





# Adaptive prescribed performance tracking control for uncertain nonlinear systems with unknown backlash-like hysteresis

Proc IMechE Part I:  
J Systems and Control Engineering  
2024, Vol. 238(7) 1206–1218  
© IMechE 2024  
Article reuse guidelines:  
sagepub.com/journals-permissions  
DOI: 10.1177/09596518241236978  
journals.sagepub.com/home/pii  


Wei Zhao<sup>1</sup> , Shan-Liang Zhu<sup>1,2,3</sup>, Ya-Feng Zhou<sup>1</sup>, Yu-Qun Han<sup>1,2,3</sup> ,  
Wen-Wu Wang<sup>1</sup> and Qing-Hua Zhou<sup>1</sup> 

## Abstract

In this paper, an adaptive prescribed performance tracking control (PPTC) problem of uncertain nonlinear systems with unknown hysteresis input is investigated. For the purpose of implementing the PPTC, a performance function and an error conversion function are introduced. Moreover, a backlash-like hysteresis model is adopted to describe the hysteresis nonlinearity, which makes the controller design feasible. Additionally, by converting the backlash-like hysteresis model into a linear model with bounded error, the difficulties caused by hysteresis behavior on controller design are settled. Thus, integrating multi-dimensional Taylor network (MTN) approximation technique into adaptive backstepping method, an adaptive control scheme for uncertain nonlinear systems is proposed. Apart from ensuring that all signals of the closed-loop system keep bounded, the proposed control scheme not only makes the tracking error converge to an arbitrarily small neighborhood around the origin, but also guarantees the tracking error trajectory within the limits set by PPTC. Ultimately, two simulations are applied to verify the validity of the proposed control scheme.

## Keywords

Adaptive control, multi-dimensional Taylor network, backlash-like hysteresis, nonlinear systems, prescribed performance

Date received: 8 October 2023; accepted: 9 February 2024

## Introduction

As is known to all, with a large number of nonlinear systems in the actual system, the research of nonlinear systems has significant theoretical and practical application value in the engineering field, and plenty of control methods have been presented, such as backstepping method,<sup>1</sup> feedback linearization,<sup>2</sup> sliding mode control,<sup>3</sup> fault-tolerant control<sup>4</sup> and adaptive control.<sup>5,6</sup> Due to the simple structure and clear control idea of adaptive backstepping method, it has become an indispensable control tool for uncertain nonlinear systems. However, the adaptive backstepping method cannot be directly applied to control systems with unknown nonlinear functions, and this is where fuzzy logic systems (FLSs) and neural networks (NNs) have compensated for the deficiency. Therefore, a new control method is formed by combining adaptive backstepping method with the two aforementioned methods, and it has gradually been utilized to various systems, such as switched nonlinear systems,<sup>7</sup> higher-order nonlinear

systems,<sup>8</sup> and large-scale nonlinear systems.<sup>9,10</sup> In particular, multi-dimensional Taylor network (MTN), a novel kind of NNs with special structure, has been widely applied in the design of nonlinear system controller, and numerous profound and lasting achievements have been obtained.<sup>11–13</sup> Although plenty of works have focused on the research of systems under uncertain nonlinear behaviors, such as stochastic disturbances,<sup>14</sup> time delays<sup>15</sup> and unknown dead-zones,<sup>16</sup> the research of

<sup>1</sup>School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao, China

<sup>2</sup>The Research Institute for Mathematics and Interdisciplinary Sciences, Qingdao University of Science and Technology, Qingdao, China

<sup>3</sup>Qingdao Innovation Center of Artificial Intelligence Ocean Technology, Qingdao, China

### Corresponding author:

Qing-Hua Zhou, School of Mathematics and Physics, Qingdao University of Science and Technology, 99 Songling Road, Laoshan District, Qingdao 266061, China.

Email: Qinghua.ZhouQKD1909@163.com

hysteresis is inadequate. In reality, both input hysteresis and output hysteresis may cause the system to exhibit varying degrees of inaccuracy or oscillation, and even severely affect the stability of the system.

It is noteworthy that hysteresis effect exists in an extensive variety of practical physical devices or phenomenon, such as electromagnetic fields, piezoceramic actuators. Therefore, the existence of hysteresis phenomenon cannot be overlooked in controller design and is a research direction of considerable practical significance. Given this, plenty of mathematical models have been proposed to describe hysteresis nonlinearity, such as Duhem model,<sup>17</sup> Krasnoselskii-Pokrovskii (KP) model<sup>18</sup> and Bouc-Wen model.<sup>19,20</sup> It is worth noting that backlash-like hysteresis model<sup>21,22</sup> as a special case of Duhem model transforms hysteresis nonlinearity into linear combination terms which makes the controller design feasible. What's more, this model is a continuous-time dynamic model with simple structure, which has been successfully utilized in the design of nonlinear system adaptive controllers. For example, authors in Bi et al.<sup>23</sup> Wang et al.,<sup>24</sup> considered the nonlinear systems with stochastic disturbance and unknown hysteresis, and backlash-like hysteresis model are applied to approach complex hysteresis behavior to design an adaptive controller. However, the above works overlook the requirements of the transient performance of the controlled systems.

In recent years, as control system structures become increasingly complex, there is a growing demand for controllers that offer higher levels of transient performance and precision. As a result, the study on transient performance of nonlinear systems has aroused increasing attention. Under such an inevitable trend, the prescribed performance tracking control (PPTC) has been gradually applied to many nonlinear systems, such as multi-input multi-output (MIMO) uncertain nonlinear systems,<sup>25</sup> nonlinear multi-agent systems<sup>26</sup> and switched nonlinear systems.<sup>27,28</sup> The above research indicates that PPTC can clearly set the transient state and steady-state processes of the system response and achieve the expected tracking accuracy and convergence rate. From this point of view, the prescribed performance control is considerably helpful to solve the problem of high-quality control for nonlinear systems. Authors in Li et al.<sup>29</sup> considered the influence of full-state constraints on the tracking performance of the system, but the impact of hysteresis input and PPTC on the tracking effect is ignored. And for nonlinear hysteretic systems, most studies focus on satisfying the steady-state performance, but pay less attention to the transient performance. Authors in Kong et al.,<sup>22</sup> Sun et al.<sup>30</sup> Liu et al.<sup>31,32</sup> considered the effects of backlash-like hysteresis, but the enhancement of controller performance by PPTC is overlooked. Therefore, the PPTC for nonlinear systems with input hysteresis has a large space for development and is a meaningful topic.

On the basis of the aforementioned research results, this paper lays more emphasis on the PPTC of

uncertain nonlinear systems with backlash-like hysteresis input. Firstly, a prescribed performance function is applied to limit the tracking error of the controlled system within the defined area. Meanwhile, by using the backlash-like hysteresis model, the hysteresis is described successfully. Moreover, by transforming the model into a combination of a linear function and a bounded term, the controller design for the nonlinear systems with hysteresis is more feasible. Additionally, by integrating MTN approximation technology into the adaptive design process, the nonlinear structures are approximated ingeniously which reduces computational complexity. In the light of the foregoing, an adaptive PPTC strategy based on MTN is designed successfully. Compared with the existing results, this paper presents several key contributions as follows:

- (1) In this paper, the MTN-based tracking control issue of uncertain nonlinear systems with hysteresis input is studied for the first time. Compared with NN-based approximation technique used in Si et al.,<sup>33</sup> MTN-based approximation technique is applied to estimate unknown nonlinear items, which enables the controller to have lower computational complexity and better tracking performance.
- (2) Although, the combination of MTN approximation method and adaptive backstepping technique has produced many excellent results,<sup>12,29,34</sup> the problem of uncertain nonlinear systems instability caused by input hysteresis still unresolved. In this paper, within the unified framework of PPTC, the backlash-like hysteresis model is introduced to tackle the difficulties of controller design caused by the hysteresis nonlinearity.
- (3) For nonlinear hysteretic systems, most studies focus on satisfying the steady-state performance, but pay less attention to the transient performance. On this basis, for uncertain nonlinear systems with backlash-like hysteresis, a PPTC method is provided to ensure that the tracking error always meets the prescribed performance boundary which indicates that the proposed control scheme is more significant in practical applications.

The remaining sections of this paper are organized as follows. Section 2 presents the problem statement and necessary background knowledge. Section 3 showcases the controller design process. Stability analysis of the system is displayed in Section 4. Section 5 provides two simulation results. Finally, Section 6 concludes the paper.

## Problem statement and preliminary knowledge

### Problem statement

The following uncertain nonlinear systems with unknown backlash-like hysteresis input is considered in this paper:

$$\begin{cases} \dot{\chi}_i = \chi_{i+1} + f_i(\bar{\chi}_i) + \tau_i(t) \\ i = 1, 2, \dots, n-1 \\ \dot{\chi}_n = \aleph(v) + f_n(\bar{\chi}_n) + \tau_n(t) \\ y = \chi_1 \end{cases} \quad (1)$$

where  $\bar{\chi}_n = [\chi_1, \dots, \chi_n]^T \in R^n$  represents the state vector with  $\bar{\chi}_i = [\chi_1, \dots, \chi_i]^T \in R^i$ , for  $i = 1, \dots, n$ .  $y \in R$  represents the output of the controlled system.  $f_i: R^i \rightarrow R$  denotes unknown function with  $f_i(0) = 0$ .  $\tau_i(t)$  ( $i = 1, \dots, n$ ) is defined as the unknown but bounded external time-varying disturbance.  $\aleph$  and  $v$  are the output and the input of the backlash-like hysteresis, respectively.

In order to address the influence of hysteresis on system stability, the following hysteresis model is introduced to characterize the nonlinear hysteresis nonlinearity:

$$\frac{d\aleph}{dt} = \hbar \left| \frac{dv}{dt} \right| (\iota v - \aleph) + \Lambda \frac{dv}{dt} \quad (2)$$

where  $v$  is the hysteresis input.  $\hbar$ ,  $\iota$ ,  $\Lambda$  are unknown constants, satisfying  $\iota > \Lambda$ .

According to the analysis in Su et al.,<sup>35</sup> Liu et al.,<sup>36</sup> the model (2) can be solved explicitly as follows

$$\aleph(t) = \iota v(t) + \varkappa(v) \quad (3)$$

with

$$\varkappa(v) = [\aleph_0 - \iota v_0] e^{\hbar(v-v_0) \operatorname{sgn} \dot{v}} + e^{-\hbar v \operatorname{sgn} \dot{v}} \int_{v_0}^v [\Lambda - \iota] e^{\hbar \xi \operatorname{sgn} \dot{v}} d\xi \quad (4)$$

where  $\aleph_0$  and  $v_0$  are the initial value of  $\aleph$  and  $v$ , respectively. Meanwhile,  $\varkappa(v)$  is bounded, satisfying

$$|\varkappa(v)| \leq \varkappa^* \quad (5)$$

where  $\varkappa^* > 0$  is the upper bound of  $\varkappa(v)$ .

**Remark 1:** Model (2) is a discontinuous function, so there may be singularities. In an effort to solve the ‘‘singularity’’ problem, the model (2) is transformed by (3) into a continuous linear function with the combination of continuous input signal and bounded term, which makes the design of adaptive MTN controller feasible.

**Remark 2:** Figure 1 shows the image of hysteresis model (2) when  $k$  takes different values, where  $\hbar = 1$ ,  $\iota = 3.1635$ ,  $\Lambda = 0.345$ . Hysteresis input is  $v(t) = k \sin(2.5t)$ , where  $k = 6.5$ ,  $k = 4.5$ ,  $k = 2.5$ , respectively.

The control objectives of this paper are to design an adaptive PPTC controller for system (1) such that

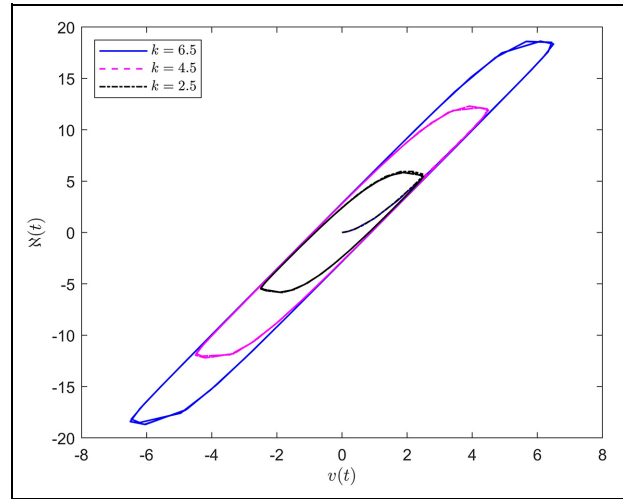


Figure 1. Hysteresis curves.

- (i) the system output tracks the designed reference signal;
- (ii) the tracking error converges to an arbitrarily small neighborhood of the origin under the PPTC;
- (iii) all signals in the closed-loop system remain bounded.

In order to facilitate the design of the controller, the following Assumption 1 is given.

**Assumption 1:** The desired reference signal  $y_r$  and its  $i$ -th time derivative  $y_r^{(i)}$  are continuous and bounded,  $i = 1, \dots, n$ .

### Multi-dimensional Taylor network

In this paper, MTN is used to approximate the unknown linear function. As stated in Han and Yan,<sup>34</sup> its mathematical expression can be defined as

$$f_{MTN}(s) = \theta^T P_m(s) \quad (6)$$

where  $s = [s_1, s_2, \dots, s_n]^T \in R^n$  and  $\theta = [\theta_1, \theta_2, \dots, \theta_l]^T \in R^l$  are the input vector and the weight vector of MTN, respectively.  $P_m(s) = [s_1, \dots, s_n, s_1^2, s_1 s_2, \dots, s_n^2, \dots, s_1^m, s_1^{m-1} s_2, \dots, s_n^m]^T \in R^l$  represents the middle intermediate input layer of MTN.

**Remark 3:** As a network structure similar to radial basis function neural network (RBFNN) in the existing studies,<sup>7,21,25</sup> MTN is composed of three layers: input layer, middle layer and output layer. Diverging from RBFNN in its intermediate layer, MTN employs a suite of polynomials as a substitute for the Gaussian function set which effectively simplifies the structure of RBFNN, thus reducing the computational complexity.

In this paper, MTN is used to approximate the unknown function in the design process of controller. The Lemma related to MTN used in the controller design is given below.

**Lemma 1**<sup>29</sup>: Supposing  $f(s) : R^n \rightarrow R$  is a continuous and unknown function defined on the compact set  $\Omega_s$ , for  $\forall I^* > 0$ , there exists a MTN used to estimate  $f(s)$  such that

$$f(s) = \theta^T P_m(s) + I(s) \tag{7}$$

where  $I(s)$  denotes the error generated in the estimation with  $|I(s)| \leq I^*$ .

**Preliminary knowledge**

The following relevant Lemmas are presented, which will play a very important role in the control scheme design process.

**Lemma 2**<sup>37</sup>: Let  $V(t) \geq 0, \forall t \in R^+$  be a continuous function and  $V(0)$  is bounded. If the following inequality holds:

$$\dot{V}(t) \leq -\mathfrak{S}_0 V(t) + \gamma_0 \tag{8}$$

where  $\mathfrak{S}_0, \gamma_0$  are constants, then it can be concluded that  $V$  is bounded.

**Lemma 3**<sup>38</sup>(Young's inequality): For  $\forall(\bar{m}, \bar{n}) \in R^2$  and  $\forall \Delta > 0$ , the following inequality holds

$$\bar{m}\bar{n} \leq \frac{\Delta^\Phi}{\Phi} |\bar{m}|^\Phi + \frac{1}{\Theta \Delta^\Theta} |\bar{n}|^\Theta \tag{9}$$

where  $\Phi > 1$  and  $\Theta > 1$  with  $(\Phi - 1)(\Theta - 1) = 1$ .

**Prescribed performance tracking control**

In order to realize the PPTC, a performance function is introduced firstly, whose concept is given below. Then, a specific performance function is proposed for the controller designed in this paper. The performance function is defined as follows

**Definition 1**<sup>39</sup>: Supposing  $\omega(t)(t > 0)$  is a smooth and bounded function defined  $R^+ \rightarrow R^+$ , if  $\omega(t)$  satisfies the following conditions:

- (1)  $\omega(t)$  is a strictly decreasing function and always greater than 0;
- (2)  $\lim_{t \rightarrow \infty} \omega(t) = \omega(\infty)$ , and  $\omega(\infty) > 0$  is a constant. then we can call  $\omega(t)$  the performance function.

According to Definition 1, the prescribed performance function can be designed as follows

$$\omega(t) = \omega_1 e^{-\omega_3 t} + \omega_2 \tag{10}$$

where  $\omega_1, \omega_2 > 0$  and  $\omega_3 > 0$  are constants.

Supposing  $\omega(t)$  is a performance function that satisfies Definition 1, and the value of initial error  $e_1(0)$  is known. Now, we put the following constraints on  $e_1(t)$

$$-\delta \omega(t) \leq e_1(t) \leq \omega(t), e_1(0) > 0 \tag{11}$$

$$-\omega(t) \leq e_1(t) \leq \delta \omega(t), e_1(0) < 0 \tag{12}$$

where  $\delta \in (0, 1)$ .

Then, (11) and (12) can be converted into the equivalent form as follows

$$e_1 = \omega(t)R(\zeta_1) \tag{13}$$

where  $\zeta_1$  is the equivalent tracking error of  $e_1$ , and  $R(\zeta_1)$  is a function described as

$$R(\zeta_1) = \frac{1}{\pi} \arctan \zeta_1 \tag{14}$$

Apparently,  $R(\zeta_1)$  is an increasing function with the following properties:

(i) if  $e_1(0) > 0$ , one has

$$\lim_{\zeta_1 \rightarrow -\infty} R(\zeta_1) = -\delta, \lim_{\zeta_1 \rightarrow +\infty} R(\zeta_1) = \delta < 1 \tag{15}$$

(ii) if  $e_1(0) < 0$ , one has

$$\lim_{\zeta_1 \rightarrow -\infty} R(\zeta_1) = -\delta > -1, \lim_{\zeta_1 \rightarrow +\infty} R(\zeta_1) = \delta \tag{16}$$

Therefore, if  $e_1(0) > 0$ , one has

$$-\delta \leq R(\zeta_1) \leq 1 \tag{17}$$

And if  $e_1(0) < 0$ , one has

$$-1 \leq R(\zeta_1) \leq \delta \tag{18}$$

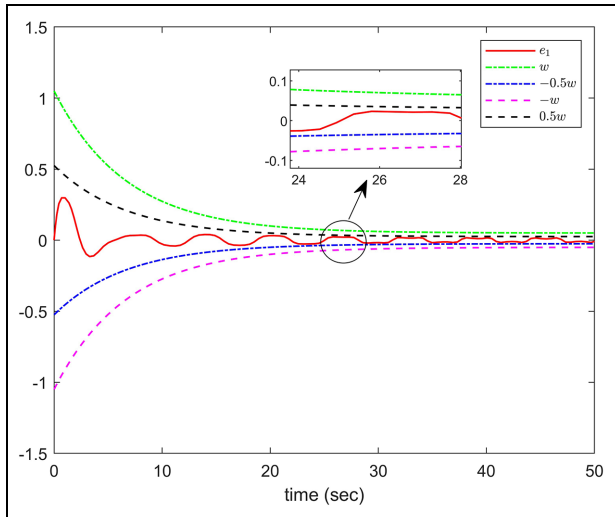
where  $\delta = 0.5$ .

Thus, when  $\zeta_1$  is bounded, (19) and (20) are established, that is, if  $e_1(0) > 0$ , one has

$$-\delta \omega(t) < \omega(t)R(\zeta_1) < \omega(t) \tag{19}$$

And if  $e_1(0) < 0$ , one has

$$-\omega(t) < \omega(t)R(\zeta_1) < \delta \omega(t) \tag{20}$$



**Figure 2.** The responses of performance function.

Then, combining (13) and (14),  $\zeta_1$  can be rewritten as

$$\zeta_1(t) = R^{-1}\left(\frac{e_1}{\omega}\right) = \tan\left(\frac{\pi e_1}{\omega}\right) \quad (21)$$

Now, by taking the derivative of (21), we can get

$$\dot{\zeta}_1(t) = \bar{Y}(t) \frac{\dot{e}_1 \omega - \dot{\omega} e_1}{\omega} = \bar{Y}(t) \left( \dot{e}_1 - \frac{\dot{\omega} e_1}{\omega} \right) \quad (22)$$

$$\text{where } \bar{Y}(t) = \frac{1}{\omega} \frac{\partial R^{-1}\left(\frac{e_1}{\omega}\right)}{\partial \left(\frac{e_1}{\omega}\right)} = \frac{\pi}{\omega \cos^2\left(\frac{\pi e_1}{\omega}\right)}.$$

**Remark 4:** It should be noticed that from (13) to (22), the system tracking error  $e_1$  is transformed into a new equivalent error  $\zeta_1$ . If  $\zeta_1$  is bounded,  $e_1$  will always stay within the range  $e_1 \in (-\delta\omega(t), \omega(t))$  or  $e_1 \in (-\omega(t), \delta\omega(t))$ . That is to say, we can choose the appropriate value of  $\omega(0)$  and  $\dot{\omega}(0)$  such that  $e_1(0)$  satisfies the following inequality:  $|e_1(0)| < \max\{\delta\omega(0), \omega(0)\}$ .

**Remark 5:** Figure 2 displays the responses of prescribed performance function control under  $\delta = 0.5$ . It can be clearly seen that PPTC method keeps the tracking error stay within the performance constraint region.

### MTN-based controller design under the PPTC

The design process of adaptive MTN controller via backstepping method for uncertain nonlinear systems with input backlash-like hysteresis will be given in this part.

Firstly, the controller design in this paper is based on the following coordinate transformation:

$$\begin{cases} e_1 = \chi_1 - y_r \\ z_i = \chi_i - \alpha_{i-1} \end{cases} \quad (23)$$

where  $i = 2, \dots, n$ .  $\alpha_{i-1}$  is the virtual control signal which will be defined in the following text.

The detailed process of controller design is given below.

*Step 1:* Considering the first Lyapunov function  $V_1$  as

$$V_1 = \frac{\zeta_1^2}{2} + \frac{\tilde{\theta}_1^T \tilde{\theta}_1}{2r_1} \quad (24)$$

where  $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ ,  $r_1$  is a design parameter.

Thus, combining (23) and (24), the time differentiation of  $V_1$  can be expressed as

$$\dot{V}_1 = \zeta_1 \dot{\zeta}_1 - \frac{\tilde{\theta}_1^T \dot{\tilde{\theta}}_1}{r_1} = \zeta_1 \bar{Y}(t) \left( \dot{e}_1 - \frac{\dot{\omega} e_1}{\omega} \right) - \frac{\tilde{\theta}_1^T \dot{\tilde{\theta}}_1}{r_1} \quad (25)$$

Then, the time differentiation of the system tracking error  $e_1$  is formulized into

$$\dot{e}_1 = \dot{\chi}_1 - \dot{y}_r = \chi_2 + f_1 + \tau_1(t) - \dot{y}_r \quad (26)$$

Substituting (26) into (25), a new form of the time derivative of  $V_1$  can be expressed as

$$\begin{aligned} \dot{V}_1 = & \zeta_1 \bar{Y}(t) \left( z_2 + \alpha_1 + f_1 + \tau_1(t) - \dot{y}_r - \frac{\dot{\omega} e_1}{\omega} \right) \\ & - \frac{\tilde{\theta}_1^T \dot{\tilde{\theta}}_1}{r_1} \end{aligned} \quad (27)$$

where  $a_1 > 0$  is a constant.

By applying Lemma 3, the following inequalities are obtained as

$$\zeta_1 \bar{Y}(t) \tau_1(t) \leq \frac{1}{2c_1^2} \zeta_1^2 \bar{Y}^2(t) \tau_1^2(t) + \frac{1}{2} c_1^2 \quad (28)$$

$$\zeta_1 \bar{Y}(t) z_2 \leq \frac{1}{2} \zeta_1^2 \bar{Y}^2(t) + \frac{1}{2} z_2^2 \quad (29)$$

where  $c_1 > 0$  is the constant.

In order to ensure the validity of the proposed control scheme and based on the adaptive backstepping design, the virtual control signal  $\alpha_1$  is designed as

$$\alpha_1 = -\frac{k_1}{\bar{Y}(t)} \zeta_1 - \frac{\dot{\theta}_1^T}{\bar{Y}(t)} P_{m_1} + \frac{\dot{\omega}}{\omega} z_1 \quad (30)$$

where  $k_1 > 0$  is a design parameter.

In the light of the first virtual control signal  $\alpha_1$  and analyzing (27)–(29), the time derivative of  $V_1$  can be rewritten as

$$\begin{aligned} \dot{V}_1 \leq & -k_1 \zeta_1^2 - \frac{\tilde{\theta}_1^T}{r_1} \zeta_1 P_{m_1} - \frac{\tilde{\theta}_1^T \dot{\tilde{\theta}}_1}{r_1} \\ & + \zeta_1 F_1 + \frac{1}{2} c_1^2 + \frac{1}{2} z_2^2 \end{aligned} \quad (31)$$

where  $F_1 = \frac{1}{2} \zeta_1 \bar{Y}^2(t) + \frac{1}{2c_1^2} \zeta_1^2 \bar{Y}^2(t) \tau_1^2(t) + \bar{Y}(t) f_1 - \bar{Y}(t) \dot{y}_r$  is a combination of nonlinear functions.

Since  $F_1$  cannot be directly used for controller design. Therefore, a MTN can be employed to approximate  $F_1$  by Lemma 1, that is to say, for  $\forall I_1^* > 0$ , the following formula can be established

$$F_1 = \boldsymbol{\theta}_1^T P_{m_1}(\mathbf{z}_1) + I_1(\mathbf{z}_1), |I_1(\mathbf{z}_1)| \leq I_1^* \quad (32)$$

where  $\mathbf{z}_1 = [z_1]^T$ ,  $I_1(\mathbf{z}_1)$  is the approximation error.

Thus, based on (32), one has

$$\zeta_1 F_1 = \zeta_1 \boldsymbol{\theta}_1^T P_{m_1} + \zeta_1 I_1^* \quad (33)$$

Then, according to Lemma 3, one has

$$\zeta_1 I_1^* \leq \frac{1}{2} \zeta_1^2 + \frac{1}{2} I_1^{*2} \quad (34)$$

Thus, combining (33) and (34) to obtain the following inequality

$$\zeta_1 F_1 \leq \zeta_1 \boldsymbol{\theta}_1^T P_{m_1} + \frac{1}{2} \zeta_1^2 + \frac{1}{2} I_1^{*2} \quad (35)$$

Making every effort to ensure the validity of the proposed control scheme, the adaptive law  $\dot{\hat{\boldsymbol{\theta}}}_1$  can be defined as

$$\dot{\hat{\boldsymbol{\theta}}}_1 = r_1 \zeta_1 P_{m_1} - \sigma_1 \hat{\boldsymbol{\theta}}_1 \quad (36)$$

where  $\sigma_1 > 0$  is a design parameter.

Now, substituting (35)–(36) to (31), the time differentiation of  $V_1$  is

$$\begin{aligned} \dot{V}_1 \leq & -k_1 \zeta_1^2 + \frac{1}{2} \zeta_1^2 + \frac{1}{2} z_2^2 \\ & + \frac{1}{2} I_1^{*2} + \frac{1}{2} c_1^2 + \frac{\sigma_1 \hat{\boldsymbol{\theta}}_1^T \hat{\boldsymbol{\theta}}_1}{r_1} \end{aligned} \quad (37)$$

In addition, since  $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$ , the following inequality holds

$$\frac{\sigma_1 \tilde{\boldsymbol{\theta}}_1^T \hat{\boldsymbol{\theta}}_1}{r_1} \leq -\frac{\sigma_1 \tilde{\boldsymbol{\theta}}_1^T \tilde{\boldsymbol{\theta}}_1}{2r_1} + \frac{\sigma_1 \|\boldsymbol{\theta}_1\|^2}{2r_1} \quad (38)$$

Thus, combining (37) and (38), the time differentiation of  $V_1$  can be rewritten in the following form

$$\begin{aligned} \dot{V}_1 \leq & -\left(k_1 - \frac{1}{2}\right) \zeta_1^2 + \frac{1}{2} z_2^2 - \frac{\sigma_1}{2r_1} \tilde{\boldsymbol{\theta}}_1^T \tilde{\boldsymbol{\theta}}_1 \\ & + \frac{1}{2} I_1^{*2} + \frac{1}{2} c_1^2 + \frac{\sigma_1}{2r_1} \|\boldsymbol{\theta}_1\|^2 \end{aligned} \quad (39)$$

Step  $i(i = 2, \dots, n - 1)$ : On the basis of step 1, the Lyapunov function  $V_i$  is designed as follows

$$V_i = \frac{z_i^2}{2} + \frac{\tilde{\boldsymbol{\theta}}_i^T \tilde{\boldsymbol{\theta}}_i}{2r_i} \quad (40)$$

where  $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ ,  $r_i$  is a design parameter.

Similar to the procedure as step 1, it follows

$$\dot{V}_i = z_i \dot{z}_i - \frac{1}{r_i} \tilde{\boldsymbol{\theta}}_i^T \dot{\tilde{\boldsymbol{\theta}}}_i \quad (41)$$

Combining  $z_i = \chi_i - \alpha_{i-1}$ , one has

$$\dot{z}_i = z_{i+1} + \alpha_i + f_i + \tau_i(t) - \dot{\alpha}_{i-1} \quad (42)$$

Then, a new form of the time differentiation of  $V_i$  can be expressed as

$$\dot{V}_i = z_i [z_{i+1} + \alpha_i + f_i + \tau_i(t) - \dot{\alpha}_{i-1}] - \frac{\tilde{\boldsymbol{\theta}}_i^T \dot{\tilde{\boldsymbol{\theta}}}_i}{r_i} \quad (43)$$

where  $\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \chi_j} (\chi_{j+1} + f_j + \tau_j) + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta_j} \dot{\theta}_j$ .

According to Lemma 3, inequalities (44) and (45) can be obtained

$$z_i z_{i+1} \leq \frac{1}{2} z_i^2 + \frac{1}{2} z_{i+1}^2 \quad (44)$$

$$z_i \tau_i(t) \leq \frac{1}{2c_i^2} z_i^2 \tau_i^2(t) + \frac{1}{2} c_i^2 \quad (45)$$

where  $c_i > 0$  is the constant.

In order to ensure the validity of the proposed control scheme and based on the adaptive backstepping design, the virtual control signal  $\alpha_i$  is designed as

$$\alpha_i = -k_i z_i - \hat{\boldsymbol{\theta}}_i^T P_{m_i} \quad (46)$$

where  $k_i$  is a positive design parameter.

In the view of the virtual control signal  $\alpha_i$  and analyzing (43)–(45), the time differentiation of  $V_i$  can be rewritten as

$$\begin{aligned} \dot{V}_i \leq & -k_i z_i^2 + \frac{1}{2} z_i^2 - \hat{\boldsymbol{\theta}}_i^T z_i P_{m_i} \\ & + z_i F_i + \frac{1}{2} c_i^2 + \frac{1}{2} z_{i+1}^2 - \frac{\tilde{\boldsymbol{\theta}}_i^T \dot{\tilde{\boldsymbol{\theta}}}_i}{r_i} \end{aligned} \quad (47)$$

where  $F_i = f_i - \dot{\alpha}_{i-1} + \frac{1}{2c_i^2} z_i \tau_i^2(t)$  is a combination of nonlinear functions.

Since  $F_i$  cannot be directly used for controller design. Therefore, a MTN can be employed to approximate  $F_i$  by Lemma 1, that is to say, for  $\forall I_i^* > 0$ , the following formula can be established

$$F_i = \boldsymbol{\theta}_i^T P_{m_i}(\mathbf{z}_i) + I_i(\mathbf{z}_i), |I_i(\mathbf{z}_i)| \leq I_i^* \quad (48)$$

where  $\mathbf{z}_i = [z_1, z_2, \dots, z_i]^T$ ,  $I_i(\mathbf{z}_i)$  is the approximation error.

Thus, based on (48), one has

$$z_i F_i = z_i \boldsymbol{\theta}_i^T P_{m_i} + z_i I_i^* \quad (49)$$

Then, according to Lemma 3, one has

$$z_i I_i^* \leq \frac{1}{2} z_i^2 + \frac{1}{2} I_i^{*2} \quad (50)$$

Thus, combining (49) and (50) to obtain the following inequality

$$z_i F_i \leq z_i \boldsymbol{\theta}_i^T P_{m_i} + \frac{1}{2} z_i^2 + \frac{1}{2} I_i^{*2} \quad (51)$$

In an effort to ensure the effectiveness of the proposed control scheme, the adaptive law can be defined as

$$\dot{\hat{\boldsymbol{\theta}}}_i = r_i z_i P_{m_i} - \sigma_i \hat{\boldsymbol{\theta}}_i \quad (52)$$

where  $\sigma_i > 0$  is a design parameter.

Now, substituting (51) and (52) into (47), the time differentiation of  $V_i$  is

$$\begin{aligned} \dot{V}_i \leq & -k_i z_i^2 + \frac{1}{2} z_i^2 + \frac{1}{2} z_{i+1}^2 + \frac{1}{2} I_i^{*2} \\ & + \frac{1}{2} c_i^2 + \frac{\sigma_i \tilde{\boldsymbol{\theta}}_i^T \hat{\boldsymbol{\theta}}_i}{r_i} \end{aligned} \quad (53)$$

In addition, since  $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ , the following inequality holds

$$\frac{\sigma_i \tilde{\boldsymbol{\theta}}_i^T \hat{\boldsymbol{\theta}}_i}{r_i} \leq -\frac{\sigma_i \tilde{\boldsymbol{\theta}}_i^T \tilde{\boldsymbol{\theta}}_i}{2r_i} + \frac{\sigma_i \|\boldsymbol{\theta}_i\|^2}{2r_i} \quad (54)$$

Thus, combining (53) and (54), the time differentiation of  $V_i$  can be rewritten in the following form

$$\begin{aligned} \dot{V}_i \leq & -\left(k_i - \frac{1}{2}\right) z_i^2 + \frac{1}{2} z_{i+1}^2 + \frac{1}{2} I_i^{*2} + \frac{1}{2} c_i^2 \\ & - \frac{\sigma_i \tilde{\boldsymbol{\theta}}_i^T \tilde{\boldsymbol{\theta}}_i}{2r_i} + \frac{\sigma_i \|\boldsymbol{\theta}_i\|^2}{2r_i} \end{aligned} \quad (55)$$

*Step n:* Considering the following Lyapunov function  $V_n$  as

$$V_n = \frac{z_n^2}{2} + \frac{\tilde{\boldsymbol{\theta}}_n^T \tilde{\boldsymbol{\theta}}_n}{2r_n} \quad (56)$$

where  $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n - \hat{\boldsymbol{\theta}}_n$ ,  $r_n$  is a design parameter.

Similar to the procedure as step 1 and step  $i$ , it follows

$$\dot{V}_n = z_n \dot{z}_n - \frac{\tilde{\boldsymbol{\theta}}_n^T \dot{\tilde{\boldsymbol{\theta}}}_n}{r_n} \quad (57)$$

Combining  $z_n = \chi_n - \alpha_{n-1}$ , one has

$$\dot{z}_n = \dot{\chi}_n - \dot{\alpha}_{n-1} = \aleph(v) + f_n(\bar{\boldsymbol{\chi}}_n) + \tau_n(t) - \dot{\alpha}_{n-1} \quad (58)$$

Then, a new form of the time differentiation of  $V_n$  can be expressed as

$$\dot{V}_n = z_n [\aleph(v) + f_n + \tau_n(t) - \dot{\alpha}_{n-1}] - \frac{\tilde{\boldsymbol{\theta}}_n^T \dot{\tilde{\boldsymbol{\theta}}}_n}{r_n} \quad (59)$$

where  $\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \chi_j} (\chi_{j+1} + f_j + \tau_j) + \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_j} \dot{\theta}_j$ .

According to Lemma 3, the following inequality can be obtained

$$z_n \tau_n(t) \leq \frac{1}{2c_n^2} z_n^2 \tau_n^2(t) + \frac{1}{2} c_n^2 \quad (60)$$

where  $c_n > 0$  is the constant.

Based on the adaptive backstepping design, the real control signal  $v$  is designed as

$$v = \frac{1}{\iota} \left( -k_n z_n - \hat{\boldsymbol{\theta}}_n^T P_{m_n} \right) \quad (61)$$

where  $k_n$  is a positive design parameter.

Thus, combining (3), (59)–(61) to have

$$\dot{V}_n \leq \iota z_n v(t) + z_n \aleph(v) + \frac{1}{2} c_n^2 + z_n F_n - \frac{\tilde{\boldsymbol{\theta}}_n^T \dot{\tilde{\boldsymbol{\theta}}}_n}{r_n} \quad (62)$$

where  $F_n = f_n - \dot{\alpha}_{n-1} + \frac{1}{2c_n^2} z_n \tau_n^2(t)$  is a combination of nonlinear functions.

Since  $F_n$  cannot be used directly for controller design. Therefore, a MTN can be employed to approximate  $F_n$  by Lemma 1, that is to say, for  $\forall I_n^* > 0$ , the following formula can be established

$$F_n = \boldsymbol{\theta}_n^T P_{m_n}(\mathbf{z}_n) + I_n(\mathbf{z}_n), |I_n(\mathbf{z}_n)| \leq I_n^* \quad (63)$$

where  $\mathbf{z}_n = [z_1, z_2, \dots, z_n]^T$ ,  $I_n(\mathbf{z}_n)$  is the approximation error.

Thus, based on (63), one has

$$z_n F_n \leq z_n \boldsymbol{\theta}_n^T P_{m_n} + z_n I_n^* \quad (64)$$

Then, according to Lemma 3 along with (5), one has

$$z_n I_n^* \leq \frac{1}{2} z_n^2 + \frac{1}{2} I_n^{*2} \tag{65}$$

$$z_n \varkappa(v) \leq \frac{1}{2} z_n^2 + \frac{1}{2} \varkappa^{*2} \tag{66}$$

Thus, substituting (65) to (64) gives

$$z_n F_n \leq z_n \theta_n^T P_{m_n} + \frac{1}{2} z_n^2 + \frac{1}{2} I_n^{*2} \tag{67}$$

Therefore, combining (66) and (67), (62) can be rewritten as

$$\begin{aligned} \dot{V}_n &\leq \nu z_n v(t) + z_n^2 + z_n \theta_n^T P_{m_n} - \frac{\tilde{\theta}_n^T \dot{\hat{\theta}}_n}{r_n} \\ &\quad + \frac{1}{2} c_n^2 + \frac{1}{2} \varkappa^{*2} + \frac{1}{2} I_n^{*2} \end{aligned} \tag{68}$$

Making every effort to ensure the validity of the proposed control scheme, the adaptive law  $\hat{\theta}_n$  can be defined as

$$\dot{\hat{\theta}}_n = r_n z_n P_{m_n} - \sigma_n \hat{\theta}_n \tag{69}$$

where  $\sigma_n$  is a positive design parameter.

Now, substituting (69) into (68), the time differentiation of  $V_n$  will change into the following form

$$\dot{V}_n \leq -(k_n - 1)z_n^2 + \frac{1}{2} \varkappa^{*2} + \frac{1}{2} I_n^{*2} + \frac{1}{2} c_n^2 + \frac{\sigma_n \tilde{\theta}_n^T \hat{\theta}_n}{r_n} \tag{70}$$

In addition, since  $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$ , the following inequality holds

$$\frac{\sigma_n \tilde{\theta}_n^T \hat{\theta}_n}{r_n} \leq -\frac{\sigma_n \tilde{\theta}_n^T \tilde{\theta}_n}{2r_n} + \frac{\sigma_n \|\theta_n\|^2}{2r_n} \tag{71}$$

Hence, it follows immediately by substituting (71) into (70) that

$$\begin{aligned} \dot{V}_n &\leq -(k_n - 1)z_n^2 - \frac{\sigma_n \tilde{\theta}_n^T \tilde{\theta}_n}{2r_n} \\ &\quad + \frac{1}{2} \varkappa^{*2} + \frac{1}{2} I_n^{*2} + \frac{1}{2} c_n^2 + \frac{\sigma_n \|\theta_n\|^2}{2r_n} \end{aligned} \tag{72}$$

### Stability analysis of the system

Up to now, the adaptive MTN-based PPTC controller design has been completed by adopting adaptive backstepping technique. The main study result will be presented by the following Theorem 1.

**Theorem 1:** Consider uncertain nonlinear system (1) with backlash-like hysteresis that satisfies Assumption

1. If the real control input (61), virtual signals (30), (46), and adaptive laws (36), (52), and (69) are selected, the designed controller guarantees that (i) all signals of the controlled system remain bounded; (ii) the system output is capable of tracking the desired reference signal; (iii) the tracking error converges to an arbitrarily small neighborhood of the origin with staying in the constraint region set by the PPTC.

**Proof:** For the stability analysis of uncertain nonlinear system (1), considering the following form of Lyapunov function  $V$ :

$$V = \sum_{i=1}^n V_i \tag{73}$$

Furthermore, the time derivative of  $V$  is

$$\begin{aligned} \dot{V} &\leq -l_1 \zeta_1^2 - \sum_{j=2}^n l_j z_j^2 - \sum_{j=1}^n \frac{\sigma_j \tilde{\theta}_j^T \tilde{\theta}_j}{2r_j} + \frac{1}{2} \varkappa^{*2} \\ &\quad + \frac{1}{2} \sum_{j=1}^n \left( I_j^{*2} + c_j^2 + \frac{\sigma_j \|\theta_j\|^2}{2r_j} \right) \end{aligned} \tag{74}$$

where  $l_1 = k_1 - \frac{1}{2} > 0$  and  $l_j = k_j - \frac{3}{2} > 0$  for  $j = 2, 3, \dots, n$ .

Let  $\mathfrak{S}_0 = \min\{2l_j, \sigma_j\}, j = 1, 2, \dots, n$  and  $\gamma_0 = \frac{1}{2} \varkappa^{*2} + \frac{1}{2} \sum_{j=1}^n \left( I_j^{*2} + c_j^2 + \frac{\sigma_j \|\theta_j\|^2}{2r_j} \right)$  such that (74) can be expressed as

$$\dot{V}(t) \leq -\mathfrak{S}_0 V(t) + \gamma_0, t \geq 0 \tag{75}$$

Multiplying  $e^{\mathfrak{S}_0 t}$  by both sides of (75) produces

$$0 \leq V(t) \leq \left( V(0) - \frac{\gamma_0}{\mathfrak{S}_0} \right) e^{-\mathfrak{S}_0 t} + \frac{\gamma_0}{\mathfrak{S}_0} \leq V(0) e^{-\mathfrak{S}_0 t} + \frac{\gamma_0}{\mathfrak{S}_0} \tag{76}$$

and

$$V(t) \leq V(0) + \frac{\gamma_0}{\mathfrak{S}_0} \tag{77}$$

where  $V(0) = \frac{1}{2} \zeta_1^2(0) + \frac{1}{2} \sum_{i=2}^n z_i^2(0) + \sum_{i=1}^n \frac{\tilde{\theta}_i^T \tilde{\theta}_i}{2r_i}(0)$ . Thus, according to (77) and the definition of  $V$ , it can be concluded that  $\zeta_1, z_i$  and  $\tilde{\theta}_i$  remain bounded. In view of  $\theta_i$  is a constant, thus  $\hat{\theta}_i$  is also bounded. Hence,  $\alpha_i$  keeps bounded. Additionally, it follows from (61) that  $v$  is bounded. In conclusion, all signals of the closed-loop system remain bounded.

Next, it can be obtained from (76) that

$$V(t) \leq \frac{\gamma_0}{\mathfrak{S}_0}, t \rightarrow \infty \tag{78}$$

Furthermore, one has

$$\zeta_1^2 + \sum_{i=2}^n z_i^2 \leq 2V(t) \leq \frac{2\gamma_0}{S_0} \tag{79}$$

Therefore,  $\zeta_1$  and  $z_i$  can eventually converge to the following compact set  $\tilde{\Omega}$

$$\tilde{\Omega} = \left\{ \zeta_1, z_i \mid \zeta_1^2 + \sum_{i=2}^n z_i^2 \leq \frac{2\gamma_0}{S_0} \right\} \tag{80}$$

It can be seen that by adopting appropriate design parameters, the tracking error can converge to the vicinity of the origin.

### Simulation results

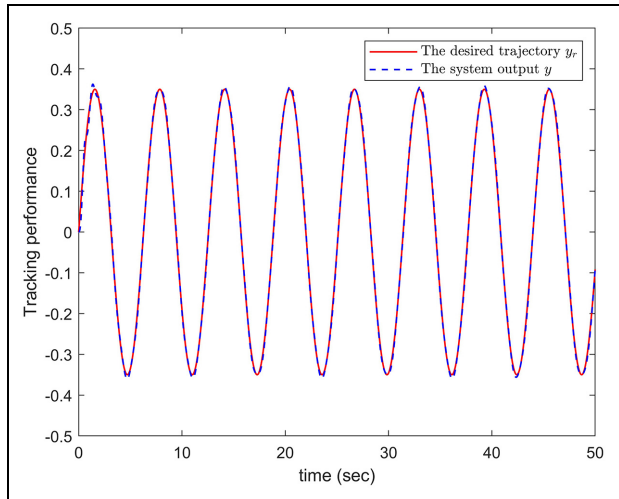
In this section, three simulation examples are employed to prove the validity of adaptive MTN-based PPTC control scheme proposed in Theorem 1.

Firstly, the required parameters during the design of the controller in Example 1 and Example 2 are shown in the following Table 1

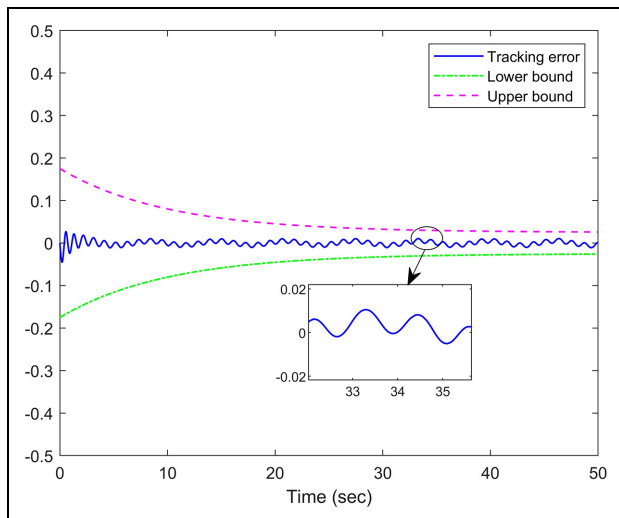
**Example 1:** Considering the following nonlinear system with unknown hysteresis input

$$\begin{cases} \dot{\chi}_1 = \chi_2 - 1.5\chi_1 + 0.7 \sin t \\ \dot{\chi}_2 = \chi_3 - \chi_1 + 0.04 \sin t \\ \dot{\chi}_3 = \aleph(v) - 50\chi_3 + 0.1 \sin t \\ y = \chi_1 \end{cases} \tag{81}$$

The simulation results are shown in Figures 3 to 6. Figure 3 shows the tracking effect, from which it can be seen that the control scheme proposed in this paper achieves excellent tracking performance. Figure 4 displays that tracking error  $z_1$  does not exceed the performance limit area under the PPTC and converges near the origin. Figure 5 illustrates the states  $\chi_1, \chi_2,$  and  $\chi_3$  of the system (81) is bounded. Figure 6 shows the backlash-like hysteresis output  $\aleph$  and input  $v$ .



**Figure 3.** The desired trajectory  $y_r$  and the system output  $y$  in Example 1.



**Figure 4.** Tracking error trajectory under the PPTC in Example 1.

**Example 2:** To further verify the feasibility of the proposed control scheme, a class of uncertain Duffing-

**Table 1.** Design parameter.

Parameter	Value in Example 1	Value in Example 2
The initial state of the controlled system	$[\chi_1(0), \chi_2(0), \chi_3(0)]^T = [0, 0, 0]^T$	$[\chi_1(0), \chi_2(0)]^T = [0, 0]^T$
Reference signals	$y_r = 0.35 \sin t$	$y_r = 0.4 \sin t$
Controller parameters	$k_1 = 50, k_2 = 10, k_3 = 25$	$k_1 = k_2 = 2$
Adaptive law parameters	$r_1 = 1, r_2 = 1.2, r_3 = 2$	$r_1 = 1, r_2 = 1.2$
The performance function	$\omega = 0.3e^{-0.1t} + 0.05$	
Backlash-like hysteresis parameters	$\hbar = 1, \iota = 3.1635, \Delta = 0.345$	

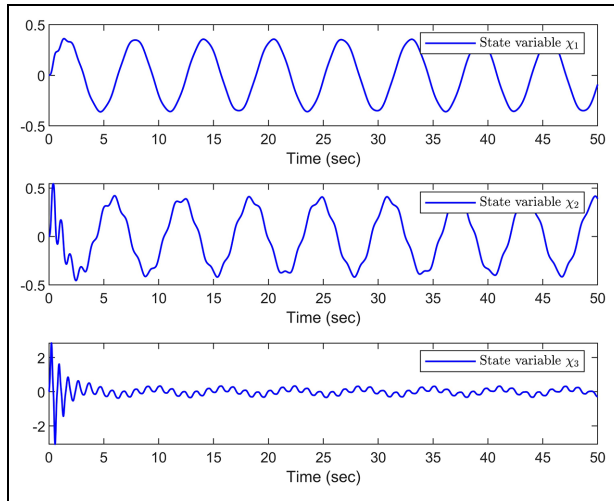


Figure 5. The system state variables in Example 1.

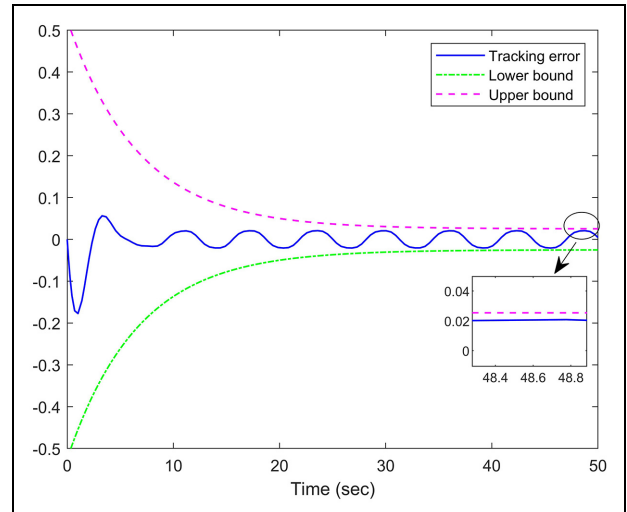


Figure 8. Tracking error trajectory under the PPTC in Example 2.

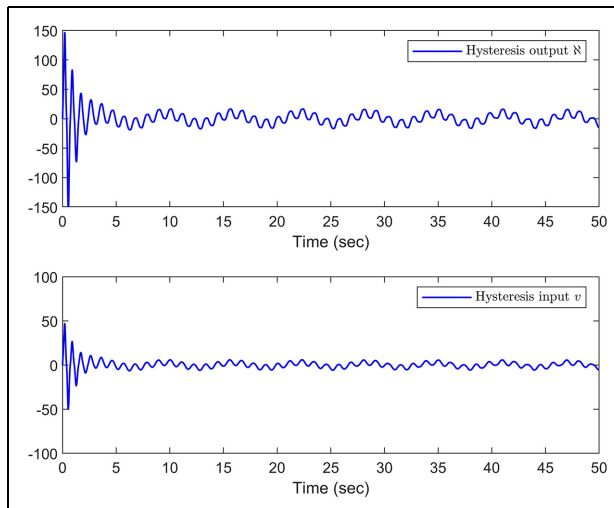


Figure 6. The backlash-like hysteresis output and input in Example 1.

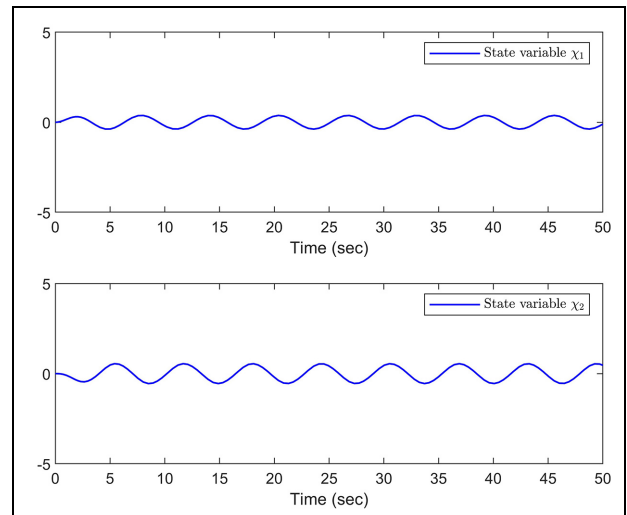


Figure 9. The system state variables in Example 2.

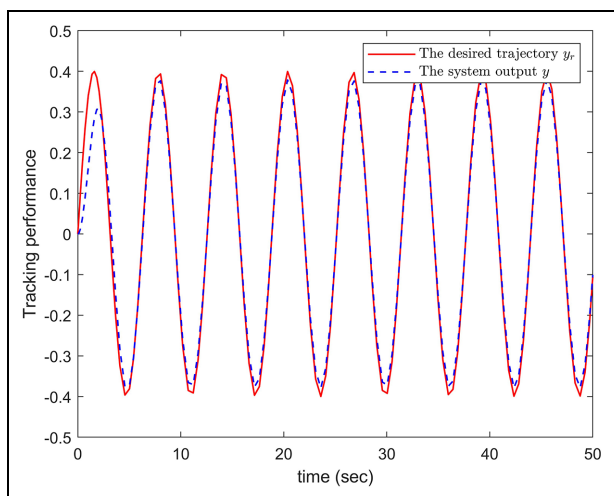
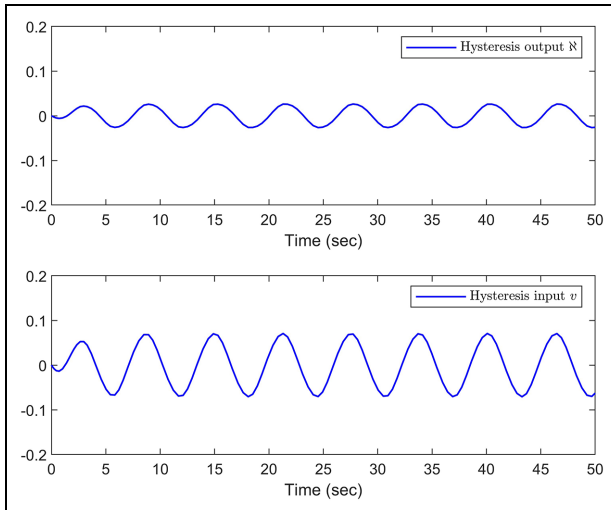


Figure 7. The desired trajectory  $y_r$  and the system output  $y$  in Example 2.

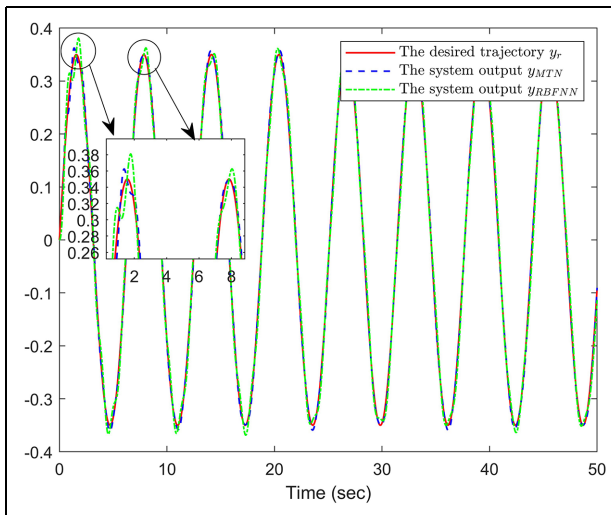
Holmes control system is considered, which is often used to describe actual systems. According to Sun,<sup>40</sup> the dynamic equation of the system is described as

$$\begin{cases} \dot{\chi}_1 = \chi_2 + 0.4 \sin t \\ \dot{\chi}_2 = \aleph - 2\chi_1 - \chi_2 - \chi_1^3 + 0.01 \sin t \\ y = \chi_1 \end{cases} \quad (82)$$

The simulation results are shown in Figures 7 to 10. Figure 7 shows the tracking effect, from which it can be seen that the control scheme proposed in this paper achieves excellent tracking performance. Figure 8 shows the trajectory of tracking error  $z_1$  under the PPTC, which is clearly to get that the trajectory of the tracking error  $z_1$  is within the performance limit area and converges near the origin. Figure 9 shows the states  $\chi_1$  and



**Figure 10.** The backlash-like hysteresis output and input in Example 2.



**Figure 11.** Tracking performance of two different control schemes.

$\chi_2$  of the system (82) is bounded. Figure 10 shows the backlash-like hysteresis output  $\aleph$  and input  $v$ .

**Example 3:** In order to highlight the superiority of the control scheme based on MTN approximation technology in this paper, a comparison test has been added by comparing the tracking performance of the MTN-based scheme with RBFNN-based control scheme in.<sup>33</sup>

As shown in Figure 11, both control schemes demonstrate good tracking performance; however, the MTN-based control scheme proposed in this paper generally exhibits a better tracking effect than the RBFNN-based control scheme in.<sup>33</sup> This further shows the superiority of the control scheme proposed in this paper.

## Conclusion

In this paper, a MTN-based adaptive PPTC controller for uncertain nonlinear systems with backlash-like hysteresis input is designed. The backlash-like hysteresis model is applied to deal with the complex hysteresis nonlinearity by converting it to a continuous linear term, which not only avoids the singularity problem but also makes the controller feasible. Meanwhile, the performance function and error transformation function are introduced to improve the tracking performance of the controlled system. Additionally, MTN is used to solve the uncertainty and input nonlinearity problems. As a result, a novel MTN-based adaptive PPTC scheme is obtained to achieve the desired tracking effect. By virtue of the simple structure of MTN, the calculation burden is greatly decreased. To conclude, the simulation results show that the scheme ensures that all signals in the closed-loop system are bounded, and the tracking error can converge to an arbitrarily small neighborhood around the origin without violating PPTC method. Future work may focus on how to implement adaptive asymptotic tracking control in finite time for constrained nonlinear systems subject to unknown hysteresis input.

## Declaration of conflicting interests


The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.


## Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

## ORCID iDs

Wei Zhao  <https://orcid.org/0009-0002-3316-1911>

Yu-Qun Han  <https://orcid.org/0000-0002-9055-2954>

Qing-Hua Zhou  <https://orcid.org/0009-0006-2610-1549>

## Data availability

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

## References

1. Li Y, Tong S, Liu L, et al. Adaptive output-feedback control design with prescribed performance for switched nonlinear systems. *Automatica* 2017; 80: 225–231.
2. Meng X, Yu H, Zhang J, et al. Disturbance observer-based feedback linearization control for a quadruple-tank liquid level system. *ISA Trans* 2022; 122: 146–162.
3. Das M and Mahanta C. Optimal second order sliding mode control for nonlinear uncertain systems. *ISA Trans* 2014; 53(4): 1191–1198.

4. Lu C, Pan Y, Liu Y, et al. Adaptive fuzzy finite-time fault-tolerant control of nonlinear systems with state constraints and input quantization. *Int J Adapt Control Signal Process* 2020; 34(9): 1199–1219.
5. He WJ, Zhu SL, Li N, et al. Adaptive finite-time control for switched nonlinear systems subject to multiple objective constraints via multi-dimensional Taylor network approach. *ISA Trans* 2023; 136: 323–333.
6. He WJ, Zhu SL, Li N, et al. Adaptive controller design for switched stochastic nonlinear systems subject to unknown dead-zone input via new type of network approach. *Int J Control Autom Syst* 2023; 21(2): 499–507.
7. Li Z, Cao G, Xie W, et al. Switched-observer-based adaptive neural networks tracking control for switched nonlinear time-delay systems with actuator saturation. *Inf Sci* 2023; 621: 36–57.
8. Kang B, Ma Z, Zhang W, et al. Fixed-time fuzzy adaptive decentralized control for high-order nonlinear large-scale systems. *Int J Control Autom Syst* 2022; 20(12): 4100–4110.
9. Li S and Xiang Z. Sampled-data decentralized output feedback control for a class of switched large-scale stochastic nonlinear systems. *IEEE Syst J* 2020; 14(2): 1602–1610.
10. Zhu SL and Han YQ. Adaptive decentralized prescribed performance control for a class of large-scale nonlinear systems subject to nonsymmetric input saturations. *Neural Comput Appl* 2022; 34(13): 11123–11140.
11. Zhu SL, Duan DY, Chu L, et al. Adaptive multi-dimensional Taylor network tracking control for a class of switched nonlinear systems with input nonlinearity. *Trans Inst Meas Contr* 2020; 42(13): 2482–2491.
12. Han YQ. Adaptive output-feedback tracking control for a class of nonlinear systems with input saturation: a multi-dimensional Taylor network-based approach. *Int J Syst Sci* 2020; 51(13): 2471–2482.
13. Yan HS and Wang GB. Adaptive tracking control for stochastic nonlinear systems with time-varying delays using multi-dimensional Taylor network. *ISA Trans* 2023; 132: 246–257.
14. Wei Y, Yu X, Feng Y, et al. Event-triggered adaptive optimal tracking control for nonlinear stochastic systems with dynamic state constraints. *ISA Trans* 2023; 139: 60–70.
15. Zhang Z, Xu B, Tan C, et al. Adaptive control of uncertain nonlinear time-delay systems with external disturbance. *IEEE Trans Syst Man Cybern Syst* 2022; 52(2): 1288–1295.
16. Li S, Ding L, Gao H, et al. Adaptive fuzzy finite-time tracking control for nonstrict full states constrained nonlinear system with coupled dead-zone input. *IEEE Trans Cybern* 2022; 52(2): 1138–1149.
17. Ikhouane F. A survey of the hysteretic Duhem model. *Arch Comput Methods Eng* 2018; 25(4): 965–1002.
18. Li Z, Shan J and Gabbert U. Inverse compensation of hysteresis using Krasnoselskii-Pokrovskii model. *IEEE/ASME Trans Mechatron* 2018; 23(2): 966–971.
19. Rakotondrabe M. Bouc–Wen modeling and inverse multiplicative structure to compensate hysteresis nonlinearity in piezoelectric actuators. *IEEE Trans Autom Sci Eng* 2011; 8(2): 428–431.
20. He WJ, Zhu SL, Li N, et al. Tracking control for switched nonlinear systems subject to output hysteresis via adaptive multi-dimensional Taylor network approach. *Int J Control* 2023; 96(7): 1724–1735.
21. Zhao Y, Zhang H, Chen Z, et al. Adaptive neural decentralised control for switched interconnected nonlinear systems with backlash-like hysteresis and output constraints. *Int J Syst Sci* 2022; 53(7): 1545–1561.
22. Kong L, Lai Q, Ouyang Y, et al. Neural learning control of a robotic manipulator with finite-time convergence in the presence of unknown backlash-like hysteresis. *IEEE Trans Syst Man Cybern Syst* 2022; 52(3): 1916–1927.
23. Bi S, Wang L, Zhao Y, et al. Operator-based robust control for nonlinear uncertain systems with unknown backlash-like hysteresis. *Int J Control Autom Syst* 2016; 14(2): 469–477.
24. Wang L, Sun W and Wu Y. Adaptive asymptotic tracking control for stochastic nonlinear systems with unknown backlash-like hysteresis. *J Syst Sci Complex* 2022; 35(5): 1824–1838.
25. Yang D, Liu W and Guo C. Command-filtered-based neuroadaptive control for multi-input multi-output saturated nonstrict-feedback nonlinear systems with prescribed tracking performance. *Int J Adapt Control Signal Process* 2023; 37(3): 617–643.
26. Xu R, Wang X and Zhou Y. Observer-based event-triggered adaptive containment control for multiagent systems with prescribed performance. *Nonlinear Dyn* 2022; 107(3): 2345–2362.
27. Wang X, Wu Q and Yin X. Adaptive finite-time prescribed performance control of switched nonlinear systems with unknown actuator dead-zone. *Int J Syst Sci* 2020; 51(1): 133–145.
28. Zhang L, Li Y and Tong S. Adaptive fuzzy output feedback control for MIMO switched nonlinear systems with prescribed performances. *Fuzzy Sets Syst* 2017; 306: 153–168.
29. Li N, Han YQ, He WJ, et al. A novel network-based controller design for a class of stochastic nonlinear systems with multiple faults and full state constraints. *Int J Control* 2024; 97(4): 651–661.
30. Sun Y, Shi P and Lim C. Event-triggered adaptive leaderless consensus control for nonlinear multi-agent systems with unknown backlash-like hysteresis. *Int J Robust Nonlinear Control* 2021; 31(15): 7409–7424.
31. Liu L and Tang L. Partial state constraints-based control for nonlinear systems with backlash-like hysteresis. *IEEE Trans Syst Man Cybern Syst* 2018; 50(8): 1–5.
32. Liu Y, Zhu Q, Zhao N, et al. Adaptive fuzzy backstepping control for nonstrict feedback nonlinear systems with time-varying state constraints and backlash-like hysteresis. *Inf Sci* 2021; 574: 606–624.
33. Si W, Dong X and Yang F. Adaptive neural prescribed performance control for a class of strict-feedback stochastic nonlinear systems with hysteresis input. *Neurocomputing* 2017; 251: 35–44.
34. Han YQ and Yan HS. Adaptive multi-dimensional Taylor network tracking control for SISO uncertain stochastic non-linear systems. *IET Control Theory Appl* 2018; 12(8): 1107–1115.
35. Su CY, Stepanenko Y, Svoboda J, et al. Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis. *IEEE Trans Automat Contr* 2000; 45(12): 2427–2432.

36. Liu S, Zhang L, Niu B, et al. Adaptive neural finite-time hierarchical sliding mode control of uncertain under-actuated switched nonlinear systems with backlash-like hysteresis. *Inf Sci* 2022; 599: 147–169.
37. Ge SS and Wang C. Adaptive neural control of uncertain MIMO nonlinear systems. *IEEE Trans Neural Netw* 2004; 15(3): 674–692.
38. Tao F, Fan P, Fu Z, et al. Adaptive fuzzy fixed time control for pure-feedback stochastic nonlinear systems with full state constraints. *J Franklin Inst* 2022; 359(10): 4642–4660.
39. Bechlioulis CP and Rovithakis GA. Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance. *IEEE Trans Automat Contr* 2008; 53(9): 2090–2099.
40. Sun YJ. Robust tracking control of uncertain Duffing–Holmes control systems. *Chaos Solitons Fractals* 2009; 40(3): 1282–1287.