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Event-triggered adaptive multi-dimensional Taylor network tracking control for stochastic nonlinear systems

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Abstract
For a class of stochastic nonlinear systems, this paper proposes a novel event-triggered adaptive control scheme by means of multi-dimensional Taylor network (MTN) approach for the first time, which has the advantages of alleviating computational burden and reducing communication frequency. In addition, the event-triggered control (ETC) strategy can effectively save network resource by alleviating the computational burden and reducing the communication frequency. Therefore, the proposed control approach can not only reduce communication frequency but also further alleviate computational burden, thereby saving network resource to a greater extent. The proposed control scheme ensures that all signals of the system are semi-global uniformly ultimately bounded (SGUUB) in probability and the tracking error can be made arbitrarily small by choosing appropriate design parameters. Meanwhile, Zeno behavior can be avoided. Finally, two simulation results are given to illustrate the effectiveness of the proposed scheme.

Keywords
Adaptive control, event-triggered, multi-dimensional Taylor network, stochastic nonlinear systems

Introduction

Stochastic disturbance is a major factor leading to system instability, which widely exists in most practical systems. Therefore, the research on stochastic nonlinear systems has become a hot research topic in recent years, and many effective control methods and theories have been reported, such as adaptive control (Liu et al., 2007, 2008), backstepping control (Deng and Krstic, 1999), Lyapunov stability theory (Florchinger, 1994), sliding mode control (Qi et al., 2020), and LaSalle's invariance principle (Mao, 1999). Recently, adaptive backstepping control has also emerged as an effective method to address the unknown parameters in stochastic nonlinear systems (Gao et al., 2014; Wu et al., 2007). It is worth noting that this method is especially suitable for control nonlinear systems with structure uncertainties since it overcomes the shortcomings of the traditional control methods. However, with the increase in systematic complexity and the occurrence of parameter uncertainty, it will no longer be applied.

To address the above problems, the approximation-based adaptive neural networks (NNs) or fuzzy logic systems (FLSs) control methods for nonlinear systems were developed in the work by Leu et al. (1999), Li et al. (2021), and Ye et al. (2020). Moreover, the above control methods have also been extended to stochastic nonstrict-feedback systems (Wang et al., 2014; Wu et al., 2022), switched nonlinear systems (Han et al., 2009), strict-feedback nonlinear systems (Chen et al., 2018; Li et al., 2021), and pure-feedback nonlinear systems (Wang and Huang, 2002). Recently, the multi-dimensional Taylor network (MTN), a new NN with special structure, has been used to deal with the complex unknown nonlinearity of the systems. Immediately, many control schemes based on the MTN approximation have been reported for different systems, including single-input single-output (SISO) stochastic nonlinear systems (Han and Yan, 2018), large-scale stochastic nonlinear systems (Yan and Han, 2019), and switched nonlinear systems (Zhu et al., 2020b). In addition, the work by Han (2018), Yan et al. (2018), and Zhu et al. (2020a) focused on the study of output-feedback control problems for different systems. Under the MTN-based approximation framework, a tracking control scheme for stochastic nonlinear systems (Han, 2020) with input delay was studied, and the input delay problem for switched nonlinear systems was further studied in the work by He et al. (2022). Although the above MTN-based schemes can save network resource to a certain extent by alleviating the computational burden, these control schemes come up against difficulties when the communication channel bandwidth of the network is limited.

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Compared with the above-traditional control schemes, the event-triggered based control scheme can save the consumption of network resource by alleviating the computational burden and reducing communication frequency. Therefore, event-triggered control (ETC) has attracted the attention of more and more researchers (Jetto and Orsini, 2014; Lehmann and Lunze, 2011; Lunze and Lehmann, 2010; Shen et al., 2016). It is worth noting that the controllers and triggering event conditions in the above-cited article were designed relatively independently. However, this design method cannot achieve the optimal control design of the controller and triggering event condition, which leads to the situation that the number of control execution is greater than the optimal control on the basis of ensuring the stability of the system. In response to the above problem, three event-triggered schemes were proposed for uncertain nonlinear systems in the work by Xing et al. (2017), in which the controllers were designed based on event-triggered mechanism. Then, this controller design method has been developed for different systems, such as strict-feedback nonlinear systems (Li and Yang, 2018; Wang and Li, 2021), non-strict-feedback nonlinear systems (Hu et al., 2021; Wang et al., 2021a), pure-feedback nonlinear systems (Zhang et al., 2020), multi-input multi-output nonlinear systems (Wang et al., 2021b), and stochastic nonlinear systems (Liu et al., 2018; Wang et al., 2019; Xia et al., 2021). To our knowledge, there have been few reports on studying event-triggered adaptive control of stochastic nonlinear systems so far, especially it has not been reported under the framework of MTN approximation.

According to the above analysis, the event-triggered adaptive MTN control problem for stochastic nonlinear systems is studied for the purpose of saving network resource. In the process of controller design, the MTNs are used to approximate the unknown nonlinearity, and then a novel event-triggered adaptive MTN control scheme is developed via backstepping technique. The main contributions are summarized as follows.

(1) A new event-triggered adaptive MTN control method is proposed to solve the control problem of stochastic nonlinear systems. In this paper, by introducing ETC strategy, the proposed control method can not only ensure the stability of the system but also simultaneously save network resource. Meanwhile, all signals of the system are semi-global uniformly ultimately bounded (SGUUB) in probability and Zeno behavior is avoided.

(2) The event-triggered adaptive controller in this paper is designed based on MTN. From the work by Han (2018) and Zhu et al. (2020a, 2020b), it is known that the controllers based on MTN can alleviate the computational burden. Therefore, compared with the event-triggered adaptive NNs controllers in the work by Li and Yang (2018), Wang et al. (2019, 2021b), and Zhang et al. (2020), the controller designed in this paper can further alleviate the computational burden, thereby saving network resource to a greater extent.

(3) Although the MTN-based controllers designed in the work by Han (2018) and Zhu et al. (2020a, 2020b) can alleviate the computational burden, they still cannot solve the problem of limited communication channel bandwidth. In this paper, an ETC strategy is applied to adaptive MTN controller design to overcome the problem of limited channel bandwidth by reducing the communication frequency.

System description and preliminary

Problem description

The following stochastic nonlinear system is considered

$$\begin{cases} dx_i = (x_{i+1} + f_i(\bar{\boldsymbol{x}}_i))dt + g_i^{\mathrm{T}}(\bar{\boldsymbol{x}}_i)d\boldsymbol{\omega} \\ \qquad i = 1, 2, \ldots, n-1 \\ dx_n = (u + f_n(\bar{\boldsymbol{x}}_n))dt + g_n^{\mathrm{T}}(\bar{\boldsymbol{x}}_n)d\boldsymbol{\omega} \\ \qquad y = x_1 \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$, $u \in R$ and $y \in R$ represent the system state vector, system input and output, respectively, and $\bar{\boldsymbol{x}}_i = [x_1, x_2, \ldots, x_i]^{\mathrm{T}} \in R^i$, $i = 1, 2, \cdots, n$. $f_i(\bar{\boldsymbol{x}}_i) : R^i \to R$ and $g_i(\bar{\boldsymbol{x}}_i) : R^i \to R^r$ are the unknown smooth nonlinear functions and satisfy $f_i(\mathbf{0}) = 0$, $g_i(\mathbf{0}) = \mathbf{0}$; $\boldsymbol{\omega}$ is a r-dimensional independent standard Wiener process.

The purpose of this paper is to design an event-triggered adaptive MTN controller for system (1) to ensure that (1) the system output y tracking the reference signal y_d; (2) all signals of the closed-loop system are SGUUB in probability; and (3) the controller avoids Zeno behavior.

Assumption 1: The reference signal y_d and up to nth derivatives are bounded and continuous.

Remark 1: Compared with the work of Leu et al. (1999), Li et al. (2021), and Ye et al. (2020), the effect of stochastic disturbance is taken into consideration in this paper. Although the authors in Wang et al., (2014) and Wu et al., (2022) studies considered the existence of stochastic disturbance, the issues of communication resource and computational burden were ignored. In fact, the external stochastic disturbance is inevitable and the network resource-constrained is objective. Therefore, the system and problem studied in this paper are of great importance both theoretically and practically.

Stability theory

To introduce the necessary definitions and lemmas, the following stochastic nonlinear system is considered

$$d\boldsymbol{x} = f(\boldsymbol{x})dt + g(\boldsymbol{x})d\boldsymbol{\omega} \tag{2}$$

where $\boldsymbol{x} \in R^n$ is the system state vector, $f(\boldsymbol{x}) : R^n \to R^n$ and $g(\boldsymbol{x}) : R^n \to R^{n \times r}$ are locally Lipschitz functions and satisfy the conditions $f(\mathbf{0}) = \mathbf{0}$, $g(\mathbf{0}) = \mathbf{0}$; $\boldsymbol{\omega}$ is a r-dimensional independent standard Wiener process.

Definition 1 (Han, 2018): Considering stochastic nonlinear system (2), for any second-order continuously differentiable function $V(\boldsymbol{x})$, the differential operator L is defined as

$$LV(\boldsymbol{x}) = \frac{\partial V}{\partial \boldsymbol{x}} f + \frac{1}{2}\mathrm{Tr}\left\{ g^{\mathrm{T}} \frac{\partial^2 V}{\partial \boldsymbol{x}^2} g \right\} \quad (3)$$

where $\mathrm{Tr}\{\bullet\}$ is the trace of $\bullet$.

Definition 2 (Han, 2020): If there exists a time constant $T = T(\varepsilon, \boldsymbol{x}_0)$ with $\varepsilon > 0$ is a constant, for the compact set Ω defined on R^n and any initial state $\boldsymbol{x}_0 = \boldsymbol{x}(t_0)$ such that $E[|\boldsymbol{x}(t)|^l] < \varepsilon$, for all $t > t_0 + T$. The trajectory $\{\boldsymbol{x}(t), t \geq 0\}$ of stochastic nonlinear system (2) is said to be SGUUB in lth moment. Especially, when $l = 2$, it is usually called SGUUB in mean square.

Lemma 1 (Wang et al., 2014): For the function $V(\boldsymbol{x}) : R^n \to R$ given in Definition 1, two constants $\beta_0 > 0$ and $\chi_0 > 0$, two class κ_∞-functions $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ such that

$$\begin{cases} \tilde{\alpha}_1(|\boldsymbol{x}|) \leq V(\boldsymbol{x}) \leq \tilde{\alpha}_2(|\boldsymbol{x}|) \\ L[V(\boldsymbol{x})] \leq -\beta_0 V(\boldsymbol{x}) + \chi_0 \end{cases}$$

For all $\boldsymbol{x} \in R^n$ and $t > t_0$, system (2) has a unique strong solution for each $\boldsymbol{x}_0 \in R^n$ and it satisfies $E[V(\boldsymbol{x})] \leq V(\boldsymbol{x}_0)e^{-\beta_0 t} + (\chi_0/\beta_0), \forall t > t_0$.

Lemma 2 (Han, 2018): (Young's inequality) For $\forall(x, y) \in R^2$, one has

$$xy \leq \frac{\upsilon^\rho}{\rho}|x|^\rho + \frac{1}{q\upsilon^q}|y|^q \quad (4)$$

where $\upsilon > 0$, $\rho > 1$, $q > 1$, and $(\rho - 1)(q - 1) = 1$.

Lemma 3 (Hu et al., 2021): The following inequality holds for any constant $\gamma > 0$ and any $\eta \in R$

$$0 \leq |\eta| - \eta \tanh\left(\frac{\eta}{\gamma}\right) \leq 0.2785\gamma \quad (5)$$

MTN

As a new type of approximation network, MTN is used to approximate unknown nonlinearity in compact set $\Omega_{\boldsymbol{Z}}$ and the related lemma is introduced as follows.

Lemma 4 (Han, 2018, 2020): For compact set $\Omega_{\boldsymbol{Z}} \subset R^n$ and $\forall\varepsilon > 0$, if a continuous function $f(\boldsymbol{Z})$ defined on $\Omega_{\boldsymbol{Z}}$, there always exists an MTN $\boldsymbol{\theta}^{\mathrm{T}} S_{m_n}(\boldsymbol{Z})$, such that

$$f(\boldsymbol{Z}) = \boldsymbol{\theta}^{\mathrm{T}} S_{m_n}(\boldsymbol{Z}) + \delta(\boldsymbol{Z}) \quad (6)$$

where $\boldsymbol{Z} = [z_1, ..., z_n]^{\mathrm{T}} \in \Omega_{\boldsymbol{Z}} \subset R^n$ is the input vector of MTN, $\boldsymbol{\theta} = [\theta_1, ..., \theta_l]^{\mathrm{T}} \in R^l$ denotes the weight vector of MTN, $S_{m_n}(\boldsymbol{Z}) = [z_1, ..., z_n, z_1^2, z_1 z_2, ..., z_1 z_n, z_2 z_3, ..., z_n^2, z_1^m, ..., z_n^m]^{\mathrm{T}} \in R^l$ denotes the intermediate input layer of MTN, and $\delta(\boldsymbol{Z})$ denotes the approximation error with $|\delta(\boldsymbol{Z})| \leq \varepsilon$.

Remark 2: MTN and NN have similar structures, including three-layer structures, which are input layer, middle layer, and output layer, respectively. In particular, the middle layer of MTN is composed of polynomial arrays that only contain addition and multiplication, which is relatively simple compared to the middle layer structure of NN composed of neural units. Therefore, the MTN-based controllers have certain advantages in alleviating the computational burden.

Controller design and stability analysis

Event-triggered adaptive MTN controller design

In this section, a new event-triggered adaptive MTN controller is designed by combining backstepping technology and ETC strategy. To implement the controller design, the coordinate transformation is defined as follows

$$\begin{aligned} z_1 &= x_1 - y_d \\ z_i &= x_i - \alpha_{i-1}, i = 2, \ldots, n \end{aligned} \quad (7)$$

where α_{i-1}, $i = 2, \ldots, n$ is virtual control signal, and z_i, $i = 1, 2, \ldots, n$ is error variable.

Step 1: Taking the derivative of $z_1 = x_1 - y_d$ with respect to time t gives

$$dz_1 = (x_2 + f_1 - \dot{y}_d)dt + g_1^{\mathrm{T}} d\boldsymbol{\omega} \quad (8)$$

Selecting the appropriate first Lyapunov function as follows

$$V_1 = \frac{1}{4}z_1^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} \boldsymbol{\Gamma}_1^{-1} \tilde{\boldsymbol{\theta}}_1 \quad (9)$$

where $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$ is parameter error and $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_1^{-1} > 0$ is any constant matrix.

Then, according to Definition 1, one can obtain the following formula

$$LV_1 = z_1^3(x_2 + f_1 - \dot{y}_d) + \frac{3}{2}z_1^2 g_1^{\mathrm{T}} g_1 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} \boldsymbol{\Gamma}_1^{-1} \dot{\hat{\boldsymbol{\theta}}}_1 \quad (10)$$

Using Young's inequality, the inequality is obtained as follows

$$\frac{3}{2}z_1^2 g_1^{\mathrm{T}} g_1 \leq \frac{3}{4a_1^2} z_1^4 \|g_1\|^4 + \frac{3}{4}a_1^2 \quad (11)$$

where $a_1 > 0$ is a constant.

Substituting equation (11) into equation (10), one gets the following inequality

$$LV_1 \leq z_1^3(x_2 + \bar{f}_1) - \frac{3}{4}z_1^4 + \frac{3}{4}a_1^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} \boldsymbol{\Gamma}_1^{-1} \dot{\hat{\boldsymbol{\theta}}}_1 \quad (12)$$

where $\bar{f}_1 = f_1 - \dot{y}_d + (3/4a_1^2) z_1 \|g_1\|^4 + (3/4) z_1$.

Because $\bar{f}_1$ is unknown function, by virtue of Lemma 4, for $\forall\varepsilon_1 > 0$, there is always an MTN that is used to estimate $\bar{f}_1$

$$\bar{f}_1 = \boldsymbol{\theta}_1{}^{\mathrm{T}} S_{m_1}(z_1) + \delta_1(z_1), \quad |\delta_1(z_1)| \leqslant \varepsilon_1 \tag{13}$$

where $z_1 = [z_1]^{\mathrm{T}}$ is the input vector of MTN and $\delta_1(z_1)$ is the estimate error.

Based on equations (12) and (13), it follows that

$$LV_1 \leqslant z_1^3\big(z_2 + \alpha_1 + \boldsymbol{\theta}_1{}^T S_{m_1}(z_1) + \delta_1(z_1)\big) - \frac{3}{4}z_1^4 + \frac{3}{4}a_1^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \tag{14}$$

The following inequality can be obtained using Young's inequality

$$z_1^3 z_2 \leqslant \frac{3}{4}z_1^4 + \frac{1}{4}z_2^4 \tag{15}$$

$$z_1^3\delta_1 \leqslant \left|z_1^3\delta_1\right| \leqslant |z_1|^3\varepsilon_1 \leqslant \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4 \tag{16}$$

By combining equations (15) and (16), equation (14) can be rewritten as

$$\begin{aligned} LV_1 \leqslant\; & z_1^3\left(\alpha_1 + \frac{3}{4}z_1 + \hat{\boldsymbol{\theta}}_1^{\mathrm{T}} S_{m_1}(z_1)\right) + \frac{1}{4}\varepsilon_1^4 + \frac{1}{4}z_2^4 + \frac{3}{4}a_1^2 \\ & + \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\left(z_1^3 S_{m_1}(z_1) - \boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1\right) \end{aligned} \tag{17}$$

Then, constructing the first virtual control signal α_1 and the first adaptive law $\hat{\boldsymbol{\theta}}_1$ as follows

$$\alpha_1 = -k_1 z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}} S_{m_1} \tag{18}$$

$$\dot{\hat{\boldsymbol{\theta}}}_1 = z_1^3 S_{m_1}(z_1) - \boldsymbol{\Gamma}_1\sigma_1\hat{\boldsymbol{\theta}}_1 \tag{19}$$

where $k_1 \geqslant (3/4)$ and $\sigma_1 \geqslant 0$ are design parameters.

Substituting equations (18) and (19) into equation (17) yields

$$LV_1 \leqslant -c_1 z_1^4 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}a_1^2 + \sigma_1\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\hat{\boldsymbol{\theta}}_1 + \frac{1}{4}z_2^4 \tag{20}$$

where $c_1 = k_1 - (3/4) \geqslant 0$ is a constant.

Step 2: Taking the derivative of $z_2 = x_2 - \alpha_1$ with respect to time t gives

$$dz_2 = (x_3 + f_2 - \ell\alpha_1)dt + \tilde{g}_2{}^{\mathrm{T}} d\boldsymbol{\omega} \tag{21}$$

where $\ell\alpha_1 = (\partial\alpha_1/\partial x_1)(f_1 + x_2) + \sum_{j=0}^{1}(\partial\alpha_1/\partial y_d^{(j)})y_d^{(j+1)} + (\partial\alpha_1/\partial\hat{\boldsymbol{\theta}}_1)\dot{\hat{\boldsymbol{\theta}}}_1 + (1/2)(\partial^2\alpha_1/\partial x_1^2)g_1^{\mathrm{T}}g_1$ and $\tilde{g}_2 = g_2 - (\partial\alpha_1/\partial x_1)g_1$.

Selecting the appropriate second Lyapunov function as follows

$$V_2 = V_1 + \frac{1}{4}z_2^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\tilde{\boldsymbol{\theta}}_2 \tag{22}$$

where $\tilde{\boldsymbol{\theta}}_2 = \boldsymbol{\theta}_2 - \hat{\boldsymbol{\theta}}_2$ is parameter error and $\boldsymbol{\Gamma}_2 = \boldsymbol{\Gamma}_2^{-1} > 0$ is any constant matrix.

Then, according to Definition 1, one can obtain the following formula

$$LV_2 = LV_1 + z_2^3(x_3 + f_2 - \ell\alpha_1) + \frac{3}{2}z_2^2\|\tilde{g}_2\|^2 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2 \tag{23}$$

Using Young's inequality, the inequality is obtained as follows

$$\frac{3}{2}z_2^2\|\tilde{g}_2\|^2 \leqslant \frac{3}{4a_2^2}z_2^4\|\tilde{g}_2\|^4 + \frac{3}{4}a_2^2 \tag{24}$$

where $a_2 > 0$ is a constant.

Substituting equations (24) into (23), one gets the following inequality

$$LV_2 \leqslant LV_1 + z_2^3\big(x_3 + \bar{f}_2\big) - \frac{3}{4}z_2^4 + \frac{3}{4}a_2^2 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2 \tag{25}$$

where $\bar{f}_2 = f_2 - \frac{\partial\alpha_1}{\partial x_1}(f_1 + x_2) - \sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(j)}}y_d^{(j+1)} - \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}_1}\dot{\hat{\boldsymbol{\theta}}}_1 - \frac{1}{2}\frac{\partial^2\alpha_1}{\partial x_1^2}g_1^{\mathrm{T}}g_1 + \frac{3}{4a_2^2}z_2\|\tilde{g}_2\|^4 + \frac{3}{4}z_2$

Because $\bar{f}_2$ is unknown function, by virtue of Lemma 4, for $\forall\varepsilon_2 > 0$, there is always an MTN that is used to estimate $\bar{f}_2$

$$\bar{f}_2 = \boldsymbol{\theta}_2^{\mathrm{T}} S_{m_2}(z_2) + \delta_2(z_2), |\delta_2(z_2)| \leqslant \varepsilon_2 \tag{26}$$

where $z_2 = [z_1, z_2]^{\mathrm{T}}$ is the input vector of MTN and $\delta_2(z_2)$ is the approximation error.

Based on equations (25) and (26), it follows that

$$\begin{aligned} LV_2 \leqslant\; & LV_1 + z_2^3\big(z_3 + \alpha_2 + \boldsymbol{\theta}_2{}^T S_{m_2}(z_2) + \delta_2(z_2)\big) \\ & - \frac{3}{4}z_2^4 + \frac{3}{4}a_2^2 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2 \end{aligned} \tag{27}$$

The following inequality can be obtained using Young's inequality

$$z_2^3 z_3 \leqslant \frac{3}{4}z_2^4 + \frac{1}{4}z_3^4 \tag{28}$$

$$z_2^3\delta_2 \leqslant \left|z_2^3\delta_2\right| \leqslant |z_2|^3\varepsilon_2 \leqslant \frac{3}{4}z_2^4 + \frac{1}{4}\varepsilon_2^4 \tag{29}$$

By combining equations (28) and (29), equation (27) can be rewritten as

$$\begin{aligned} LV_2 \leqslant\; & LV_1 + z_2^3\left(\alpha_2 + \frac{3}{4}z_2 + \hat{\boldsymbol{\theta}}_2^{\mathrm{T}} S_{m_2}(z_2)\right) + \frac{1}{4}\varepsilon_2^4 + \frac{1}{4}z_3^4 \\ & + \frac{3}{4}a_2^2 + \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}(z_2^3 S_{m_2}(z_2) - \boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2) \end{aligned} \tag{30}$$

Then, constructing the second virtual control signal α_2 and the second adaptive law $\hat{\boldsymbol{\theta}}_2$ as follows

$$\alpha_2 = -k_2 z_2 - \hat{\boldsymbol{\theta}}_2^{\mathrm{T}} S_{m_2} \tag{31}$$

$$\dot{\hat{\boldsymbol{\theta}}}_2 = z_2^3 S_{m_2}(z_2) - \boldsymbol{\Gamma}_2\sigma_2\hat{\boldsymbol{\theta}}_2 \tag{32}$$

where $k_2 \geqslant 1$ and $\sigma_2 \geqslant 0$ are design parameters.

Substituting equations (31) and (32) into equation (30) yields

$$LV_2 \leqslant -\sum_{j=1}^{2} c_j z_j^4 + \frac{1}{4}\sum_{j=1}^{2}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{2} a_j^2 + \sum_{j=1}^{2}\sigma_j \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j + \frac{1}{4}z_3^4 \tag{33}$$

where $c_1 = k_1 - (3/4) \geqslant 0$, $c_2 = k_2 - 1 \geqslant 0$.

Step *i* $(3 \leqslant i \leqslant n-1)$ **:** Taking the derivative of $z_i = x_i - \alpha_{i-1}$ with respect to time t gives

$$dz_i = (x_{i+1} + f_i - \ell\alpha_{i-1})dt + \tilde{g}_i^{\mathrm{T}} d\boldsymbol{\omega} \tag{34}$$

where $\ell\alpha_{i-1} = \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)(f_j + x_{j+1}) + \sum_{j=0}^{i-1}(\partial\alpha_{i-1}/\partial y_d^{(j)}) y_d^{(j+1)} + \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\hat{\boldsymbol{\theta}}_j)\dot{\hat{\boldsymbol{\theta}}}_j + (1/2)\sum_{j,k=1}^{i-1}(\partial^2\alpha_{i-1}/\partial x_j \partial x_k)g_j^{\mathrm{T}} g_k$ and $\tilde{g}_i = g_i - \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)g_j$.

Selecting the appropriate ith Lyapunov function as follows

$$V_i = V_{i-1} + \frac{1}{4}z_i^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i \tag{35}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ is parameter error and $\boldsymbol{\Gamma}_i = \boldsymbol{\Gamma}_i^{-1} > 0$ is any constant matrix.

Then, according to Definition 1, one can obtain the following formula

$$LV_i = LV_{i-1} + z_i^3(x_{i+1} + f_i - \ell\alpha_{i-1}) + \frac{3}{2}z_i^2\|\tilde{g}_i\|^2 - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i \tag{36}$$

Using Young's inequality, the inequality is obtained as follows

$$\frac{3}{2}z_i^2\|\tilde{g}_i\|^2 \leqslant \frac{3}{4a_i^2}z_i^4\|\tilde{g}_i\|^4 + \frac{3}{4}a_i^2 \tag{37}$$

where $a_i > 0$ is a constant.

Substituting equation (37) into equation (36), one gets the following inequality

$$LV_i \leqslant LV_{i-1} + z_i^3\left(x_{i+1} + \bar{f}_i\right) - \frac{3}{4}z_i^4 + \frac{3}{4}a_i^2 - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i \tag{38}$$

where

$$\begin{aligned}\bar{f}_i = f_i &- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(f_j + x_{j+1}\right) - \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} \\ &- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j - \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}g_j^{\mathrm{T}}g_k \\ &+ \frac{3}{4a_i^2}z_i\|\tilde{g}_i\|^4 + \frac{3}{4}z_i\end{aligned}$$

Similarly, by virtue of Lemma 4, for $\forall\varepsilon_i > 0$, there is always an MTN that is used to estimate $\bar{f}_i$

$$\bar{f}_i = \boldsymbol{\theta}_i^{\mathrm{T}} S_{m_i}(\boldsymbol{z}_i) + \delta_i(\boldsymbol{z}_i), |\delta_i(\boldsymbol{z}_i)| \leqslant \varepsilon_i \tag{39}$$

where $\boldsymbol{z}_i = [z_1, \ldots, z_i]^{\mathrm{T}}$ is the input vector of MTN and $\delta_i(\boldsymbol{z}_i)$ is the approximation error.

Based on equations (38) and (39), it follows that

$$\begin{aligned}LV_i \leqslant LV_{i-1} &+ z_i^3\left(z_{i+1} + \alpha_i + \boldsymbol{\theta}_i{}^T S_{m_i}(\boldsymbol{z}_i) + \delta_i(\boldsymbol{z}_i)\right) - \frac{3}{4}z_i^4 \\ &+ \frac{3}{4}a_i^2 - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i\end{aligned} \tag{40}$$

The following inequality can be obtained using Young's inequality

$$z_i^3 z_{i+1} \leqslant \frac{3}{4}z_i^4 + \frac{1}{4}z_{i+1}^4 \tag{41}$$

$$z_i^3\delta_i \leqslant \left|z_i^3\delta_i\right| \leqslant |z_i|^3\varepsilon_i \leqslant \frac{3}{4}z_i^4 + \frac{1}{4}\varepsilon_i^4 \tag{42}$$

By combining equations (41) and (42), equation (40) can be rewritten as

$$\begin{aligned}LV_i \leqslant LV_{i-1} &+ z_i^3\left(\alpha_{i+1} + \frac{3}{2}z_i + \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} S_{m_i}(\boldsymbol{z}_i)\right) + \frac{1}{4}\varepsilon_i^4 + \frac{1}{4}z_{i+1}^4 \\ &+ \frac{3}{4}a_i^2 \\ &+ \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_i^3 S_{m_i}(\boldsymbol{z}_i) - \boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i)\end{aligned} \tag{43}$$

Then, constructing the ith virtual control signal α_i and the ith adaptive law $\dot{\hat{\boldsymbol{\theta}}}_i$ as follows

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} S_{m_i} \tag{44}$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i^3 S_{m_i}(\boldsymbol{z}_i) - \boldsymbol{\Gamma}_i\sigma_i\hat{\boldsymbol{\theta}}_i \tag{45}$$

where $k_i \geqslant 1$ and $\sigma_i \geqslant 0$ are design parameters.

Substituting equations (44) and (45) into equation (43) yields

$$LV_i \leqslant -\sum_{j=1}^{i} c_j z_j^4 + \frac{1}{4}\sum_{j=1}^{i}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{i} a_j^2 + \sum_{j=1}^{i}\sigma_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j + \frac{1}{4}z_{i+1}^4 \tag{46}$$

where $c_1 = k_1 - (3/4) \geqslant 0$, $c_i = k_i - 1 \geqslant 0$, $i = 1, 2, ..., n-1$.

Step *n* : Taking the derivative of $z_n = x_n - \alpha_{n-1}$ with respect to time t gives

$$dz_n = (u + f_n - \ell\alpha_{n-1})dt + \tilde{g}_n^{\mathrm{T}} d\boldsymbol{\omega} \tag{47}$$

where $\ell\alpha_{n-1} = \sum_{j=1}^{n-1}(\partial\alpha_{n-1}/\partial x_j)(f_j + x_{j+1}) + \sum_{j=0}^{n-1}(\partial\alpha_{n-1}/\partial y_d^{(j)}) y_d^{(j+1)} + \sum_{j=1}^{n-1}(\partial\alpha_{i-1}/\hat{\boldsymbol{\theta}}_j)\dot{\hat{\boldsymbol{\theta}}}_j + (1/2)\sum_{j,k=1}^{n-1}(\partial^2\alpha_{n-1}/\partial x_j\partial x_k)g_j^{\mathrm{T}} g_k$ and $\tilde{g}_n = g_n - \sum_{j=1}^{n-1}(\partial\alpha_{n-1}/\partial x_j)g_j$.

Selecting the appropriate nth Lyapunov function as follows

$$V_n = V_{n-1} + \frac{1}{4}z_n^4 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\tilde{\boldsymbol{\theta}}_n \tag{48}$$

where $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n - \hat{\boldsymbol{\theta}}_n$ is parameter error and $\boldsymbol{\Gamma}_n = \boldsymbol{\Gamma}_n^{-1} > 0$ is any constant matrix.

Then, according to Definition 1, one can obtain the following formula

$$LV_n = LV_{n-1} + z_n^3(u + f_n - \ell\alpha_{n-1}) + \frac{3}{2}z_n^2\|\tilde{g}_n\|^2 - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n \tag{49}$$

Using Young's inequality, the inequality is obtained as follows

$$\frac{3}{2}z_n^2\|\tilde{g}_n\|^2 \leqslant \frac{3}{4a_n^2}z_n^4\|\tilde{g}_n\|^4 + \frac{3}{4}a_n^2 \tag{50}$$

where $a_n > 0$ is a constant.

Substituting equation (37) into equation (36), one gets the following inequality

$$LV_n \leqslant LV_{n-1} + z_n^3\left(u + \bar{f}_n\right) - z_n^4 + \frac{3}{4}a_n^2 - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n \tag{51}$$

where

$$\begin{aligned}\bar{f}_n &= f_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(f_j + x_{j+1}\right)\\ &- \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)} - \sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j - \frac{1}{2}\sum_{j,k=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_j\partial x_k}g_j^{\mathrm{T}}g_k\\ &+ \frac{3}{4a_n^2}z_n\|\tilde{g}_n\|^4 + z_n\end{aligned}$$

Similarly, by virtue of Lemma 4, for $\forall\varepsilon_n > 0$, there is always an MTN that is used to estimate $\bar{f}_n$

$$\bar{f}_n = \boldsymbol{\theta}_n^{\mathrm{T}}S_{m_n}(\boldsymbol{z}_n) + \delta_n(\boldsymbol{z}_n), |\delta_n(\boldsymbol{z}_n)| \leqslant \varepsilon_n \tag{52}$$

where $\boldsymbol{z}_n = [z_1, \ldots, z_n]^{\mathrm{T}}$ is the input vector of MTN and $\delta_n(\boldsymbol{z}_n)$ is the approximation error.

The following inequality can be obtained using Young's inequality

$$z_n^3\delta_n \leqslant \left|z_n^3\delta_n\right| \leqslant |z_n|^3\varepsilon_n \leqslant \frac{3}{4}z_n^4 + \frac{1}{4}\varepsilon_n^4 \tag{53}$$

By combining equations (52) and (53), equation (51) can be rewritten as

$$\begin{aligned}LV_n \leqslant LV_{n-1} &+ z_n^3\left(u + \hat{\boldsymbol{\theta}}_n^{\mathrm{T}}S_{m_n}(\boldsymbol{z}_n)\right) + \frac{1}{4}\varepsilon_n^4 + \frac{3}{4}a_n^2\\ &+ \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\left(z_n^3S_{m_n}(\boldsymbol{z}_n) - \boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n\right) - \frac{1}{4}z_n^4\end{aligned} \tag{54}$$

According to the above analysis, the event-triggered adaptive MTN controller and the adaptive law $\hat{\boldsymbol{\theta}}_n$ are designed as follows

$$W(t) = \alpha_n - \bar{M}_1\tanh\left(\frac{z_n^3\bar{M}_1}{\gamma}\right) \tag{55}$$

$$\dot{\hat{\boldsymbol{\theta}}}_n = z_n^3S_{m_n}(\boldsymbol{z}_n) - \boldsymbol{\Gamma}_n\sigma_n\hat{\boldsymbol{\theta}}_n \tag{56}$$

The triggering event condition using fixed threshold is designed as

$$\begin{cases}u(t) = W(t_k), \forall t \in [t_k, t_{k+1})\\ t_{k+1} = \inf\{t \in R\,||e(t)| \geqslant M_1\}\end{cases} \tag{57}$$

where γ, σ_n, t_k, $k \in z^+$, M_1, and $\bar{M}_1 > M_1$ are positive design parameters, $e(t) = W(t) - u(t)$ represents the measurement error. t_k is the triggering event time. During the time $t \in [t_k, t_{k+1})$, the control signal u holds as a constant $W(t_k)$. From equation (57), when the event is triggered at $t = t_{k+1}$, the control input value of the system will be changed to $u(t_{k+1})$.

Remark 3: The positive parameter M_1 can determine the event update frequency and update time. When the M_1 value is larger, the signal transmission can obtain a larger execution interval, thus saving communication resource to a greater extent on the premise of ensuring system stability. In fact, different M_1 can be selected for different control objectives to achieve a compromise between system performance and resource saving. Besides, the positive parameter M_1 ensures that Zeno behavior is avoided when the tracking error approaches zero.

Remark 4: Compared with the event-triggered adaptive NNs controllers in the work by Li and Yang (2018), Wang et al. (2019, 2021b), and Zhang et al. (2020), the event-triggered adaptive MTN controller designed in this paper can further alleviate the computational burden due to the simple structure of MTN, thus saving the consumption of network resource to a greater extent.

From equation (57), obviously, $|W(t) - u(t)| \leqslant M_1$ within the time interval $t \in [t_k, t_{k+1})$. Therefore, we can get $W(t) = u(t) + \lambda(t)M_1$ where $\lambda(t)$ is a continuous time-varying parameter satisfying $|\lambda(t)| \leqslant 1$. Then, one has

$$\begin{aligned}LV_n \leqslant LV_{n-1} &+ z_n^3(W(t) - \lambda(t)M_1) + z_n^3\hat{\boldsymbol{\theta}}_n^{\mathrm{T}}S_{m_n}(\boldsymbol{z}_n)\\ &+ \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\left(z_n^3S_{m_n}(\boldsymbol{z}_n) - \boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n\right)\\ &+ \frac{1}{4}\varepsilon_n^4 + \frac{3}{4}a_n^2 - \frac{1}{4}z_n^4\end{aligned} \tag{58}$$

Substituting equation (55) into equation (58) yields

$$\begin{aligned}LV_n \leqslant LV_{n-1} &+ z_n^3\alpha_n - z_n^3\bar{M}_1\tanh\left(\frac{z_n^3\bar{M}_1}{\gamma}\right)\\ &- z_n^3\lambda(t)M_1 + \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\left(z_n^3S_{m_n}(\boldsymbol{z}_n) - \boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n\right)\\ &+ z_n^3\hat{\boldsymbol{\theta}}_n^{\mathrm{T}}S_{m_n}(\boldsymbol{z}_n) + \frac{1}{4}\varepsilon_n^4 + \frac{3}{4}a_n^2 - \frac{1}{4}z_n^4\end{aligned} \tag{59}$$

By applying Lemma 3, equation (59) can be transformed into the following form

$$\begin{aligned}LV_n \leqslant LV_{n-1} &+ z_n^3\alpha_n + 0.2785\gamma - \left|z_n^3\bar{M}_1\right| - z_n^3\lambda(t)M_1\\ &+ \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\left(z_n^3S_{m_n}(\boldsymbol{z}_n) - \boldsymbol{\Gamma}_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n\right)\\ &+ z_n^3\hat{\boldsymbol{\theta}}_n^{\mathrm{T}}S_{m_n}(\boldsymbol{z}_n) + \frac{1}{4}\varepsilon_n^4 + \frac{3}{4}a_n^2 - \frac{1}{4}z_n^4\end{aligned} \tag{60}$$

Selecting $\alpha_n = -k_nz_n - \hat{\boldsymbol{\theta}}_n^{\mathrm{T}}S_{m_n}$, together with equation (56), inequality (60) can be rewritten as follows

$$LV_n \leqslant LV_{n-1} - k_n z_n^4 + 0.2785\gamma - \left|z_n^3 \bar{M}_1\right| - z_n^3 \lambda(t) M_1 + \frac{1}{4}\varepsilon_n^4 + \frac{3}{4}a_n^2 + \sigma_n \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} \hat{\boldsymbol{\theta}}_n - \frac{1}{4}z_n^4 \leqslant -\sum_{j=1}^{n} c_j z_j^4 + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{n} a_j^2 + \sum_{j=1}^{n}\sigma_j \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j + 0.2785\gamma \tag{61}$$

where $c_1 = k_1 - (3/4) \geqslant 0$, $c_i = k_i - 1 \geqslant 0$, $i = 1, 2, ..., n-1$, $c_n = k_n \geqslant 0$.

Stability analysis

Theorem 1: Considering closed-loop stochastic nonlinear system (1), under the condition of Assumption 1, event-triggered adaptive MTN controller (55), adaptive laws (19), (32), (45), (56), and triggering event condition (57). Then, for any bounded initial conditions, the proposed method guarantees that all signals of the system are SGUUB in probability and the tracking error can be made arbitrarily small by choosing appropriate design parameters. Meanwhile, Zeno behavior can be avoided.

Proof: According to the control design process, the following Lyapunov function is considered

$$V = V_n = \frac{1}{4}\sum_{j=1}^{n} z_j^4 + \frac{1}{2}\sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\Gamma}_j^{-1}\tilde{\boldsymbol{\theta}}_j \tag{62}$$

According to equation (61), we can get

$$LV \leqslant -\sum_{j=1}^{n} c_j z_j^4 + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{n} a_j^2 + \sum_{j=1}^{n}\sigma_j \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j + 0.2785\gamma \tag{63}$$

with $\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}$ and $\hat{\boldsymbol{\theta}}_j$ are given by

$$\sigma_j \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j = \sigma_j \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(\boldsymbol{\theta} - \tilde{\boldsymbol{\theta}}_j\right) \leqslant -\frac{\sigma_j}{2}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j + \frac{\sigma_j}{2}\left\|\boldsymbol{\theta}_j\right\|^2 \tag{64}$$

with

$$-\frac{\sigma_j}{2}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j \leqslant -\frac{\sigma_j}{2\lambda_{\max}\left(\boldsymbol{\Gamma}_j^{-1}\right)}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\Gamma}_j^{-1}\tilde{\boldsymbol{\theta}}_j \tag{65}$$

Substituting equations (64) and (65) into equation (63) yields

$$LV \leqslant -\sum_{j=1}^{n} c_j z_j^4 - \frac{1}{2}\bar{\sigma}_j \sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{\Gamma}_j^{-1}\tilde{\boldsymbol{\theta}}_j + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{n} a_j^2 + \frac{1}{2}\sum_{j=1}^{n}\sigma_j\left\|\boldsymbol{\theta}_j\right\|^2 + 0.2785\gamma \tag{66}$$

where $\bar{\sigma}_j = \min\{(\sigma_j/(\lambda_{\max}(\boldsymbol{\Gamma}_j^{-1})))|j = 1, 2, ..., n\}$.

Let $\beta_0 = \min\{4c_j, \bar{\sigma}_j | j = 1, 2, ..., n\}$, $\chi_0 = (1/4)\sum_{j=1}^{n}\varepsilon_j^4 + (3/4)\sum_{j=1}^{n} w_j^2 + (1/2)\sum_{j=1}^{n}\sigma_j\left\|\boldsymbol{\theta}_j\right\|^2 + 0.2785\gamma$, yields

$$LV \leqslant -\beta_0 V + \chi_0 \tag{67}$$

Then, according to Lemma 1 and for $\forall t \geqslant 0$, the following inequality holds

$$E[V(t)] \leqslant V(0)e^{-\beta_0 t} + \frac{\chi_0}{\beta_0} \tag{68}$$

Therefore, we have

$$\frac{1}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i \leqslant V(0)e^{-\beta_0 t} + \frac{\chi_0}{\beta_0}, i = 1, 2, ..., n \tag{69}$$

$$\frac{1}{4}z_i^4 \leqslant V(0)e^{-\beta_0 t} + \frac{\chi_0}{\beta_0}, i = 1, 2, ..., n \tag{70}$$

From equations (69) and (70), we know that $z_i (i = 1, 2, ..., n)$ and $\left\|\tilde{\boldsymbol{\theta}}_i\right\| (i = 1, 2, ..., n)$ are bounded in probability. Therefore, we can further deduce that $\left\|\hat{\boldsymbol{\theta}}_i\right\|$ and α_i are also bounded in probability. Due to $z_i = x_i - \alpha_{i-1}$, x_i is bounded in probability. Finally, controller $W(t)$ (55) and system input u (57) are bounded in probability can be inferred, too. Therefore, all signals of stochastic nonlinear system (1) are SGUUB in probability.

Then we need to prove that $\exists t^* > 0$ such that $\{t_{k+1} - t_k\} \geqslant t^*$, for $\forall k \in z^+$. Recalling $e(t) = W(t) - u(t)$, $\forall t \in [t_k, t_{k+1})$, we have

$$\frac{d}{dt}|e| = \frac{d}{dt}(e * e)^{\frac{1}{2}} = \mathrm{sign}(e)\dot{e} \leqslant \left|\dot{W}\right| \tag{71}$$

From equation (55), the time derivative of $W(t)$ can be expressed as

$$\dot{W}(t) = \dot{\alpha}_n - \frac{\bar{M}_1}{\gamma} \cdot \frac{3z_n^2 \dot{z}_n \bar{M}_1}{\cosh^2\left(\frac{z_n^3 \bar{M}_1}{\gamma}\right)} \tag{72}$$

Obviously, $\dot{W}(t)$ is a bounded function composed of α_n and z_n. Therefore, it can be presented as $\left|\dot{W}(t)\right| \leqslant \vartheta$, where the constant $\vartheta > 0$. In addition, noting that $e(t_k) = 0$, $\lim_{t \to t_{k+1}} e(t) = M_1$, so we can get that the lower bound of the execution interval t^* satisfies $t^* \geqslant (M_1/\vartheta)$. Therefore, Zeno behavior is avoided.

Remark 5: Under the event-triggered adaptive MTN controller designed in this paper, it needs to be proved that the tracking error z_1 converges to an arbitrarily small range.

From equation (70), we can get

$$z_1^4 \leqslant 4V(0)e^{-\beta_0 t} + \frac{4\chi_0}{\beta_0} \tag{73}$$

According to the above inequality, we can further get

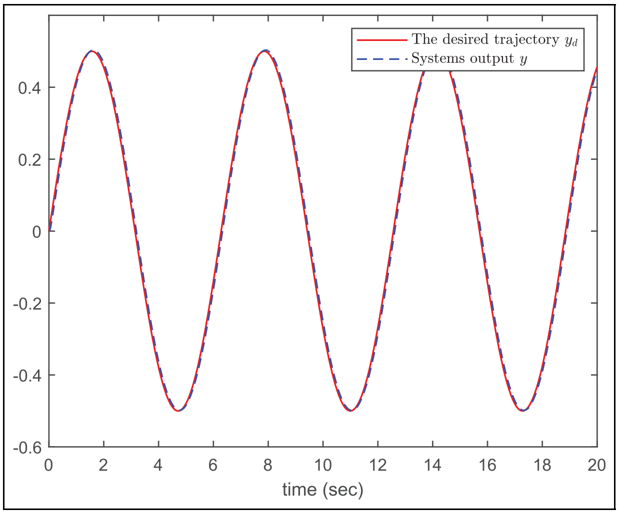

Figure 1. The trajectories of y and y_d of equation (75).

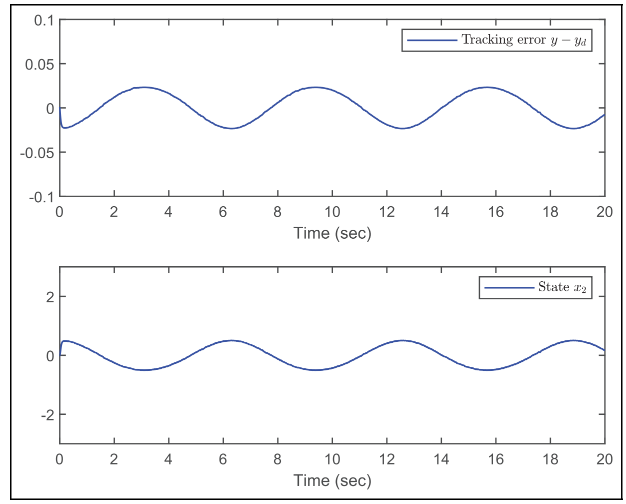

Figure 2. The trajectories of tracking error $y - y_d$ and state x_2 of equation (75).

$$|z_1| = |x_1 - y_d| \leqslant \sqrt[4]{\frac{4\chi_0}{\beta_0}} \tag{74}$$

Therefore, we can make the tracking error converges to an arbitrarily small range by increasing the control parameters $c_1, c_2, ..., c_n$ and simultaneously decreasing control parameters $(\lambda_{\max}(\boldsymbol{\Gamma}_i^{-1})), i = 1, 2, ...n, \gamma$.

Simulation results

In this note, a numerical example and a practical example are used to demonstrate the effectiveness of the design strategy.

Example 1 (Numerical example): Considering the second-order stochastic nonlinear system as follows

$$\begin{cases} dx_1 = (x_2 + 0.2x_1 \sin(x_1))dt + 0.1x_1 d\boldsymbol{\omega} \\ dx_2 = \left(u - x_1 \cos\left(x_2^2\right) + \sin(x_1)\right)dt + 0.1x_2 d\boldsymbol{\omega} \\ y = x_1 \end{cases} \tag{75}$$

with the initial states $[x_1(0), x_2(0)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, the desired reference signal is selected as $y_d = 0.5\sin(t)$.

According to Theorem 1, the MTN-based event-triggered adaptive tracking control strategy of system (75) can be designed as follows

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} S_{m_i}, i = 1, 2$$

$$W(t) = \alpha_2 - \bar{M}_1 \tanh\left(\frac{z_2^3 \bar{M}_1}{\gamma}\right)$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i^3 S_{m_i}(z_i) - \boldsymbol{\Gamma}_i \sigma_i \hat{\boldsymbol{\theta}}_i, \sigma_i \geqslant 0, i = 1, 2$$

$$\begin{cases} u(t) = W(t_k), \forall t \in [t_k, t_{k+1}) \\ t_{k+1} = \inf\{t \in R \| e(t)| \geqslant M_1\} \end{cases}$$

where $\boldsymbol{z}_1 = [z_1]^{\mathrm{T}}$, $\boldsymbol{z}_2 = [z_1, z_2]^{\mathrm{T}}$.

In simulation, the design parameters of the system controller and the triggering event condition are selected as $M_1 = 1$, $\bar{M}_1 = 7$, $k_1 = 21.5$, $k_2 = 100$, $\gamma = 0.01$, $\sigma_1 = 4$, $\sigma_2 = 10$, $\boldsymbol{\Gamma}_1 = 0.7\boldsymbol{I}_5$, $\boldsymbol{\Gamma}_2 = 0.2\boldsymbol{I}_9$. The simulation results are shown in Figures 1–4.

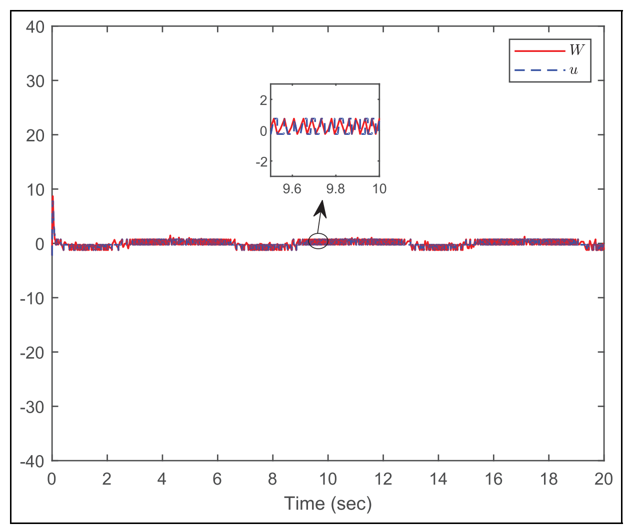

Figure 3. The trajectories of W and u of equation (75).

Figure 1 depicts the curves of the desired signal y_d and the output signal y. Figure 2 demonstrates the tracking error $y - y_d$ and the state trajectory x_2. The inputs W and u are shown in Figure 3. It can be seen that the curve of the control input signals fluctuates greatly in the initial stage, and then the fluctuations tend to stabilize. All the triggering instants and triggering time intervals are shown in Figure 4. The proposed control scheme not only ensures a satisfactory tracking control result of the system but also avoids the Zeno behavior.

Example 2 (Practical example): Considering a class of single-link manipulator system, according to the work of

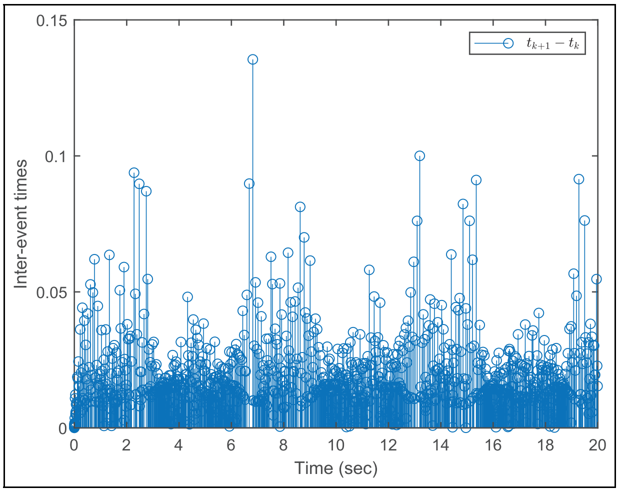

Figure 4. The time interval of triggering events of equation (75).

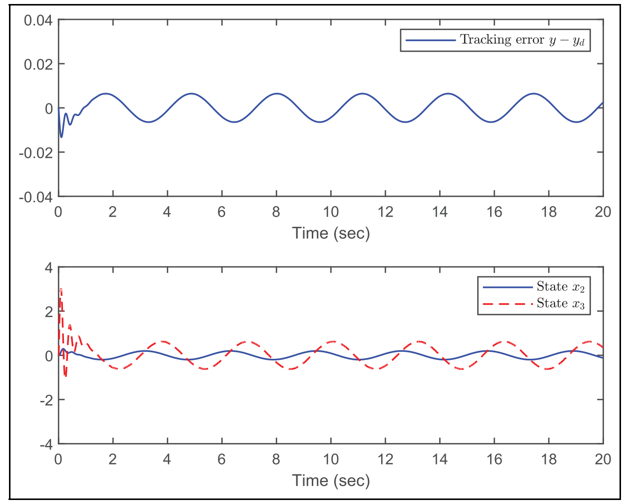

Figure 6. The trajectories of tracking error $y-y_d$ and states of x_2 and x_3 of equation (76).

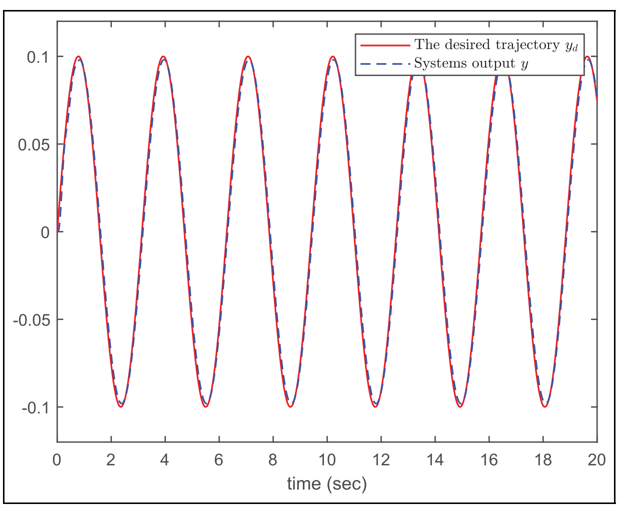

Figure 5. The trajectories of y and y_d of equation (76).

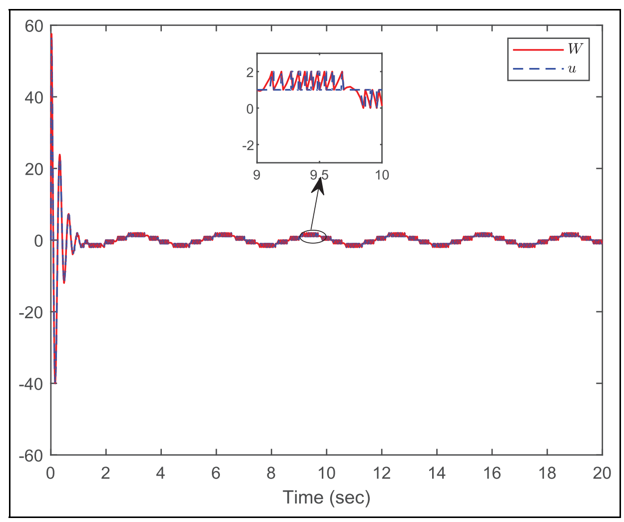

Figure 7. The trajectories of W and u of equation (76).

Wang and Chen (2021), it can be described by the following third-order stochastic nonlinear system

$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = \left(\frac{1}{B}x_3 - \frac{H}{B}x_2 - \frac{C}{B}\sin(x_1)\right)dt + \frac{G(x_1^2)}{B}d\boldsymbol{\omega} \\ dx_3 = \left(\frac{u}{L} - \frac{K_m}{L}x_2 - \frac{J}{L}x_3\right)dt \\ y = x_1 \end{cases} \tag{76}$$

with the initial values of the system are defined as $[x_1(0), x_2(0), x_3(0)]^{\mathrm{T}} = [0,0,0]^{\mathrm{T}}$ and the reference signal is selected as $y_d = 0.1\sin(2t)$. In addition, the system parameters are selected as $B=1$, $H=1$, $C=10$, $G(x_1^2) = 0.1\sin(x_1^2)$, $L=1$, $J=0.5$, and $K_m = 0.2$.

According to Theorem 1, the MTN-based event-triggered adaptive tracking control strategy of system (76) can be designed as follows

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} S_{m_i}, i = 1,2,3$$

$$W(t) = \alpha_3 - \bar{M}_1 \tanh\left(\frac{z_3^3 \bar{M}_1}{\gamma}\right)$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i^3 S_{m_i}(z_i) - \boldsymbol{\Gamma}_i \sigma_i \hat{\boldsymbol{\theta}}_i, \sigma_i \geqslant 0, i = 1,2,3$$

$$\begin{cases} u(t) = W(t_k), \forall t \in [t_k, t_{k+1}) \\ t_{k+1} = \inf\{t \in R \| e(t)| \geqslant M_1\} \end{cases}$$

where $z_1 = [z_1]^{\mathrm{T}}$, $z_2 = [z_1, z_2]^{\mathrm{T}}$, $z_3 = [z_1, z_2, z_3]^{\mathrm{T}}$.

In simulation, the design parameters of the system controller and the triggering event condition are selected as $M_1 = 1$, $\bar{M}_1 = 5$, $k_1 = 35$, $k_2 = 10$, $k_3 = 90$, $\gamma = 1$, $\sigma_1 = 2$, $\sigma_2 = 20$, $\sigma_3 = 1$, $\boldsymbol{\Gamma}_1 = 0.2\boldsymbol{I}_5$, $\boldsymbol{\Gamma}_2 = 5\boldsymbol{I}_9$, $\boldsymbol{\Gamma}_3 = 0.2\boldsymbol{I}_{19}$. The simulation results are exhibited in Figures 5–8.

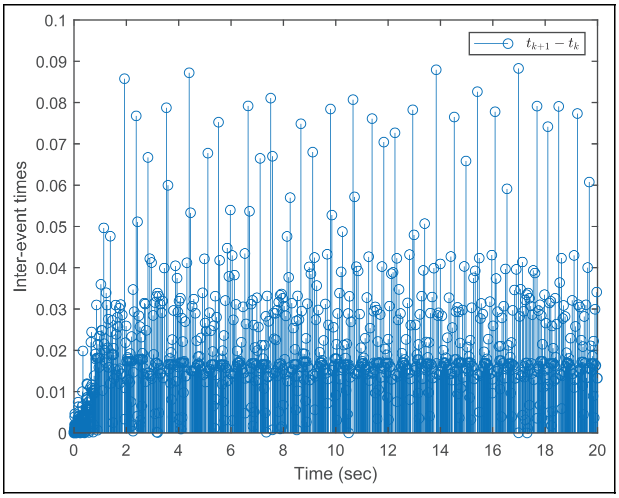

Figure 8. The time interval of triggering events of equation (76).

Figure 5 depicts the curves of the desired signal and the output signal. The responses of the tracking error $y - y_d$ and state trajectory x_2, x_3 are described in Figure 6. The inputs W and u are shown in Figure 7. All the triggering instants and triggering time intervals are shown in Figure 8.

Conclusion

In this paper, an MTN-based event-triggered adaptive tracking control scheme for stochastic nonlinear systems is proposed, which successfully saves network resource to a greater extent by alleviating computational burden and reducing communication frequency. Meanwhile, the proposed control scheme can avoid the Zeno behavior and achieve the satisfactory tracking control performance. Finally, the effectiveness of the proposed controller is demonstrated by two examples.

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