The provided scheme can ensure that the closed-loop system is semi-global finite-time stable in probability (SGFSP). Compared with the existing results, the main innovation points of this paper are listed as follows:

(1) For the first time, this paper simultaneously considers the issues of finite-time control and FTC of stochastic nonlinear systems subject to multiple faults, and presentes an adaptive MTN finite-time FTC scheme. Although several MTN-based control schemes for stochastic nonlinear systems have been developed in [14], [15], [16], [17], [18], [19], and [20], most of the results can not be directly used to solve the finite-time control problem of stochastic nonlinear systems with multiple faults.

(2) For stochastic nonlinear systems, the abrupt system faults and the actuator faults are considered at the same time in this paper. It should be noted that the multiple faults problem has been studied in [43], [44], and [51]. However, their control plants are nonlinear systems rather than stochastic nonlinear systems. In addition, although [47], [48], [50], [52] also addressed the fault problem of stochastic nonlinear systems, they only focus on actuator faults and forget the abrupt system faults. Therefore, the issue and the systems studied in this paper is more general.

(3) Different from the FTC schemes proposed in [46] and [49], a novel adaptive MTN FTC scheme is proposed to realize the tracking control of the systems, which can not only achieve the tracking error converges to a small neighborhood around the origin in the finite-time, but also guarantee the closed-loop control system is SGFSP.

The rest of this paper is organized as follows. Section II provides the problem formulation and preliminaries. The main results of this paper are developed in Section III. To validate the proposed theoretic results, the simulations are made in Section IV. Finally, the conclusion of this paper is given in Section V.

II. Problem Formulation and Preliminaries

A. System Descriptions

Considering the stochastic nonlinear system with λ inputs as follows

$$\begin{cases} dx_i &= (x_{i+1} + \zeta_i(\bar{\boldsymbol{x}}_i))dt + \varsigma_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)d\boldsymbol{\omega} \\ i &= 1, 2, \ldots, n-1 \\ dx_n &= (\boldsymbol{a}^{\mathrm{T}}\boldsymbol{v} + \zeta_n(\bar{\boldsymbol{x}}_n) + H(t)F(\boldsymbol{x}))dt + \varsigma_n^{\mathrm{T}}(\bar{\boldsymbol{x}}_n)d\boldsymbol{\omega} \\ y &= x_1 \end{cases} \tag{1}$$

where $x_1, x_2, \ldots, x_n$ are the state variables with $\bar{\boldsymbol{x}}_i = [x_1, x_2, \ldots, x_i]^{\mathrm{T}} \in R^i$. $\boldsymbol{a} = [a_1, a_2, \ldots, a_\lambda]^{\mathrm{T}} \in R^\lambda$ with $a_j(j = 1, 2, \ldots, \lambda)$ are known constants, $\boldsymbol{v} = [v_1, v_2, \ldots v_\lambda]^{\mathrm{T}} \in R^\lambda$ is the system input vector, whose components may breakdown, $\zeta_i(\cdot) : R^i \to R$ and $\varsigma_i(\cdot) : R^i \to R$ denote nonlinear continuous functions and satisfy $\zeta_i(\mathbf{0}) = 0$, $\varsigma_i(\mathbf{0}) = 0$, $y \in R$ denotes the system output, ω is a standard Wiener process. Function $F(\boldsymbol{x})$ denotes the system external fault, and $H(t) \in R^n$ denotes a diagonal matrix, which is designed as

$$H(t) = \begin{cases} 0, & t < \mathrm{T}_{\text{fault}} \\ 1, & t \geq \mathrm{T}_{\text{fault}} \end{cases} \tag{2}$$

where $\mathrm{T}_{\text{fault}}$ denotes the system external fault occurrence time.

For the system (1), the aim of this paper is to propose a MTN-based adaptive finite-time FTC scheme such that: (i) The closed-loop system is SGFSP; (ii) In the finite-time, the tracking error $y - y_d$ eventually converges to a small neighborhood of the origin.

With the exception of the system external fault, two types of actuator faults are considered in this paper, namely, the lock-in-place faults and the loss of effectiveness faults. According to [49], [54], the above actuator faults can be described as follows, respectively

Lock-in-place faults model:

$$v_i(t) = \bar{v}_i, \quad t \geq t_i \tag{3}$$

where $i \in \{i_1, i_2, \cdots, i_q\} \subset \{1, 2, \cdots, \lambda\}$, $\bar{v}_i$ is a constant and t_i denotes the time when the actuator seizes up.

Loss of effectiveness faults model:

$$v_j(t) = \sigma_j u_j(t), \quad t \geq t_j \tag{4}$$

where $j \in \overline{\{i_1, i_2, \cdots, i_q\}} \cap \{1, 2, \cdots, \lambda\}$, $\sigma_j \in [\underline{\sigma}_j, 1]$ is the still effective ratio when the loss of effectiveness fault happens, $\underline{\sigma}_j$ denotes the minimum of σ_j, t_j is the time instant at which the loss of effectiveness fault occurs, $u_j(t)$ is the control framework, which will be determined later.

Based on (3) and (4), the jth control input $v_j(t)$ can be expressed as $v_j(t) = \left(1 - \rho_j\right)\sigma_j u_j(t) + \rho_j \bar{v}_j$, where $j = 1, 2, \ldots, \lambda$ and ρ_j is defined as

$$\rho_j = \begin{cases} 1, & \text{if the } j\text{th actuator seizes up} \\ 0, & \text{the other situation.} \end{cases} \tag{5}$$

Therefore, the input vector $\boldsymbol{v}(t)$ of system (1) can be expressed as follows

$$\boldsymbol{v}(t) = \boldsymbol{\sigma}\boldsymbol{u}(t) + \boldsymbol{\rho}(\bar{\boldsymbol{v}} - \boldsymbol{\sigma}\boldsymbol{u}(t)) \tag{6}$$

where $\boldsymbol{u}(t) = [u_1(t), \ldots, u_\lambda(t)]^{\mathrm{T}}$ is the control vector, $\bar{\boldsymbol{v}} = [\bar{v}_1, \ldots \bar{v}_\lambda]^{\mathrm{T}}$ is the constant vector, and $\boldsymbol{\rho} = \mathrm{diag}\{\rho_1, \rho_2, \ldots, \rho_\lambda\}$, $\boldsymbol{\sigma} = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_\lambda\}$.

Remark 1: The actuator faults considered in this paper, as shown as (3) and (4), which exist widely in practical industrial systems, such as manipulator systems, aircraft systems, and so on. The considered faults can be abrupt appearance and enter into the systems without fault diagnosis information. Consequently, it is more universal and has a wider range of applications.

Assumption 1 [17]: The reference signal y_d and its time derivatives up to the nth order are bounded and continuous.

Assumption 2 [55]: The system (1) can achieve the control goal when some actuators seize up and the others may lose effectiveness.

For the stochastic nonlinear system (1), all inputs are conducive to achieve the control aim and the control framework u_j can be designed as follows

$$u_j = b_j(x)v_0 \tag{7}$$

where $j = 1, 2, \dots, \lambda$, $x \in \Omega_n \subset R^n$, the function $b_j(x)$ satisfies $0 \le \underline{b}_j \le b_j(x) \le \bar{b}_j$, $\underline{b}_j$ is the minimum of b_j, $\bar{b}_j$ is the maximum of b_j, v_0 stands for the actual control law, which will be designed later.

B. Correlation Theory

To introduce the definition and theorem of the stochastic nonlinear system, considering the following general stochastic system

$$d\boldsymbol{x}(t) = \zeta(\boldsymbol{x}(t))dt + \varsigma(\boldsymbol{x}(t))d\boldsymbol{\omega} \tag{8}$$

where $\boldsymbol{x} \in R^n$ is the system state, $\zeta : R^n \to R^n$ and $\varsigma : R^n \to R^{n\times r}$ stand for unknown smooth nonlinear functions and satisfy $\zeta(\mathbf{0}) = \mathbf{0}$, $\varsigma(\mathbf{0}) = \mathbf{0}$, and $\boldsymbol{\omega}$ is a standard Wiener process.

Definition 1 [15]: Considering the system (8), for any function $V(\boldsymbol{x}) \in C^2$, the differential operator $\mathcal{L}$ is defined as:

$$\mathcal{L}V(\boldsymbol{x}) = \frac{\partial V(\boldsymbol{x})}{\partial \boldsymbol{x}}\zeta + \frac{1}{2}\mathrm{Tr}\left\{\varsigma^{\mathrm{T}}\frac{\partial^2 V(\boldsymbol{x})}{\partial \boldsymbol{x}^2}\varsigma\right\} \tag{9}$$

where C^2 represents the set of all functions with continuous 2-th partial derivative.

Definition 2 [35]: The equilibrium $\boldsymbol{x} = 0$ of the system (8) is said to be SGFSP, if for any $\varepsilon > 0$, there exists a setting time $T(\varepsilon, \boldsymbol{x}_0) < \infty$, such that $E[\|\boldsymbol{x}(t)\|] < \varepsilon$ when $t \ge t_0 + T$, for all $\boldsymbol{x}(t_0, \boldsymbol{\omega}) = \boldsymbol{x}_0$.

Lemma 1 [43]: For any $l_i \in R$ and $0 < \gamma \le 1$, we have $\left(\sum_{i=1}^{n} |l_i|\right)^{\gamma} \le \sum_{i=1}^{n} |l_i|^{\gamma}$.

Lemma 2 [43]: If α and β are real variables, the following inequality holds:

$$|\alpha|^{\tau}|\beta|^{o} \le \frac{\tau}{\tau+o}\, s|\alpha|^{\tau+o} + \frac{o}{\tau+o}s^{-\frac{\tau}{o}}|\beta|^{\tau+o} \tag{10}$$

where τ, o and s are any positive constants.

Lemma 3 [35]: The system (8) is SGFSP, if there exist $V(\boldsymbol{x}) \in C^2$ and $\bar{\Lambda}_1(\cdot)$, $\bar{\Lambda}_2(\cdot) \in \kappa_\infty$ for all $\boldsymbol{x} \in R^n$ and $t > t_0$, such that

$$\begin{cases} \bar{\Lambda}_1(|\boldsymbol{x}|) & \le V(\boldsymbol{x}) \le \bar{\Lambda}_2(|\boldsymbol{x}|) \\ \mathcal{L}V(\boldsymbol{x}) & \le -aV^{\gamma}(\boldsymbol{x}) + b \end{cases} \tag{11}$$

where $a > 0$, $b > 0$ and $0 < \gamma < 1$ are constants.

In this paper, the unknown smooth nonlinear function is approached by MTN. More details of the MTN are available at the research results [15], [16], [17], [18], and the approximate theory of MTN can be summarized as follows:

Lemma 4 [17]: Supposing $\zeta(\mathbf{Z})$ is a continuous function defined on a compact set $\Omega_{\mathbf{Z}} \subset R^n$, which can be approximated by MTN with any precision. That is to say, for $\forall\varepsilon > 0$, there exists a MTN $\boldsymbol{\theta}^{*\mathrm{T}} S_{m_n}(\mathbf{Z})$, such that

$$\zeta(\mathbf{Z}) = \boldsymbol{\theta}^{*\mathrm{T}} S_{m_n}(\mathbf{Z}) + \delta(\mathbf{Z}), \quad |\delta(\mathbf{Z})| \le \varepsilon \tag{12}$$

with $\mathbf{Z} = [z_1, z_1, \dots, z_n]^{\mathrm{T}} \in \Omega_{\mathbf{Z}}$, $S_{m_n}(\mathbf{Z}) = [z_1, \dots, z_n, z_1^2, z_1 z_2, \dots, z_n^2, z_1^m, z_1^{m-1} z_2, \dots, z_n^m]^{\mathrm{T}}$, and $\delta(\mathbf{Z})$ is the approximation error. $\boldsymbol{\theta}^* = [\theta_1, \theta_2, \dots, \theta_l]^{\mathrm{T}}$ is the optimal weight vector and defined as $\boldsymbol{\theta}^* := \arg\min_{\boldsymbol{\theta}\in R^l}\left\{\sup_{\mathbf{Z}\in\Omega_{\mathbf{Z}}}\left|\zeta(\mathbf{Z}) - \boldsymbol{\theta}^{\mathrm{T}} S_{m_n}(\mathbf{Z})\right|\right\}$.

Remark 2: The structure of MTN has been given in [11], [15], and [16]. It is worth pointing out that MTN is a three-layer feed-forward NN, which can be applied to the control issue of nonlinear systems. In fact, MTN can be seen as radial basis function neural network (RBFNN) with special architecture, The major difference between the MTN and RBFNN is the way of processing information of the middle-layer. Specifically, MTN uses the polynomial combination of inputs instead of the traditional radial basis function in the middle layer, which can realize the approximation of nonlinear function with less computation.

III. Main Results

This section focuses on developing a MTN-based adaptive finite-time FTC scheme, and analysing the stability of the closed-loop system.

First of all, the following coordinate transformation is employed

$$\begin{aligned} z_1 &= x_1 - y_d \\ z_i &= x_i - \alpha_{i-1},\ i = 2, \dots, n \end{aligned} \tag{13}$$

where α_{i-1} denote the virtual control signals, which will be constructed later.

According to (1) and (13), we obtain

$$\begin{cases} dz_1 & = (x_2 + \zeta_1 - \dot{y}_d)dt + \varsigma_1^{\mathrm{T}} d\boldsymbol{\omega} \\ dz_i & = (x_{i+1} + \zeta_i - \Delta\alpha_{i-1})dt + \bar{\varsigma}_i^{\mathrm{T}} d\boldsymbol{\omega} \\ i & = 2, \cdots, n-1 \\ dz_n & = (\boldsymbol{a}^{\mathrm{T}}\boldsymbol{v} + \zeta_n + H(t)F(\boldsymbol{x}) - \Delta\alpha_{n-1})dt + \bar{\varsigma}_n^{\mathrm{T}} d\boldsymbol{\omega} \end{cases} \tag{14}$$

where $\bar{\varsigma}_i = \varsigma_i - \sum_{j=1}^{i-1} \frac{\partial\alpha_{i-1}}{\partial x_j}\varsigma_j$, and $\Delta\alpha_{i-1}$ denotes the derivative of α_{i-1}, which will be determined later.

A. MTN-Based Backstepping Design

Step 1: Choosing the first Lyapunov function as follows

$$V_1 = \frac{1}{4}z_1^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\tilde{\boldsymbol{\theta}}_1 \tag{15}$$

where $\boldsymbol{\Gamma}_1 \in$ M and M denotes the set of symmetric positive definite matrices, $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$ denotes the parameter error vector.

From (14) and (15), and using the Definition 1, we have

$$\mathcal{L}V_1 = z_1^3(x_2 + \zeta_1 - \dot{y}_d) + \frac{3}{2}z_1^2\|\varsigma_1\|^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \tag{16}$$

According to Young's Inequality, we can get

$$\frac{3}{2}z_1^2\|\varsigma_1\|^2 \leq \frac{3}{4\xi_1^2}z_1^4\|\varsigma_1\|^4 + \frac{3}{4}\xi_1^2 \tag{17}$$

where $\xi_1 > 0$ is a constant.

Substituting (17) into (16) yields

$$\mathcal{L}V_1 \leq z_1^3\big(x_2 + \bar{\zeta}_1\big) - \frac{3}{2}z_1^4 + \frac{3}{4}\xi_1^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \tag{18}$$

where $\bar{\zeta}_1 = \zeta_1 + \frac{3}{2}z_1 + \frac{3}{4\xi_1^2}z_1^4\|\varsigma_1\|^4 - \dot{y}_d$.

It's easy to see that $\bar{\zeta}_1$ is an unknown function. According to Lemma 4, a MTN can be used to estimate $\bar{\zeta}_1$. Namely, for $\forall\varepsilon_1 > 0$, there exists a $\boldsymbol{\theta}_1^{\mathrm{T}}S_{m_1}(z_1)$ satisfying

$$\bar{\zeta}_1 = \boldsymbol{\theta}_1^{\mathrm{T}}S_{m_1}(\boldsymbol{z}_1) + \delta_1(\boldsymbol{z}_1), |\delta_1(\boldsymbol{z}_1)| \leq \varepsilon_1 \tag{19}$$

where $\boldsymbol{z}_1 = [z_1]^{\mathrm{T}}$ and $\delta_1(\boldsymbol{z}_1)$ is the approximation error.

With the help of Young's Inequality, we obtain

$$z_1^3 z_2 \leq \frac{1}{4}z_2^4 + \frac{3}{4}z_1^4 \tag{20}$$

$$z_1^3 \delta_1 \leq \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}z_1^4 \tag{21}$$

Substituting (13), (19), (20) and (21) into (18), we obtain

$$\mathcal{L}V_1 \leq z_1^3(\alpha_1 + \boldsymbol{\theta}_1^{\mathrm{T}}S_{m_1}) + \frac{3}{4}\xi_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{1}{4}z_2^4 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \tag{22}$$

Designing the virtual control signal α_1 as follows

$$\alpha_1 = -k_1 z_1^{4\gamma-3} - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}}S_{m_1} \tag{23}$$

where $0 < \gamma < 1$ and $k_1 > 0$ are design parameters.

Then, from (22) and (23), we have

$$\mathcal{L}V_1 \leq -k_1 z_1^{4\gamma} + \frac{1}{4}z_2^4 + \frac{3}{4}\xi_1^2 + \frac{1}{4}\varepsilon_1^4 + \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Big(z_1^3 S_{m_1} - \boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1\Big) \tag{24}$$

Based on (24), choosing the adaptive law $\dot{\hat{\boldsymbol{\theta}}}_1$ as

$$\dot{\hat{\boldsymbol{\theta}}}_1 = \boldsymbol{\Gamma}_1 S_{m_1}(\boldsymbol{z}_1)z_1^3 - \eta_1\boldsymbol{\Gamma}_1\hat{\boldsymbol{\theta}}_1 \tag{25}$$

where $\eta_1 > 0$ is a design parameter.

Substituting (25) into (24) yields

$$\mathcal{L}V_1 \leq -k_1 z_1^{4\gamma} + \frac{1}{4}z_2^4 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}\xi_1^2 + \eta_1\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\hat{\boldsymbol{\theta}}_1 \tag{26}$$

Step 2: Choosing the second Lyapunov function as follows

$$V_2 = V_1 + \frac{1}{4}z_2^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\tilde{\boldsymbol{\theta}}_2 \tag{27}$$

where $\boldsymbol{\Gamma}_2 \in$ M, $\tilde{\boldsymbol{\theta}}_2 = \boldsymbol{\theta}_2 - \hat{\boldsymbol{\theta}}_2$ denotes the parameter error vector.

From (14) and (27), and using the Definition 1, we have

$$\mathcal{L}V_2 = \mathcal{L}V_1 + z_2^3(x_3 + \zeta_2 - \Delta\alpha_1) + \frac{3}{2}z_2^2\|\bar{\varsigma}_2\|^2 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2 \tag{28}$$

where

$$\Delta\alpha_1 = \sum_{j=1}^{1}\frac{\partial\alpha_1}{\partial x_j}(\zeta_j + x_{j+1}) + \sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(j)}}y_d^{(j+1)} + \sum_{j=1}^{1}\frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j + \frac{1}{2}\sum_{j,k=1}^{1}\frac{\partial^2\alpha_1}{\partial x_j\partial x_k}\varsigma_j^{\mathrm{T}}\varsigma_k.$$

Using Young's Inequality, we obtain

$$\frac{3}{2}z_2^2\|\bar{\varsigma}_2\|^2 \leq \frac{3}{4\xi_2^2}z_2^4\|\bar{\varsigma}_2\|^4 + \frac{3}{4}\xi_2^2 \tag{29}$$

where $\xi_2 > 0$ is a constant.

Substituting (29) into (28) yields

$$\mathcal{L}V_2 \leq \mathcal{L}V_1 + z_2^3\big(x_3 + \bar{\zeta}_2\big) - \frac{7}{4}z_2^4 + \frac{3}{4}\xi_2^2 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2 \tag{30}$$

where $\bar{\zeta}_2 = \zeta_2 - \sum_{j=1}^{1}\frac{\partial\alpha_1}{\partial x_j}(\zeta_j + x_{j+1}) + \sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(j)}}y_d^{(j+1)} + \sum_{j=1}^{1}\frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j + \frac{3}{4\xi_2^2}z_2^4\|\bar{\varsigma}_2\|^4 + \frac{1}{2}\sum_{j,k=1}^{1}\frac{\partial^2\alpha_1}{\partial x_j\partial x_k}\varsigma_j^{\mathrm{T}}\varsigma_k + \frac{7}{4}z_2$.

Similarly, $\bar{\zeta}_2$ also is an unknown function that can be approximated by a MTN. In other words, for $\forall\varepsilon_2 > 0$, there is a $\boldsymbol{\theta}_2^{\mathrm{T}}S_{m_2}(z_2)$ satisfying

$$\bar{\zeta}_2 = \boldsymbol{\theta}_2^{\mathrm{T}}S_{m_2}(\boldsymbol{z}_2) + \delta_2(\boldsymbol{z}_2), |\delta_2(\boldsymbol{z}_2)| \leq \varepsilon_2 \tag{31}$$

where $\boldsymbol{z}_2 = [z_1, z_2]^{\mathrm{T}}$ and $\delta_2(\boldsymbol{z}_2)$ is the approximation error.

With the help of Young's Inequality, we can get

$$z_2^3 z_3 \leq \frac{3}{4}z_2^4 + \frac{1}{4}z_3^4 \tag{32}$$

$$z_2^3 \delta_2 \leq \frac{3}{4}z_2^4 + \frac{1}{4}\varepsilon_2^4 \tag{33}$$

Substituting (13), (31), (32) and (33) into (30), we have

$$\mathcal{L}V_2 \leq \mathcal{L}V_1 + z_2^3\big(\alpha_2 + \boldsymbol{\theta}_2^{\mathrm{T}}S_{m_2}(\boldsymbol{z}_2)\big) + \frac{3}{4}\xi_2^2 + \frac{z_3^4}{4} + \frac{1}{4}\varepsilon_2^4 - \frac{1}{4}z_2^4 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2 \tag{34}$$

Designing the virtual control signal α_2 as follows

$$\alpha_2 = -k_2 z_2^{4\gamma-3} - \hat{\boldsymbol{\theta}}_2^{\mathrm{T}}S_{m_2} \tag{35}$$

where $k_2 > 0$ is a design parameter.

Then, from (34) and (35), we have

$$\mathcal{L}V_2 \leq \mathcal{L}V_1 - k_2 z_2^{4\gamma} + \frac{1}{4}z_3^4 + \frac{3}{4}\xi_2^2 + \frac{1}{4}\varepsilon_2^4 + \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\Big(z_2^3 S_{m_2} - \boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2\Big) \tag{36}$$

Based on (36), choosing the adaptive law $\dot{\hat{\boldsymbol{\theta}}}_2$ as

$$\dot{\hat{\boldsymbol{\theta}}}_2 = \boldsymbol{\Gamma}_2 S_{m_2}(\boldsymbol{z}_2)z_2^3 - \eta_2\boldsymbol{\Gamma}_2\hat{\boldsymbol{\theta}}_2 \tag{37}$$

where $\eta_2 > 0$ is a design parameter.

Substituting (26) and (37) into (36) yields

$$\mathcal{L}V_2 \leq -\sum_{j=1}^{2} k_j z_j^{4\gamma} + \frac{1}{4}z_3^4 + \frac{1}{4}\sum_{j=1}^{2}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{2}\xi_j^2 + \sum_{j=1}^{2}\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j \tag{38}$$

Step $i(i = 3, \ldots, n-1)$: Choosing the i-th Lyapunov function as follows

$$V_i = V_{i-1} + \frac{1}{4}z_i^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i \tag{39}$$

where $\boldsymbol{\Gamma}_i \in \mathrm{M}$, $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ denotes the parameter error vector.

From (14) and (39), and using the Definition 1, we have

$$\mathcal{L}V_i = \mathcal{L}V_{i-1} + z_i^3(x_{i+1} + \zeta_i - \Delta\alpha_{i-1}) + \frac{3}{2}z_i^2\|\bar{\varsigma}_i\|^2 - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i \tag{40}$$

where $\Delta\alpha_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(\zeta_j + x_{j+1}) + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j + \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}\varsigma_j^{\mathrm{T}}\varsigma_k$.

By Young's Inequality, we obtain

$$\frac{3}{2}z_i^2\|\bar{\varsigma}_i\|^2 \leq \frac{3}{4\xi_i^2}z_i^4\|\bar{\varsigma}_i\|^4 + \frac{3}{4}\xi_i^2 \tag{41}$$

where $\xi_i > 0$ is a constant.

Substituting (41) into (40) yields

$$\mathcal{L}V_i \leq \mathcal{L}V_{i-1} + z_i^3(x_{i+1} + \bar{\zeta}_i) + \frac{3}{4}\xi_i^2 - \frac{7}{4}z_i^4 - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i \tag{42}$$

where $\bar{\zeta}_i = \zeta_i + \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}\varsigma_j^{\mathrm{T}}\varsigma_k + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\theta}}_j + \frac{3}{4\xi_i^2}z_i^4\|\bar{\varsigma}_i\|^4 - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(\zeta_j + x_{j+1}) + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \frac{7}{4}z_i$.

Similarly, $\bar{\zeta}_i$ also is an unknown function. By using the Lemma 4, a MTN can be used to estimate $\bar{\zeta}_i$. Namely, for $\forall\varepsilon_i > 0$, there exists a MTN $\boldsymbol{\theta}_i^{\mathrm{T}}S_{m_i}(z_i)$, such that

$$\bar{\zeta}_i = \boldsymbol{\theta}_i^{\mathrm{T}}S_{m_i}(\boldsymbol{z}_i) + \delta_i(\boldsymbol{z}_i), |\delta_i(\boldsymbol{z}_i)| \leq \varepsilon_i \tag{43}$$

where $\delta_i(\boldsymbol{z}_i)$ is the approximation error and $\boldsymbol{z}_i = [z_1, \cdots, z_i]^{\mathrm{T}}$.

According to Young's Inequality, and combining (13) with (42) and (43) gives

$$\mathcal{L}V_i \leq \mathcal{L}V_{i-1} + z_i^3\left(\alpha_i + \boldsymbol{\theta}_i^{\mathrm{T}}S_{m_i}\right) + \frac{3}{4}\xi_i^2 + \frac{1}{4}z_{i+1}^4 + \frac{1}{4}\varepsilon_i^4 - \frac{1}{4}z_i^4 - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i \tag{44}$$

Designing the virtual control signal α_i as

$$\alpha_i = -k_i z_i^{4\gamma-3} - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}S_{m_i} \tag{45}$$

where $k_i > 0$ is the design parameter.

Then, substituting (45) into (44), we have

$$\mathcal{L}V_i \leq \mathcal{L}V_{i-1} - k_i z_i^{4\gamma} + \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\left(z_i^3 S_{m_i} - \boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i\right) - \frac{1}{4}z_i^4 + \frac{1}{4}z_{i+1}^4 + \frac{3}{4}\xi_i^2 + \frac{1}{4}\varepsilon_i^4 \tag{46}$$

Based on (46), choosing the adaptive law $\dot{\hat{\boldsymbol{\theta}}}_i$ as follows

$$\dot{\hat{\boldsymbol{\theta}}}_i = \boldsymbol{\Gamma}_i S_{m_i} z_i^3 - \eta_i\boldsymbol{\Gamma}_i\hat{\boldsymbol{\theta}}_i \tag{47}$$

where $\eta_i > 0$ is a design parameter.

Next, substituting (47) into (46) yields

$$\mathcal{L}V_i \leq -\sum_{j=1}^{i}k_j z_j^{4\gamma} + \frac{1}{4}z_{i+1}^4 + \sum_{j=1}^{i}\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j + \frac{1}{4}\sum_{j=1}^{i}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{i}\xi_j^2 \tag{48}$$

Step n: According to (6) and (7), the following equation holds

$$\boldsymbol{a}^{\mathrm{T}}\boldsymbol{v} = a'v_0 + \sum_{j=i_1}^{i_\lambda}a_j\bar{v}_j \tag{49}$$

where $a' = \sum_{j\neq i_1\ldots i_\lambda}\sigma_j a_j b_j$.

It follows from (14) and (49), we can obtain

$$dz_n = [a'v_0 + \sum_{j=i_1}^{i_\lambda}a_j\bar{v}_j + \zeta_n + H(t)F(\boldsymbol{x}) - \Delta\alpha_{n-1}]dt + \bar{\varsigma}_n^{\mathrm{T}}d\boldsymbol{\omega} \tag{50}$$

Choosing the n-th Lyapunov function as follows

$$V_n = V_{n-1} + \frac{1}{4}z_n^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\tilde{\boldsymbol{\theta}}_n + \frac{1}{2}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\vartheta}} \tag{51}$$

where $\boldsymbol{\Gamma}_n \in \mathrm{M}$ and $\boldsymbol{\Gamma} \in \mathrm{M}$, $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n - \hat{\boldsymbol{\theta}}_n$ and $\tilde{\boldsymbol{\vartheta}} = \boldsymbol{\vartheta} - \hat{\boldsymbol{\vartheta}}$ are the parameter error vectors.

From (50) and (51), and using the Definition 1, we have

$$\mathcal{L}V_n = \mathcal{L}V_{n-1} - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n - \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\vartheta}}} + z_n^3(-\Delta\alpha_{n-1} + \zeta_n + H(t)F(\boldsymbol{x}) + z_n^3(a'v_0 + \sum_{j=i_1}^{i_\lambda}a_j\bar{v}_j)) + \frac{3}{2}z_n^2\|\bar{\varsigma}_n\|^2 \tag{52}$$

where $\Delta\alpha_{n-1} = \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(\zeta_j + x_{j+1}) + \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j + \frac{1}{2}\sum_{j,k=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_j\partial x_k}\varsigma_j^{\mathrm{T}}\varsigma_k$.

By using the Young's Inequality, we obtain

$$\frac{3}{2}z_n^2\|\bar{\varsigma}_n\|^2 \leq \frac{3}{4\xi_n^2}z_n^4\|\bar{\varsigma}_n\|^4 + \frac{3}{4}\xi_n^2 \tag{53}$$

where $\xi_n > 0$ is a constant.

Substituting (53) into (52), we can get

$$\mathcal{L}V_n \leq \mathcal{L}V_{n-1} - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n - \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\vartheta}}} + \frac{3}{4}\xi_n^2 - \frac{7}{4}z_n^4 + z_n^3(a'v_0 + \sum_{j=i_1}^{i_\lambda}a_j\bar{v}_j + H(t)F(\boldsymbol{x}) + \bar{\zeta}_n) \tag{54}$$

where $\bar{\zeta}_n = \zeta_n + \frac{7}{4}z_n + \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j + \frac{3}{4\xi_n^2}z_n^4\|\bar{\varsigma}_n\|^4 + \frac{1}{2}\sum_{j,k=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_j\partial x_k}\varsigma_j^{\mathrm{T}}\varsigma_k - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(\zeta_j + x_{j+1})$.

Similarly, $\bar{\zeta}_n$ also is an unknown function. By using the Lemma 4, $\bar{\zeta}_n$ can be estimated by a MTN. Namely, for $\forall\varepsilon_n > 0$, there exists a MTN $\boldsymbol{\theta}_n^{\mathrm{T}} S_{m_n}(\boldsymbol{z}_n)$ satisfying

$$\bar{\zeta}_n = \boldsymbol{\theta}_n^{\mathrm{T}} S_{m_n}(\boldsymbol{z}_n) + \delta_n(\boldsymbol{z}_n), |\delta_n(\boldsymbol{z}_n)| \le \varepsilon_n \tag{55}$$

where $\boldsymbol{z}_n = [z_1, \cdots, z_n]^{\mathrm{T}}$ and $\delta_n(\boldsymbol{z}_n)$ denotes the approximation error.

By using the Young's Inequality, we can obtain

$$z_n^3 \delta_n \le \frac{1}{4}\varepsilon_n^4 + \frac{3}{4} z_n^4 \tag{56}$$

Substituting (48), (55) and (56) into (54), we obtain

$$\begin{aligned}\mathcal{L}V_n \le & -\sum_{j=1}^{n-1} k_j z_j^{4\gamma} + \sum_{j=1}^{n-1} \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} (z_j^3 S_j - \boldsymbol{\Gamma}_j^{-1} \dot{\hat{\boldsymbol{\theta}}}_j) - \frac{3}{4} z_n^4 \\ & + z_n^3 (a' v_0 + \sum_{j=i_1 \dots i_p} a_j \bar{v}_j + H(t) F(\boldsymbol{x}) + \boldsymbol{\theta}_n^{\mathrm{T}} S_n) \\ & - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} \boldsymbol{\Gamma}_n^{-1} \dot{\hat{\boldsymbol{\theta}}}_n - \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \dot{\hat{\boldsymbol{\vartheta}}} + \frac{1}{4} \sum_{j=1}^{n} \varepsilon_j^4 + \frac{3}{4} \sum_{j=1}^{n} \xi_j^2 \end{aligned} \tag{57}$$

where $F(\boldsymbol{x})$ denotes the system external fault, which can be estimate by a MTN. According to Lemma 4, for $\forall\varepsilon > 0$ there exists a MTN $\boldsymbol{\vartheta}^{\mathrm{T}} \bar{S}_{\bar{m}_n}(\boldsymbol{x})$ satisfying

$$F(\boldsymbol{x}) = \boldsymbol{\vartheta}^{\mathrm{T}} \bar{S}_{\bar{m}_n}(\boldsymbol{x}) + \delta(\boldsymbol{x}), |\delta(\boldsymbol{x})| \le \varepsilon \tag{58}$$

where $\delta(\boldsymbol{x})$ denotes the approximation error and $\boldsymbol{x} = [x_1, \cdots, x_n]^{\mathrm{T}}$.

According to Young's Inequality, and taking (58) into account, we have

$$z_n^3 H(t) F(\boldsymbol{x}) \le z_n^3 \boldsymbol{\vartheta}^{\mathrm{T}} \bar{S}_{\bar{m}_n}(\boldsymbol{x}) + \frac{3}{4} z_n^4 + \frac{1}{4}\varepsilon^4 \tag{59}$$

Substituting (58) and (59) into (57) yields

$$\begin{aligned}\mathcal{L}V_n \le & -\sum_{j=1}^{n-1} k_j z_j^{4\gamma} + \sum_{j=1}^{n} \frac{\varepsilon_j^4}{4} - \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \dot{\hat{\boldsymbol{\vartheta}}} - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} \boldsymbol{\Gamma}_n^{-1} \dot{\hat{\boldsymbol{\theta}}}_n \\ & + z_n^3 (a' v_0 + \sum_{j=i_1}^{i_\lambda} a_j \bar{v}_j + \boldsymbol{\theta}_n^{\mathrm{T}} S_{m_n} + \boldsymbol{\vartheta}^{\mathrm{T}} \bar{S}_{\bar{m}_n}) \\ & + \sum_{j=1}^{n-1} \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} (z_j^3 S_{m_n} - \boldsymbol{\Gamma}_j^{-1} \dot{\hat{\boldsymbol{\theta}}}_j) + \frac{3}{4} \sum_{j=1}^{n} \xi_j^2 + \frac{\varepsilon^4}{4} \end{aligned} \tag{60}$$

Based on (60), designing the actual control input v_0 as follows

$$v_0 = -\frac{1}{a'} (k_n z_n^{4\gamma - 3} + \sum_{j=i_1}^{i_\lambda} a_j \bar{v}_j + \hat{\boldsymbol{\theta}}_n^{\mathrm{T}} S_{m_n} + \hat{\boldsymbol{\vartheta}}^{\mathrm{T}} \bar{S}_{\bar{m}_n}) \tag{61}$$

where $k_n > 0$ is the design parameter.

Then, from (60) and (61), it follows that

$$\begin{aligned}\mathcal{L}V_n \le & -\sum_{j=1}^{n} k_j z_j^{4\gamma} + \sum_{j=1}^{n-1} \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} \left(z_j^3 S_{m_n} - \Gamma_j^{-1} \dot{\hat{\boldsymbol{\theta}}}_j \right) \\ & + \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} (z_n^3 S_{m_n} - \Gamma_n^{-1} \dot{\hat{\boldsymbol{\theta}}}_n) + \frac{3}{4} \sum_{j=1}^{n} \xi_j^2 \\ & + \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}} (z_n^3 \bar{S}_{\bar{m}_n} - \Gamma^{-1} \dot{\hat{\boldsymbol{\vartheta}}}) + \frac{1}{4} \sum_{j=1}^{n} \varepsilon_j^4 + \frac{1}{4}\varepsilon^4 \end{aligned} \tag{62}$$

According to (62), choosing the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_n$ and $\dot{\hat{\boldsymbol{\vartheta}}}$ as follows

$$\dot{\hat{\boldsymbol{\theta}}}_n = \boldsymbol{\Gamma}_n S_{m_n}(\boldsymbol{z}_n) z_n^3 - \eta_n \boldsymbol{\Gamma}_n \hat{\boldsymbol{\theta}}_n \tag{63}$$

$$\dot{\hat{\boldsymbol{\vartheta}}} = \boldsymbol{\Gamma} \bar{S}_{\bar{m}_n}(\boldsymbol{x}) z_n^3 - \eta \boldsymbol{\Gamma} \hat{\boldsymbol{\vartheta}} \tag{64}$$

where $\eta > 0$ and $\eta_n > 0$ are the design parameters.

Substituting (63) and (64) into (62), we can get

$$\begin{aligned}\mathcal{L}V_n \le & -\sum_{j=1}^{n} k_j z_j^{4\gamma} + \frac{1}{4} \sum_{j=1}^{n} \varepsilon_j^4 + \frac{1}{4}\varepsilon^4 + \frac{3}{4} \sum_{j=1}^{n} \xi_j^2 \\ & + \sum_{j=1}^{n} \eta_j \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} \hat{\boldsymbol{\theta}}_j + \eta \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}} \hat{\boldsymbol{\vartheta}} \end{aligned} \tag{65}$$

Remark 3: Although the multiple faults problem for nonlinear systems has been addressed in [43], the stochastic disturbances are not taken into account. Compared with the control structure proposed in [43], that of this paper is more simpler, even though more complex issues are considered.

B. Stability Analysis

Theorem 1: For the stochastic nonlinear system (1) satisfying Assumptions 1 and 2. Then, for any initial condition, the proposed control strategy, including the control input (61), the virtual control signals (23), (35), (45) and the adaptive laws (25), (37), (47), (63) and (64), can guarantee that tracking error converges to a small neighborhood around the origin in the finite-time as well as the closed-loop control system is SGFSP.

Proof: For the entire system, considering the following Lyapunov function

$$V = V_n = \frac{1}{4} \sum_{i=1}^{n} z_i^4 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \boldsymbol{\Gamma}_i^{-1} \tilde{\boldsymbol{\theta}}_i + \frac{1}{2} \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \tilde{\boldsymbol{\vartheta}} \tag{66}$$

According to (65) and (66), we can obtain

$$\begin{aligned}\mathcal{L}V \le & -\sum_{i=1}^{n} k_i z_i^{4\gamma} + \frac{1}{4} \sum_{i=1}^{n} \varepsilon_i^4 + \frac{1}{4}\varepsilon^4 + \frac{3}{4} \sum_{i=1}^{n} \xi_i^2 \\ & + \sum_{i=1}^{n} \eta_i \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \hat{\boldsymbol{\theta}}_i + \eta \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}} \hat{\boldsymbol{\vartheta}} \end{aligned} \tag{67}$$

According to the definition of $\hat{\boldsymbol{\theta}}_i$ and $\hat{\boldsymbol{\vartheta}}$ yields

$$\sum_{i=1}^{n} \eta_i \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \hat{\boldsymbol{\theta}}_i \le \frac{1}{2} \sum_{i=1}^{n} \eta_i \|\boldsymbol{\theta}_i\|^2 - \tilde{\eta} \sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{\boldsymbol{\theta}}_i \tag{68}$$

$$\eta \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}} \hat{\boldsymbol{\vartheta}} \le \frac{\eta}{2} \|\boldsymbol{\vartheta}\|^2 - \bar{\eta} \tilde{\boldsymbol{\vartheta}}^{\mathrm{T}} \Gamma^{-1} \tilde{\boldsymbol{\vartheta}} \tag{69}$$

where $\bar{\eta} = \frac{\eta}{2\lambda_{\max}(\boldsymbol{\Gamma}^{-1})}$, $\tilde{\eta} = \min\{\tilde{\eta}_1, \cdots, \tilde{\eta}_n\}$ with $\tilde{\eta}_i = \frac{\eta_i}{2\lambda_{\max}(\Gamma_i^{-1})}$.

Substituting (68) and (69) into (67), and subtracting and adding the terms $(\bar{\eta}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}})^{\gamma}$ and $(\tilde{\eta}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i)^{\gamma}$, we have

$$\begin{aligned}\mathcal{L}V \leq & -\sum_{i=1}^{n}k_i z_i^{4\gamma} - \left(\bar{\eta}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}}\right)^{\gamma} + \left(\bar{\eta}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}}\right)^{\gamma} \\ & -(\tilde{\eta}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i)^{\gamma} + (\tilde{\eta}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i)^{\gamma} \\ & -\tilde{\eta}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i - \bar{\eta}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}} + \frac{1}{2}\sum_{i=1}^{n}\eta_i\|\boldsymbol{\theta}_i\|^2 \\ & +\frac{\eta}{2}\|\boldsymbol{\vartheta}\|^2 + \frac{1}{4}\sum_{i=1}^{n}\varepsilon_i^4 + \frac{1}{4}\varepsilon^4 + \frac{3}{4}\sum_{i=1}^{n}\xi_i^2 \end{aligned} \tag{70}$$

According to Lemma 2, we obtain

$$(\bar{\eta}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}})^{\gamma} \leq (1-\gamma)e^{\frac{\gamma\ln\gamma}{1-\gamma}} + \bar{\eta}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}} \tag{71}$$

$$(\tilde{\eta}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i)^{\gamma} \leq (1-\gamma)e^{\frac{\gamma\ln\gamma}{1-\gamma}} + \tilde{\eta}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i \tag{72}$$

Substituting (71) and (72) into (70) yields

$$\begin{aligned}\mathcal{L}V \leq & -\sum_{i=1}^{n}k_i z_i^{4\gamma} - (\tilde{\eta}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i)^{\gamma} \\ & -(\bar{\eta}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}})^{\gamma} + b \end{aligned} \tag{73}$$

where $b = \frac{1}{2}\sum_{i=1}^{n}\eta_i\|\boldsymbol{\theta}_i\|^2 + \frac{\eta}{2}\|\boldsymbol{\vartheta}\|^2 + \frac{1}{4}\sum_{i=1}^{n}\varepsilon_i^4 + \frac{1}{4}\varepsilon^4 + \frac{3}{4}\sum_{i=1}^{n}\xi_i^2 + 2(1-\gamma)e^{\frac{\gamma\ln\gamma}{1-\gamma}}$.

According to Lemma 1, we have

$$\begin{aligned} & -\sum_{i=1}^{n}k_i z_i^{4\gamma} - (\bar{\eta}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}})^{\gamma} - (\tilde{\eta}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i)^{\gamma} \\ & \leq -a((\sum_{i=1}^{n}z_i^4)^{\gamma} + (\frac{1}{2}\tilde{\boldsymbol{\vartheta}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{\vartheta}})^{\gamma} + (\frac{1}{2}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i)^{\gamma}) \\ & \leq -aV^{\gamma} \end{aligned} \tag{74}$$

where $a = \min\left\{4\underline{k}, (2\bar{\eta})^{\gamma}, (2\tilde{\eta})^{\gamma}\right\}$ with $\underline{k} = \min\{k_i | i = 1, \ldots, n\}$.

Substituting (74) into (73), yields

$$\mathcal{L}V \leq -aV^{\gamma} + b \tag{75}$$

According to Lemma 3 and inequality (75), we can obtain that the closed-loop control system is SGFSP.

In addition, according to Lemma 3 and the Theorem 1 of the work of [31], we have

$$E\left[\left(\frac{1}{2}|y - y_d|^2\right)^{\gamma}\right] \leq \left(\frac{b}{(1-u)a}\right), \forall t \geq T^* \tag{76}$$

where $0 < \mu < 1$ and $T^* = \frac{1}{(1-\gamma)\mu a} \times \left\{V^{1-\gamma}\left(\boldsymbol{x}(\boldsymbol{0}), \tilde{\boldsymbol{\theta}}_n, \tilde{\boldsymbol{\vartheta}}\right) - \left(\frac{b}{(1-\mu)a}\right)^{\frac{1-\gamma}{\gamma}}\right\}$.

This theorem completes the proof.

Remark 4: Based on the above process, a distinctive MTN-based adaptive finite-time FTC approach is presented for the stochastic nonlinear system with actuator and abrupt system faults, the details of the control procedure and signals are show in Fig. 1. In every step of the backstepping technology, the combination of nonlinear functions is estimated by the MTN, and then the computational complexity is reduced. Therefore, the constructed adaptive finite-time controller is comparatively simple, and the proposed control scheme has good application values.

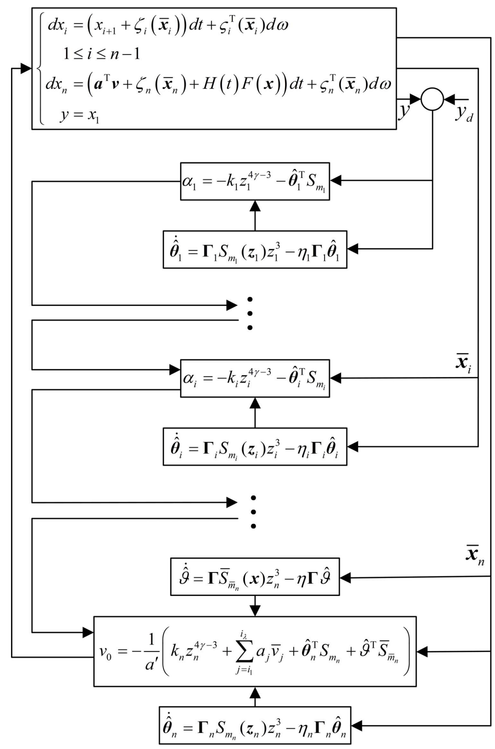

Fig. 1. Block diagram of control system.

IV. SIMULATION RESEARCH

Three examples will be given in this section to show the validity of the constructed controller.

Example 1: Considering the three-order stochastic nonlinear system with multiple faults as follows:

$$\begin{cases} dx_1 &= (x_2 + 0.2x_1\sin x_1)dt + x_1^2 d\boldsymbol{\omega} \\ dx_2 &= \left(x_3 - x_1^2\cos x_2\right)dt + x_2\sin x_1 d\boldsymbol{\omega} \\ dx_3 &= \left(\boldsymbol{a}^{\mathrm{T}}\boldsymbol{v} + x_2 x_3^2 + H(t)F(\boldsymbol{x})\right)dt + x_1 d\boldsymbol{\omega} \\ y &= x_1 \end{cases} \tag{77}$$

where $\boldsymbol{a} = [1, 2]^{\mathrm{T}}, H(t)$ is defines as (2) and $\mathrm{T}_{\text{fault}} = 25s$, the system fault $F(\boldsymbol{x}) = 5x_1x_2$, the initial state $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0$.

In simulation, the reference signal is selected as $y_d = 0.5\sin t$. The parameters of control structure are taken as follows: $b_1 = b_2 = 1$, $\eta_1 = 2$, $\eta_2 = 0.2$, $\eta_3 = 0.6$, $k_1 = 12$, $k_2 = 20$, $k_3 = 20$, $\boldsymbol{\Gamma}_1 = 0.6\boldsymbol{I}_5$, $\boldsymbol{\Gamma}_2 = 10\boldsymbol{I}_9$, $\boldsymbol{\Gamma}_3 = 0.1\boldsymbol{I}_9$ and $\gamma = 0.99$. The actuator faults are expressed by $\upsilon_1 = 0.8\upsilon_0$ and $\upsilon_2 = \bar{\upsilon}_2 = 6$ for $t > 14s$.

Figs. 2-5 show the simulation results of Example 1, respectively. Fig. 2 illustrates the trajectories of y and y_d, so it is clear that a satisfactory tracking effect has been obtained. Fig. 3 displays the trajectories of control signals υ_1 and υ_2. Fig. 4

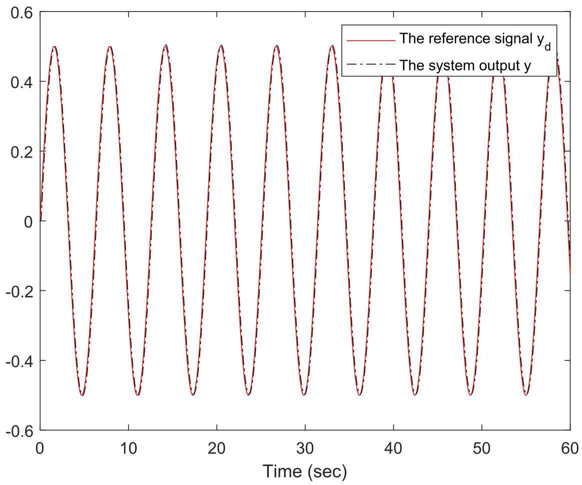

Fig. 2. The trajectories of y and y_d of the system (77).

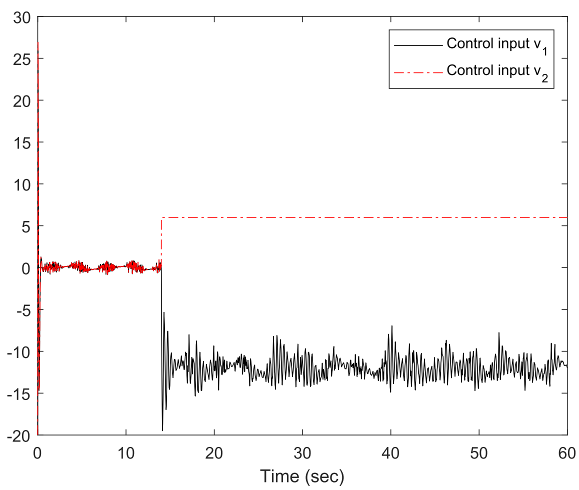

Fig. 3. The trajectories of control inputs υ_1 and υ_2 of the system (77).

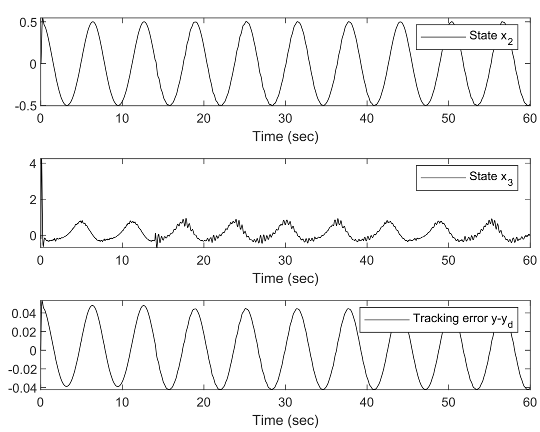

Fig. 4. The trajectories of state variables x_2, x_3 and tracking error $y - y_d$ of the system (77).

depicts the trajectories of the state variables x_2 and x_3 and the tracking error $y - y_d$, which we can see that the state variables x_2 and x_3 are bounded as well as the tracking error $y - y_d$ converges to a small neighborhood around the origin in the finite-time. Fig. 5 illustrates the trajectory of the system fault function $F(\boldsymbol{x})$. From Figs. 2-5, we can draw the conclusion that the control scheme proposed in this paper can guarantee that the closed-loop control system is SGFSP.

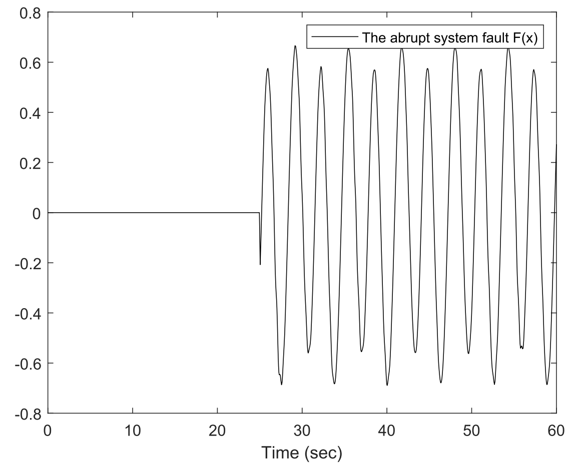

Fig. 5. The fault function $F(\boldsymbol{x})$ of the system (77).

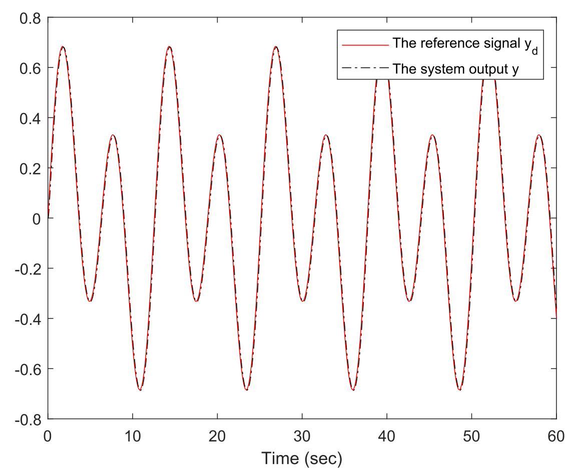

Fig. 6. The trajectories of y and y_d of the system (78).

As shown as the simulation results of the numerical example, two classes of actuator faults, the Lock-in-place fault and the Loss of effectiveness fault, occur at the same time. When $t > 14s$, the control input υ_1 loses 20% efficiency and the control input υ_2 becomes a constant. Although the multiple faults affect the system performance, we can conclude that the control scheme can still maintain the stability of the system and achieve the control goal.

Example 2: Considering a third-order one-link manipulator system with stochastic disturbances and multiple faults, according to [56], the system can be described as:

$$\begin{cases} dx_1 &= x_2 dt + x_1 d\boldsymbol{\omega} \\ dx_2 &= (x_3 - 2\sin x_1 - x_2)dt + x_2 \sin x_1 d\boldsymbol{\omega} \\ dx_3 &= \big(\boldsymbol{a}^{\mathrm{T}}\boldsymbol{v} - 2x_2 - x_3 + H(t)F(\boldsymbol{x})\big)dt + x_2 d\boldsymbol{\omega} \\ y &= x_1 \end{cases} \tag{78}$$

where x_1, x_2 and x_3 denote the link angular position, velocity and the motor shaft angle, the initial state $x_1(0) = 0$, $x_2(0) = 0, x_3(0) = 0$, $\boldsymbol{a} = [2, 1]^{\mathrm{T}}$, $H(t)$ is defines as (2) and $\mathrm{T}_{\mathrm{fault}} = 25s$, the system fault $F(\boldsymbol{x}) = 5x_2 \sin x_1$, and the given reference signal $y_d = 0.5(\sin t + 0.5\sin(0.5t))$.

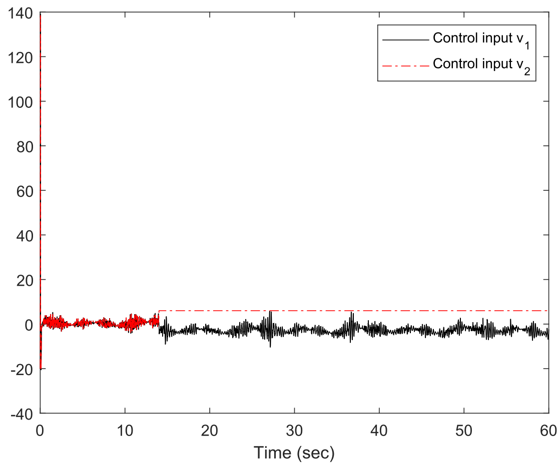

Fig. 7. The actual control inputs υ_1 and υ_2 of the system (78).

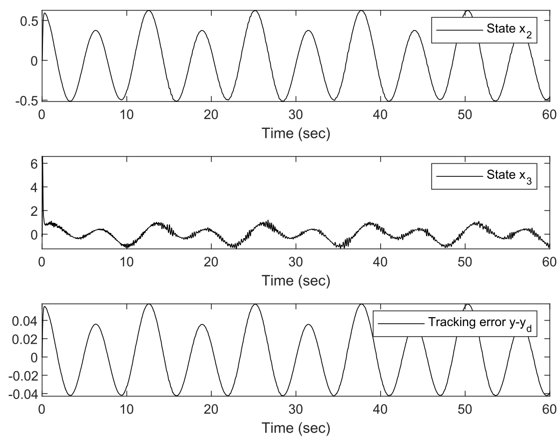

Fig. 8. The trajectories of state variables x_2, x_3 and tracking error $y - y_d$ of the system (78).

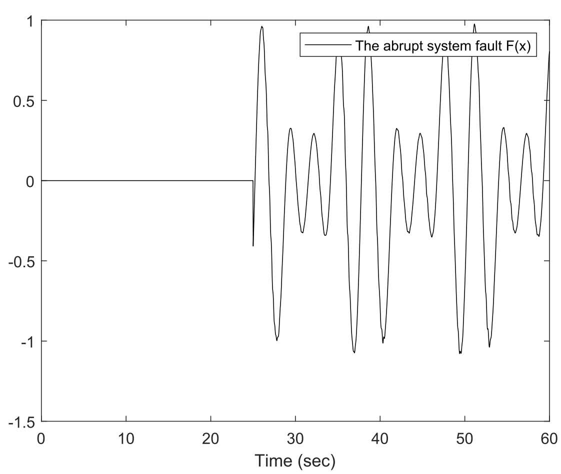

Fig. 9. The fault function $F(\boldsymbol{x})$ of the system (78).

The design parameters are taken as follows: $b_1 = b_2 = 1$, $\eta_1 = 1$, $\eta_2 = 6$, $\eta_3 = 1.8$, $k_1 = 12$, $k_2 = 50$, $k_3 = 8$, $\boldsymbol{\Gamma}_1 = 0.6\boldsymbol{I}_5$, $\boldsymbol{\Gamma}_2 = 6\boldsymbol{I}_9$, $\boldsymbol{\Gamma}_3 = 0.4\boldsymbol{I}_9$ and $\gamma = 0.99$. The actuator faults are expressed by $\upsilon_1 = 0.6\upsilon_0$ and $\upsilon_2 = \bar{\upsilon}_2 = 6$ for $t > 14s$.

The simulation results are given in Figs. 6-9, respectively. Fig. 6 illustrates the trajectories of y and y_d. Fig. 7 displays the trajectories of control signals υ_1 and υ_2. Fig. 8 depicts the trajectories of the state variables x_2 and x_3 and the tracking error $y - y_d$. Fig. 9 illustrates the trajectory of the system fault function $F(\boldsymbol{x})$. From Figs. 6-9, we can draw the conclusion that the control scheme designed in this paper can guarantee that the one-link manipulator closed-loop control system is SGFSP.

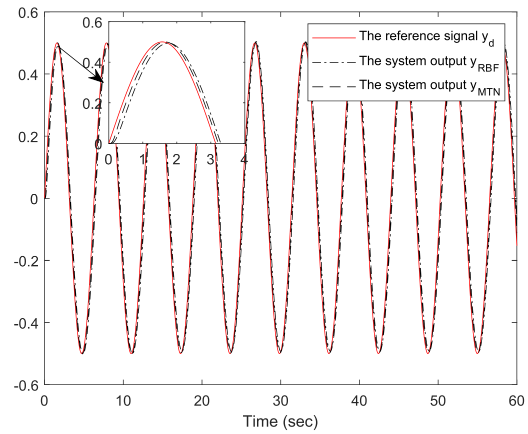

Fig. 10. The tracking performances of Example 1 under two cases.

Remark 5: The above two simulation examples show that the adaptive finite-time MTN controller designed in this paper can ensure the realization of control objectives. However, due to the singularity problem of finite-time control, it is necessary to carefully select the design parameters η_i, k_i and $\boldsymbol{\Gamma}_i$ to obtain good tracking performance.

Remark 6: It is worth pointing out that the aim of this paper can be obtained by selecting the appropriate design parameters. In the controller provided, there are many design parameters, such as η_i, k_i, a_i, b_i, γ and $\boldsymbol{\Gamma}_i$, which should be satisfy corresponding requirements respectively. In general, the tracking error can be as close to zero as possible by properly adjusting the parameters η_i, k_i and matrix $\boldsymbol{\Gamma}_i$.

Example 3: Considering the following comparative experiment between the MTN and RBFNN based on the Example 1.

According to the local enlarged diagram of Fig. 10, we can conclude that the MTN-based controller possesses more satisfactory performance and lower computational complexity than the RBFNN-based controller, which further demonstrate the superiority of the MTN control scheme.

V. Conclusion

A novel MTN-based adaptive finite-time FTC scheme is proposed for the stochastic nonlinear systems subjected to multiple faults in this paper. For the first time, the abrupt system fault and the actuator faults of the stochastic nonlinear systems are considered in the same theoretical framework. In addition, by using the MTN method, an adaptive finite-time fault-tolerant controller with the simple structure is constructed. It is shown that the designed controller can ensure that the tracking error converges to a small neighborhood of the origin in the finite-time and the closed-loop system is SGFSP. Lastly, the effectiveness of the proposed control scheme is demonstrated by three examples. Based on the research results of this paper, the event-triggered-based finite-time FTC problems of stochastic nonlinear systems with multiple faults will be considered.

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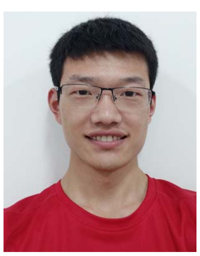
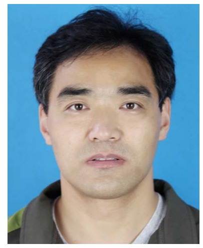


Ming-Xin Wang received the B.S. degree in information and computing science and the M.S. degree in mathematics from the Qingdao University of Science and Technology in 2015 and 2019, respectively. He is currently pursuing the Ph.D. degree with the China University of Mining and Technology, since August 2022.

His current research interests include adaptive control, synchronous control, neural networks, and stochastic systems.



Shan-Liang Zhu received the M.S. degree from the School of Mathematical Sciences, Ocean University of China, in 2004, and the Ph.D. degree from the College of Electromechanical Engineering, Qingdao University of Science and Technology, in 2020.

He is currently an Associate Professor with the Qingdao University of Science and Technology. His research interests include differential dynamic systems, data driven control, machine learning, and their applications.

Si-Min Liu received the B.S. degree in applied statistics from the Qingdao University of Science and Technology, Qingdao, China, in 2019, where she is currently pursuing the M.S. degree.

Her research interests include nonlinear system control, switched nonlinear system control, adaptive control, and their applications.

Yang Du received the B.S. degree in mathematics and applied mathematics from Binzhou University, Binzhou, China, in 2019. He is currently pursuing the M.S. degree with the Qingdao University of Science and Technology.

His current research interests include event-triggered control, adaptive control, neural networks, and stochastic nonlinear systems.

Yu-Qun Han received the B.S. degree in mathematics and applied mathematics and the M.S. degree in applied mathematics from the Qingdao University of Science and Technology, Qingdao, China, in 2010 and 2013, respectively, and the Ph.D. degree in control theory and control engineering from Southeast University, Nanjing, China, in 2018.

He has been with the School of Mathematics and Physics, Qingdao University of Science and Technology, since December 2018. His current research interests include nonlinear system control, stochastic nonlinear system control, adaptive control, and neural networks.