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# Design of event-triggered adaptive finite-time controller for full-state constrained stochastic nonlinear systems with unknown control directions

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## ABSTRACT

In this article, the problem of event-triggered (ET) adaptive finite-time tracking control is investigated for full-state constrained stochastic nonlinear systems with unknown control directions. In the backstepping, multi-dimensional Taylor networks are employed to estimate the unknown nonlinearities and the command filter is used to overcome the problem of explosion of complexity. Subsequently, a novel adaptive updated law is created for co-designing the controller and the switching threshold-based ET mechanism by introducing the Nussbaum-type functions, which effectively handles the effects of the measurement errors due to the ET mechanism, and the challenges of controller design because of the presence of unknown control directions. The proposed control strategy can achieve that the closed-loop system is semi-global finite-time stable in probability, the system states are maintained in the predefined compact sets in a finite time and the system output tracking expectation signal is commendable while easing the communications burden. Finally, two examples are employed to illustrate the validity of the proposed control strategy.

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Event-triggered control; finite-time control; multi-dimensional Taylor networks; full-state constraints; stochastic nonlinear systems

## 1. Introduction

Stochastic disturbances, uncertainties, and nonlinearities are often present in practical systems, which are important factors leading to system instability. As a consequence, the investigations on stochastic nonlinear systems have received great amount of attentions in the past few decades (Deng et al., 2001; Ji & Xi, 2006; Jia & He, 2023; Tsukamoto & Chung, 2021). Since the nonlinearities and uncertainties in the system cannot be described by an exact mathematical model, a number of promising approximation-based methods have been developed. The three most popular methods used are neural networks (NNs) (Sun et al., 2016; H. Q. Wang et al., 2014), fuzzy logic systems (FLSs) (Li & Yu, 2022; Liu & Ma, 2020), and multi-dimensional Taylor networks (MTNs) (He et al., 2023; Lu et al., 2024). Particularly, the MTN-based method, in combination with Lyapunov stability theory and adaptive backstepping technique, has led to remarkable results in the control design of stochastic systems (Han, 2020; Han & Yan, 2018; D. M. Wang et al., 2024; Yan & Han, 2019).

The control objects of the above mentioned articles are stochastic nonlinear systems without state constraints. In the operation of the practical system, it is often important to impose certain state constraints on the system to ensure the safety and efficiency of the controlled system. Hence, it is essential to take state constraints into account when designing a controller. At present, the barrier Lyapunov function (BLF) is a very effective tool to handle the constrained system, whose

common forms are as follows: logarithmic-type BLFs (Zhang & He, 2022), tangent-type BLFs (Tang et al., 2019) and integral-type BLFs (Li & Li, 2018). Many valuable results of control design with BLFs for stochastic systems have been reported (Jin & Li, 2021; Li et al., 2023; Y. J. Liu et al., 2018). Specifically, based on MTN and logarithmic-type BLFs, the adaptive decentralised prescribed performance control was further taken into account in Li et al. (2023) for large-scale stochastic nonlinear systems with input saturation and full state constraints, which did not concern unknown control directions. In addition, the system's control directions often cannot be known in a prior, which is challenging to design the controllers. Therefore, it is very valuable to study full-state constrained stochastic nonlinear systems (FCSNSs) with unknown control directions, while two aspects should be emphasised: limited communication resources and the convergence performance of the controlled system.

In recent years, it has become a trend to connect controlled systems and factories through network communication channels. Therefore, reducing communication transmission to save resources has become a key focus of research in the field of control. In this background, event-triggered (ET) control strategies have been developed thanks to their unique dominance in saving communication resources (Chen et al., 2023; Liu & Li, 2022). In addition, it is very difficult to guarantee this assumption of input-to-state stable (ISS) for stochastic nonlinear systems. This has prompted research into how to avoid this ISS hypothesis (Du et al., 2023; Li & Yang, 2018; Ma et al., 2019;

Xing et al., 2017), in which the controller and ET mechanism are designed simultaneously to avoid the ISS assumption. On the basis of this theory, existing articles mainly involve three kinds of ET mechanisms: fixed-threshold-based ET mechanism (Z. R. Zhang et al., 2022), relative-threshold-based ET mechanism (E. Y. Wang et al., 2022; Zhao et al., 2022), and switching-threshold-based ET mechanism (Du et al., 2023). Unfortunately, a little research has been done on switching-threshold-based ET mechanism for stochastic nonlinear systems. This is a motivation for this article.

It is noteworthy that the above studies on ET control did not concern the transient performance of the system, but only considered asymptotic stability which can ensure the closed-loop system was convergent as time tended to infinity. Compared with asymptotic stability (Liu & Zhu, 2021; Su et al., 2021), the finite-time control has higher accuracy and better robustness (X. F. Zhang et al., 2012; Zhu et al., 2011), which can reach a steady state in a relatively short time. Moreover, it is the expected control objective to guarantee the controlled systems to converge to near zero in a finite time for stochastic nonlinear systems. For this reason, the authors in Yin et al. (2011) firstly mentioned the main idea of finite-time stability. Subsequently, many significant results about different stochastic systems have been reported based on finite-time stability theory, such as switched stochastic nonlinear systems (Huang & Xiang, 2016), non-triangular stochastic nonlinear systems (Sui et al., 2019), high-order stochastic nonlinear systems (Wang & Zhu, 2015). Unfortunately, there is little finite-time control results on FCSNSs with unknown control directions. This is another motivation for this article.

Based on the aforementioned literatures and inspired by the conclusion and future challenging of Liu, Li, et al. (2022), this article constructs a novel ET adaptive MTN controller for FCSNSs with unknown control directions. In comparison with the existing results, the highlights of this article are categorised into the following three:

- (1) To the best of authors' knowledge, this article is the first to study the finite-time tracking control for FCSNSs by means of MTN estimation method, the command filter and the switching threshold-based ET mechanism. It should be noted that the finite-time control and ET control have been studied in Xie et al. (2023) and Zhang and Yang (2020). However, their control plants are nonlinear systems which ignored the existence of stochastic factors. Additionally, although (Du et al., 2023; Yue et al., 2023) also considered the ET control for FCSNSs, they ignore the finite time convergence. Therefore, the system and the problems considered in this article are more general and can enlarge the practical application range.
- (2) A novel adaptive law is developed to estimate the upper bound of the actual control gain. The command-filtered adaptive backstepping control is presented to avoid the 'explosion of complexity' problem and decrease the effect of the filtering errors by designing compensating signals simultaneously. Moreover, the controller and ET mechanism are jointly designed to successfully compensate the measurement error caused by the ET mechanism and unknown control directions. Therefore, the proposed

control scheme can achieve the expected control goal while conserving communication resources.

- (3) Different from the controller designed in Xia et al. (2021) and You et al. (2023), the proposed controller has a simpler structure and less computational effort with the aid of MTN and the command filter. Moreover, in comparison with the asymptotic tracking (Du et al., 2023; Su et al., 2021), the finite-time tracking studied possesses the better control performance and disturbance rejection property.

## 2. Preliminaries and formulation

### 2.1 Research objects and objectives

Consider a category of strict-feedback stochastic nonlinear systems:

$$\begin{cases} dx_i = (h_i(\bar{\mathbf{x}}_i) x_{i+1} + f_i(\bar{\mathbf{x}}_i)) dt + g_i^T(\bar{\mathbf{x}}_i) d\omega \\ i = 1, \dots, n-1 \\ dx_n = (h_n(\mathbf{x}) u + f_n(\mathbf{x})) dt + g_n^T(\mathbf{x}) d\omega \\ y = x_1 \end{cases} \quad (1)$$

where  $x_i$  and  $y$  stand for system states and system output constrained in the compact sets; i.e.  $|x_i| < k_{bi}$  ( $i = 1, \dots, n$ ) with  $k_{bi}$  being known positive constants;  $\bar{\mathbf{x}}_i = [x_1, \dots, x_i]^T$ , ( $\mathbf{x} = \bar{\mathbf{x}}_n$ ) denote the system state vectors;  $\omega$  stands for an independent  $r$ -dimensional standard Brownian motion;  $h_i(\bar{\mathbf{x}}_i)$  represents smooth nonlinear functions, the control directions are referred to as the signs of  $h_i(\bar{\mathbf{x}}_i)$ , which are assumed to be unknown.  $f_i(\bar{\mathbf{x}}_i)$  and  $g_i(\bar{\mathbf{x}}_i)$ ,  $i = 1, 2, \dots, n$  represent unknown smooth nonlinear functions with satisfying  $f_i(\mathbf{0}) = 0$  and  $g_i(\mathbf{0}) = \mathbf{0}$ .

The control goal of this article is to design an ET-based adaptive MTN controller for the controlled stochastic system (1) such that:

- (1) The system (1) can achieve the property of SGFSP while avoiding Zeno phenomenon.
- (2) All of the system states satisfy their respective constraint bounds for the provided state constraints.
- (3) It is achievable to get the satisfying tracking performance for the chosen expectation signal  $y_r$  in a finite time.

In addition, the following assumptions are necessary for the design of the controller.

**Assumption 2.1 (Min et al., 2021):** *The tracking expectation signal  $y_r$  and its  $i$ th-order derivatives  $y_r^{(i)}$  ( $i = 1, \dots, n$ ) fulfil  $\max\{\bar{Y}_{rm}, \bar{Y}_{rM}\} \leq \bar{A}_0 < k_{b1}$ ,  $-\bar{Y}_{rm} \leq y_r \leq \bar{Y}_{rM}$  and  $|y_r^{(i)}| \leq \bar{A}_i$ , in which  $\bar{Y}_{rm}$ ,  $\bar{A}_i$ ,  $\bar{A}_0$  and  $\bar{Y}_{rM}$  represent positive real numbers.*

**Assumption 2.2 (Liu, Zhang, et al., 2022):** *The control coefficients  $h_i(\bar{\mathbf{x}}_i)$  with unknown signs, satisfy  $h_{i1} \leq h_i(\bar{\mathbf{x}}_i) \leq h_{i2}$  and  $h_i(\bar{\mathbf{x}}_i) \neq 0$ , where  $h_{i1}$  and  $h_{i2}$  are positive constants.*

**Remark 2.1:** In Assumption 2.1, the differentiability and boundedness of the tracking expectation signal  $y_r$  are used to meet the requirement of the backstepping method. In Assumption 2.2, the value of  $h_i(\bar{\mathbf{x}}_i)$  is defined as a bounded interval without containing zero, which is used to ensure the controllability of the system (1). Therefore, Assumptions 2.1–2.2 are common in most existing results.

## 2.2 Stochastic systems theory

To introduce the definitions and theorems of stochastic nonlinear systems, take the following general stochastic system into consideration:

$$d\vartheta = \Phi(\vartheta) dt + \Lambda(\vartheta) d\omega \quad (2)$$

where  $\vartheta \in R^n$  denotes the state vector,  $\omega$  has the same definition as in (1), and  $\Phi(\cdot)$ ,  $\Lambda(\cdot)$  are locally Lipschitz functions with satisfying  $\Phi(\mathbf{0}) = \mathbf{0}$  and  $\Lambda(\mathbf{0}) = \mathbf{0}$ .

**Definition 2.1 (H. Q. Wang et al., 2014):** For any given  $V(\vartheta)$  with second-order continuous partial derivative, the differential operator  $L$  relating to the stochastic system (2) is defined as the following mathematical expression:

$$LV = \frac{\partial V}{\partial \vartheta} \Phi + \frac{1}{2} \text{Tr} \left\{ \Lambda^T \frac{\partial^2 V}{\partial \vartheta^2} \Lambda \right\} \quad (3)$$

where  $\text{Tr}\{\cdot\}$  stands for the trace of  $\cdot$ .

**Definition 2.2 (Min et al., 2021):** The solution of stochastic system (2) is considered to be semi-globally finite-time stability in probability (SGFSP), if it satisfies

- (i) For any initial value  $\vartheta_0 \in R^n$ , the system (2) has a unique solution;
- (ii) For each  $\vartheta_0 \in R^n \setminus \{\Omega_1\}$ , the stochastic setting time  $T(\vartheta_0, \epsilon) = \inf\{t : \vartheta(t; \vartheta_0) \in \Omega_1\}$  is finite almost surely; in other words,  $P\{T(\vartheta_0, \epsilon) < \infty\} = 1$ , in which  $\epsilon$  represents a positive constant,  $\Omega_1$  stands for a compact set and  $T(\vartheta_0, \epsilon)$  is the first time at which the set  $\Omega_1$  is arrived;
- (iii) For given positive constant  $\epsilon$  and all  $t \geq T(\vartheta_0, \epsilon)$ , the solution meets  $E(\|\vartheta(t; \vartheta_0)\|) < \epsilon$ .

**Definition 2.3 (Wang & Wang, 2022):** A smooth function  $\eta(\varphi)$  can be defined as Nussbaum-type function when it satisfies  $\lim_{s \rightarrow \infty} \sup \frac{1}{s} \int_0^s \eta(\varphi) d\varphi = +\infty$  and  $\lim_{s \rightarrow \infty} \inf \frac{1}{s} \int_0^s \eta(\varphi) d\varphi = -\infty$ .

**Remark 2.2:** In this article, let  $\eta(\varphi) = e^{\varphi^2} \cos(\varphi)$ , where  $\varphi$  is a smooth function which will be designed later. The Nussbaum-type functions is introduced, which can effectively handle the difficulty in the controller design because of the unknown control directions.

At the end of the section, a few Lemmas are presented, which can be used in the process of designing the controller.

**Lemma 2.1 (Lin & Qian, 2000):** For any given  $\iota, j \in R$ , positive real numbers  $p, q, b$  and continuous function  $a(\cdot) \geq 0$ , the inequality holds  $a(\cdot)^p j^q \leq \frac{q}{p+q} a(\cdot)^{\frac{p+q}{q}} \left(\frac{p+q}{p} b\right)^{-\frac{p}{q}} |j|^{p+q} + b|\iota|^{p+q}$ .

**Lemma 2.2 (Yang & Lin, 2005):** For  $z_i \in R, i = 1, \dots, n$  and  $0 < \iota < 1$ , it holds  $\left(\sum_{i=1}^n |z_i|\right)^\iota \leq \sum_{i=1}^n |z_i|^\iota \leq n^{1-\iota} \left(\sum_{i=1}^n |z_i|\right)^\iota$ .

**Lemma 2.3 (Li et al., 2023):** For any positive constants  $\bar{k}$  and any variable  $\varsigma \in R$  satisfies  $|\varsigma| < \bar{k}$ , one has  $\log \frac{\bar{k}^{2p}}{\bar{k}^{2p} - \varsigma^{2p}} \leq \frac{\varsigma^{2p}}{\bar{k}^{2p} - \varsigma^{2p}}$ , where  $p$  is a positive integer.

**Lemma 2.4 (Du et al., 2023):** For  $\forall \nabla \in R$  and  $\delta > 0$ , one can get the inequality:  $0 \leq |\nabla| - \nabla \tanh\left(\frac{\nabla}{\delta}\right) \leq 0.2785\delta$ .

**Lemma 2.5 (Min et al., 2021):** Consider system (2) and assume  $\Phi(\vartheta)$ ,  $\Lambda(\vartheta)$  are locally bounded and locally Lipschitz continuous in  $\vartheta$  with satisfying  $\Phi(0) = 0$  and  $\Lambda(0) = 0$ . If for any given  $\vartheta_0 \in R^n$  and all  $\vartheta \in R^n$  with  $t \geq 0$ , there exists a positive definite,  $C^2$  function  $V(\vartheta) \in R^n$ ,  $\mathcal{K}_\infty$  class functions  $\Theta_1$  and  $\Theta_2$ , real numbers  $c > 0, 0 < \Delta < \infty$  and  $0 < \iota < 1$ , the two inequalities hold with the following

$$\begin{aligned} \Theta_1(\|\vartheta\|) &\leq V(\vartheta) \leq \Theta_2(\|\vartheta\|) \\ LV(\vartheta) &\leq -cV^\iota(\vartheta) + \Delta \end{aligned} \quad (4)$$

after that, the system (2) is SGFSP with the setting time  $T$  satisfying

$$E(T) \leq \frac{1}{\gamma c(1-\iota)} \left[ E(V^{1-\iota}(0)) - \left(\frac{\Delta}{(1-\gamma)c}\right)^{\frac{1-\iota}{\iota}} \right] \quad (5)$$

where  $0 < \gamma < 1$  is a constant.

## 2.3 Multi-dimensional Taylor network approximation

As a neural network with a straightforward structure, MTN has good nonlinear approximation ability. Since stochastic systems typically do not allow for the direct use of unknown nonlinear functions in the controller design, MTN is employed to estimate the unknown functions.

**Lemma 2.6 (He et al., 2023):** Assume  $H(\bar{z})$  is a continuous nonlinear function which is defined on a compact set  $\Omega$ , then for  $\forall \tau > 0$ ,  $H(\bar{z})$  can be estimated by  $\theta^T S_{m_n}(\bar{z})$  as follows

$$H(\bar{z}) = \theta^T S_{m_n}(\bar{z}) + \Xi(\bar{z}), |\Xi(\bar{z})| \leq \tau \quad (6)$$

where  $\theta = [\theta_1, \theta_2, \dots, \theta_l]^T$  and  $\bar{z} = [z_1, z_2, \dots, z_n]^T$  stand for the weight vector and input vector of MTN, respectively.  $S_{m_n}(\bar{z}) = [z_1, \dots, z_n, z_1^2, z_1 z_2, \dots, z_1^m, z_1^{m-1} z_2, \dots, z_n^m]^T$  represents the middle layer of MTN, where  $n$  and  $m$  are the input number and dimension of  $S_{m_n}(\bar{z})$ , respectively.

## 2.4 Command filter

To reduce the computational burden, the command filter is introduced to solve the problem of ‘explosion of complexity’ caused by the repeated differentiation of virtual control signal in the backstepping control design process.

**Lemma 2.7 (Qiu et al., 2022):** For  $\forall p > 0$ , if the input signal  $\alpha_j$  satisfies  $|\dot{\alpha}_j| \leq c_1$  and  $|\ddot{\alpha}_j| \leq c_2$ , where  $c_1$  and  $c_2$  are positive constants, there exist  $\mu_{j,n} > 0$  and  $0 < k_j \leq 1$ , the following equations are satisfied

$$\begin{cases} \dot{\zeta}_{j,1} = \mu_{j,n} \zeta_{j,2} \\ \dot{\zeta}_{j,2} = -2k_j \mu_{j,n} \zeta_{j,2} - \mu_{j,n} (\zeta_{j,1} - \alpha_j) \end{cases} \quad (7)$$

where  $\zeta_{j,1}$  denotes the output of the command filter,  $\alpha_j$  is the input of the command filter which will be designed with  $j = 1, \dots, n$ . The initial conditions are defined as  $\zeta_{j,1}(0) = \alpha_j(0)$  and  $\dot{\zeta}_{j,1}(0) = 0$ . Then, the filter error satisfies  $|\zeta_{j,1} - \alpha_j| \leq p$ . Moreover,  $|\dot{\zeta}_{j,1}|$ ,  $|\ddot{\zeta}_{j,1}|$ , and  $|\ddot{\zeta}_{j,1}|$  are bounded.

### 3. Main results

This section includes not only the process of the controller design via adaptive backstepping methodology, but also the formulation and demonstration of a new theorem.

#### 3.1 Controller design

An adaptive finite-time controller will be designed in view of the command filter and ET mechanism. Before that, it is essential to make the following error transformation:

$$\begin{aligned} z_1 &= x_1 - y_r, z_i = x_i - x_{i,c}, \quad i = 2, \dots, n \\ v_1 &= z_1 - \zeta_1, v_i = z_i - \zeta_i, \quad i = 2, \dots, n \end{aligned} \quad (8)$$

where  $y_r$  stands for the expectation signal and  $x_{i,c}$  represents the output of the command filter.  $\zeta_i$  denotes the compensating signal will be designed later.

At present, it is shown the design procedure of the controller for system (1).

*Step 1:* First of all, design the BLF function with the following specific form:

$$V_1 = \frac{1}{4} \log \left( \frac{k_{c1}^4}{k_{c1}^4 - v_1^4} \right) + \frac{1}{2} \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1 \quad (9)$$

where  $k_{c1} = k_{b1} - \bar{A}_0$ ;  $\Gamma_1 = \Gamma_1^T > 0$  stands for a constant matrix;  $\hat{\theta}_1$  denotes the estimate of unknown parameter  $\theta_1$ .  $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$  represents the parameter estimate error. It is not difficult to know that  $V_1$  is continuous differentiable in  $\Omega_{v_1} := \{v_1 : |v_1| < k_{c1}\}$ .

Then, based on Definition 2.1 and (1), one arrives at

$$\begin{aligned} LV_1 &\leq \frac{v_1^3}{k_{c1}^4 - v_1^4} (h_1 x_2 + f_1 - \dot{y}_r - \dot{\zeta}_1) \\ &\quad + \frac{v_1^2 (3k_{c1}^4 + v_1^4)}{2(k_{c1}^4 - v_1^4)^2} \text{Tr} \{g_1^T g_1\} - \tilde{\theta}_1^T \Gamma_1^{-1} \dot{\hat{\theta}}_1 \end{aligned} \quad (10)$$

With the aid of Lemma 2.1, it is not hard to obtain the following inequality

$$\frac{v_1^2 (3k_{c1}^4 + v_1^4)}{2(k_{c1}^4 - v_1^4)^2} \text{Tr} \{g_1^T g_1\} \leq \varepsilon_{11} + \frac{\phi_{11} v_1^4}{(k_{c1}^4 - v_1^4)^4} \quad (11)$$

where  $\varepsilon_{11} > 0$  is a design constant,  $\phi_{11} = \frac{1}{2}(2\varepsilon_{11})^{-1} \left( \frac{3k_{c1}^4 + v_1^4}{2} \|g_1\|^2 \right)^2$ .

Substituting (11) into (10), one has

$$LV_1 \leq \frac{v_1^3}{k_{c1}^4 - v_1^4} (h_1 (z_2 + x_{2,c}) + H_1 - \dot{\zeta}_1) + \varepsilon_{11} - \tilde{\theta}_1^T \Gamma_1^{-1} \dot{\hat{\theta}}_1 \quad (12)$$

where  $H_1 = f_1 - \dot{y}_r + \frac{\phi_{11} v_1}{(k_{c1}^4 - v_1^4)^3}$ . Obviously,  $H_1$  can not be used to design controller because it contains an unknown function.

By introducing Lemma 2.6, MTN is employed to approximate the unknown function  $H_1$ . Specifically, for  $\forall \tau_1 > 0$ , there is a MTN as  $\theta_1^T S_{m_1}(\bar{v}_1)$ , such that

$$H_1 = \theta_1^T S_{m_1}(\bar{v}_1) + \Xi_1(\bar{v}_1), |\Xi_1(\bar{v}_1)| \leq \tau_1 \quad (13)$$

where  $\bar{v}_1 = [v_1]^T$ ,  $\Xi_1(\bar{v}_1)$  represents the estimation error.

Using Lemma 2.1 again, one obtains

$$\frac{v_1^3}{k_{c1}^4 - v_1^4} \Xi_1(\bar{v}_1) \leq \varepsilon_{12} \tau_1^4 + \frac{\phi_{12} v_1^4}{(k_{c1}^4 - v_1^4)^{\frac{4}{3}}} \quad (14)$$

where  $\varepsilon_{12}$  denotes a positive design constant,  $\phi_{12} = \frac{3}{4}(4\varepsilon_{12})^{-\frac{1}{3}}$ . Next, substituting (13) and (14) into (12), gives

$$\begin{aligned} LV_1 &\leq \frac{v_1^3}{k_{c1}^4 - v_1^4} (h_1 (z_2 + x_{2,c}) + \theta_1^T S_{m_1} - \dot{\zeta}_1) + \varepsilon_{11} \\ &\quad + \frac{\phi_{12} v_1^4}{(k_{c1}^4 - v_1^4)^{\frac{4}{3}}} - \tilde{\theta}_1^T \Gamma_1^{-1} \dot{\hat{\theta}}_1 + \varepsilon_{12} \tau_1^4 \end{aligned} \quad (15)$$

Based on (15), design the error compensating signal  $\dot{\zeta}_1$ , the virtual control signal  $\alpha_1$  and the adaptive law  $\dot{\hat{\theta}}_1$  as follows

$$\dot{\zeta}_1 = -\kappa_1 \zeta_1 + h_1 \zeta_2 + h_1 (x_{2,c} - \alpha_1) \quad (16)$$

$$\alpha_1 = \eta(\varphi_1) \psi_1 \quad (17)$$

$$\psi_1 = \kappa_1 z_1 + \hat{\theta}_1^T S_{m_1} + \frac{\phi_{12} v_1}{(k_{c1}^4 - v_1^4)^{\frac{1}{3}}} \quad (18)$$

$$\dot{\varphi}_1 = \frac{v_1^3}{k_{c1}^4 - v_1^4} \psi_1 \quad (19)$$

$$\dot{\hat{\theta}}_1 = \Gamma_1 \left( \frac{v_1^3}{k_{c1}^4 - v_1^4} S_{m_1} - \sigma_1 \hat{\theta}_1 \right) \quad (20)$$

where  $\kappa_1$  and  $\sigma_1$  represent positive design constants.

Then, substituting (17)-(20) into (15), the following inequality holds

$$\begin{aligned} LV_1 &\leq -\kappa_1 \frac{v_1^4}{k_{c1}^4 - v_1^4} + \frac{v_1^3}{k_{c1}^4 - v_1^4} h_1 z_2 + \sigma_1 \tilde{\theta}_1^T \hat{\theta}_1 \\ &\quad + (h_1 \eta(\varphi_1) + 1) \dot{\varphi}_1 + \varepsilon_{11} + \varepsilon_{12} \tau_1^4 \end{aligned} \quad (21)$$

Further considering  $\hat{\theta}_1 = \theta_1 - \tilde{\theta}_1$  and Lemma 2.1, the following three inequalities are true

$$\sigma_1 \tilde{\theta}_1^T \hat{\theta}_1 \leq -\frac{\sigma_1}{2} \|\tilde{\theta}_1\|^2 + \frac{\sigma_1}{2} \|\theta_1\|^2 \quad (22)$$

$$\left( \frac{1}{2} \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1 \right)^l \leq \iota^{\frac{l}{1-\iota}} (1-\iota) + \frac{1}{2} \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1 \quad (23)$$

$$\left( \frac{v_1^4}{k_{c1}^4 - v_1^4} \right)^l \leq \frac{v_1^4}{k_{c1}^4 - v_1^4} + \iota^{\frac{l}{1-\iota}} (1-\iota) \quad (24)$$

where  $0 < \iota < 1$ . Additionally, with the aid of Lemma 2.3, when  $|v_1| < k_{c1}$ ,  $\log \frac{k_{c1}^4}{k_{c1}^4 - v_1^4} \leq \frac{v_1^4}{k_{c1}^4 - v_1^4}$  holds, which implies

$\left(\log \frac{k_{c1}^4}{k_{c1}^4 - v_1^4}\right)^l \leq \left(\frac{v_1^4}{k_{c1}^4 - v_1^4}\right)^l$ . Then, with the aid of Lemma 2.2, combining (9) with (21)–(24), one results in

$$LV_1 \leq -c_1 V_1^l + \Delta_1 + (h_1 \eta(\varphi_1) + 1) \dot{\phi}_1 + \frac{v_1^3}{k_{c1}^4 - v_1^4} h_1 v_2 \quad (25)$$

where  $c_1 = \min\{4^l \kappa_1, \sigma_1 \lambda_{\min}(\Gamma_1)\}$  and  $\Delta_1 = (\sigma_1 \lambda_{\min}(\Gamma_1) + 1)(1 - l)^{\frac{1}{1-l}} + \frac{\sigma_1}{2} \|\theta_1\|^2 + \varepsilon_{11} + \varepsilon_{12} \tau_1^4$ .

*Step i* ( $2 \leq i \leq n-1$ ): Construct the  $i$ th BLF function, the specific form of which is as follows

$$V_i = \frac{1}{4} \log \left( \frac{k_{ci}^4}{k_{ci}^4 - v_i^4} \right) + \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \quad (26)$$

where  $\Gamma_i = \Gamma_i^T > 0$  stands for a constant matrix;  $k_{ci}$  represents a positive design constant;  $\hat{\theta}_i$  denotes the estimate of unknown parameter  $\theta_i$ .  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$  is the estimate error. It is not difficult to know that  $V_i$  is continuous differentiable in  $\Omega_{v_i} := \{v_i : |v_i| < k_{ci}\}$ .

Then, based on Definition 2.1 and (1), one arrives at

$$LV_i \leq \frac{v_i^3}{k_{ci}^4 - v_i^4} (h_i x_{i+1} + f_i - \dot{\zeta}_i - \dot{x}_{i,c}) + \frac{v_i^2 (3k_{ci}^4 + v_i^4)}{2(k_{ci}^4 - v_i^4)^2} \|g_i\|^2 - \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i \quad (27)$$

By introducing Lemma 2.1, one in (27) can be transformed into the following form

$$\frac{z_i^2 (3k_{ci}^4 + v_i^4)}{2(k_{ci}^4 - v_i^4)^2} \|g_i\|^2 \leq \varepsilon_{i1} + \frac{\phi_{i1} v_i^4}{(k_{ci}^4 - v_i^4)^4} \quad (28)$$

where  $\varepsilon_{i1} > 0$  is a design constant;  $\phi_{i1} = \frac{1}{2} (2\varepsilon_{i1})^{-1} \left( \frac{3k_{ci}^4 + v_i^4}{2} \|g_i\|^2 \right)^2$ .

Substituting (28) into (27), one has

$$LV_i \leq \frac{v_i^3}{k_{ci}^4 - v_i^4} (h_i (z_{i+1} + x_{i+1,c}) + H_i - \dot{\zeta}_i - \dot{x}_{i,c}) + \varepsilon_{i1} - \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i \quad (29)$$

where  $H_i = f_i + \frac{\phi_{i1} v_i}{(k_{ci}^4 - v_i^4)^3} + h_{i-1} v_i$ . Since  $H_i$  contains unknown nonlinear terms, it will be estimated with the help of MTN. Specifically, for  $\forall \tau_2 > 0$ , there is a MTN as  $\theta_i^T S_{m_i}(\bar{v}_i)$ , such that

$$H_i = \theta_i^T S_{m_i}(\bar{v}_i) + \Xi_i(\bar{v}_i), |\Xi_i(\bar{v}_i)| \leq \tau_i \quad (30)$$

where  $\bar{v}_i = [v_1, \dots, v_i]^T$ ,  $\Xi_i(\bar{v}_i)$  represents the estimation error.

By using Lemma 2.1, the following inequalities hold

$$\frac{v_i^3}{k_{ci}^4 - v_i^4} \Xi_i(\bar{v}_i) \leq \varepsilon_{i2} \tau_i^4 + \frac{\phi_{i2} v_i^4}{(k_{ci}^4 - v_i^4)^3} \quad (31)$$

where  $\varepsilon_{i2}$  denotes a positive design constant,  $\phi_{i2} = \frac{3}{4} (4\varepsilon_{i2})^{-\frac{1}{3}}$ .

Substituting (30) and (31) into (29), one has

$$LV_i \leq \frac{\phi_{i2} v_i^4}{(k_{ci}^4 - v_i^4)^{\frac{4}{3}}} + \frac{v_i^3}{k_{ci}^4 - v_i^4} \left( h_i (z_{i+1} + x_{i+1,c}) + \theta_i^T S_{m_i} - \dot{\zeta}_i - \dot{x}_{i,c} \right) + \varepsilon_{i1} - \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\tilde{\theta}}_i + \varepsilon_{i2} \tau_i^4 \quad (32)$$

Based on (32), design the error compensating signal  $\dot{\zeta}_i$ , the virtual control signal  $\alpha_i$ , and the adaptive law  $\dot{\tilde{\theta}}_i$  as follows

$$\dot{\zeta}_i = -\kappa_i \zeta_i + h_i \zeta_{i+1} + h_i (x_{i+1,c} - \alpha_i) \quad (33)$$

$$\alpha_i = \eta(\varphi_i) \psi_i \quad (34)$$

$$\psi_i = \kappa_i z_i + \hat{\theta}_i^T S_{m_i} + \frac{\phi_{i2} v_i}{(k_{ci}^4 - v_i^4)^{\frac{1}{3}}} - \dot{x}_{i,c} \quad (35)$$

$$\dot{\phi}_i = \frac{v_i^3}{k_{ci}^4 - v_i^4} \psi_i \quad (36)$$

$$\dot{\tilde{\theta}}_i = \Gamma_i \left( \frac{v_i^3}{k_{ci}^4 - v_i^4} S_{m_i} - \sigma_i \tilde{\theta}_i \right) \quad (37)$$

where  $\kappa_i$  and  $\sigma_i$  represent positive design constants.

Substituting (34)–(37) into (32), we have

$$LV_i \leq -\kappa_i \frac{v_i^4}{k_{ci}^4 - v_i^4} + \sigma_i \tilde{\theta}_i^T \tilde{\theta}_i + \varepsilon_{i1} + \varepsilon_{i2} \tau_i^4 + \frac{v_i^3}{k_{ci}^4 - v_i^4} h_i v_{i+1} + (h_i \eta(\varphi_i) + 1) \dot{\phi}_i \quad (38)$$

Then, by taking a similar approach as in (22)–(24), and considering Lemmas 2.2 and 2.3, (38) can be converted into the following form

$$LV_i \leq -c_i V_i^l + \Delta_i + \frac{v_i^3}{k_{ci}^4 - v_i^4} h_i v_{i+1} + (h_i \eta(\varphi_i) + 1) \dot{\phi}_i \quad (39)$$

where  $c_i = \min\{4^l \kappa_i, \sigma_i \lambda_{\min}(\Gamma_i)\}$  and  $\Delta_i = (1 - l)^{\frac{1}{1-l}} (1 + \sigma_i \lambda_{\min}(\Gamma_i)) + \frac{\sigma_i}{2} \|\theta_i\|^2 + \varepsilon_{i1} + \varepsilon_{i2} \tau_i^4$ .

*Step n*: Construct the last BLF function, the specific form of which is as follows:

$$V_n = \frac{1}{4} \log \left( \frac{k_{cn}^4}{k_{cn}^4 - v_n^4} \right) + \frac{1}{2} \tilde{\theta}_n^T \Gamma_n^{-1} \tilde{\theta}_n + \frac{1}{2D} \tilde{h}_{n2}^2 \quad (40)$$

where  $\Gamma_n = \Gamma_n^T > 0$  stands for a constant matrix;  $k_{cn}$  and  $D$  represent positive design constants;  $\hat{\theta}_n$  denotes the estimate of unknown parameter  $\theta_n$ .  $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$  is the estimate error.  $h_{n2}$  denotes an unknown design parameter.  $\tilde{h}_{n2} = h_{n2} - \hat{h}_{n2}$  with  $\hat{h}_{n2}$  being the estimation of  $h_{n2}$ . It is not difficult to know that  $V_n$  is continuous differentiable in  $\Omega_{v_n} := \{v_n : |v_n| < k_{cn}\}$ .

Then, based on Definition 2.1 and (1), one arrives at

$$LV_n \leq \frac{v_n^3}{k_{cn}^4 - v_n^4} (h_n u + f_n - \dot{\zeta}_n - \dot{x}_{n,c}) - \tilde{\theta}_n^T \Gamma_n^{-1} \dot{\tilde{\theta}}_n + \frac{v_n^2 (3k_{cn}^4 + v_n^4)}{2(k_{cn}^4 - v_n^4)^2} \|g_n\|^2 - D^{-1} \tilde{h}_{n2} \dot{\tilde{h}}_{n2} \quad (41)$$

By resorting to Lemma 2.1, one in (41) can be changed as follows

$$\frac{v_n^2 (3k_{cn}^4 + v_n^4)}{2(k_{cn}^4 - v_n^4)^2} \|g_n\|^2 \leq \varepsilon_{n1} + \frac{\phi_{n1} v_n^4}{(k_{cn}^4 - v_n^4)^4} \quad (42)$$

where  $\varepsilon_{n1} > 0$  is a design constant,  $\phi_{n1} = \frac{1}{2}(2\varepsilon_{n1})^{-1} \left( \frac{3k_{cn}^4 + v_n^4}{2} \|g_n\|^2 \right)^2$ .

Substituting (42) into (41), yields

$$\begin{aligned} LV_n \leq & \frac{v_n^3}{k_{cn}^4 - v_n^4} (h_n u + H_n - \dot{\zeta}_n - \dot{x}_{n,c}) - \tilde{\theta}_n^T \Gamma_n^{-1} \dot{\hat{\theta}}_n \\ & - D^{-1} \tilde{h}_{n2} \dot{\hat{h}}_{n2} + \varepsilon_{n1} \end{aligned} \quad (43)$$

where  $H_n = f_n + \frac{\phi_{n1} v_n}{(k_{cn}^4 - v_n^4)^3} + h_{n-1} v_n$ . Since  $H_i$  contains unknown nonlinear terms, it will be estimated with the aid of MTN. Specifically, for  $\forall \tau_n > 0$ , there is a MTN as  $\theta_n^T S_{m_n}(\bar{v}_n)$ , such that

$$H_n = \theta_n^T S_{m_n}(\bar{v}_n) + \Xi_n(\bar{v}_n), |\Xi_n(\bar{v}_n)| \leq \tau_n \quad (44)$$

where  $\bar{v}_n = [v_1, \dots, v_n]$ ,  $\Xi_n(\bar{v}_n)$  represents the estimation error.

By using Lemma 2.1, one has

$$\frac{v_n^3}{k_{cn}^4 - v_n^4} \Xi_n(\bar{v}_n) \leq \varepsilon_{n2} \tau_n^4 + \frac{\phi_{n2} v_n^4}{(k_{cn}^4 - v_n^4)^{\frac{4}{3}}} \quad (45)$$

where  $\varepsilon_{n2}$  denotes a positive design constant,  $\phi_{n2} = \frac{3}{4}(4\varepsilon_{n2})^{-\frac{1}{3}}$ .

Substituting (44) and (45) into (43), we have

$$\begin{aligned} LV_n \leq & \frac{v_n^3}{k_{cn}^4 - v_n^4} (h_n u + \theta_n^T S_{m_n} - \dot{\zeta}_n - \dot{x}_{n,c}) + \varepsilon_{n2} \tau_n^4 + \varepsilon_{n1} \\ & + \frac{\phi_{n2} v_n^4}{(k_{cn}^4 - v_n^4)^{\frac{4}{3}}} - \tilde{\theta}_n^T \Gamma_n^{-1} \dot{\hat{\theta}}_n - D^{-1} \tilde{h}_{n2} \dot{\hat{h}}_{n2} \end{aligned} \quad (46)$$

Design the error compensating signal  $\dot{\zeta}_n$ , the controller  $\varpi$ , and the adaptive law  $\dot{\hat{\theta}}_n$ ,  $\dot{\hat{h}}_{n2}$  as follows

$$\dot{\zeta}_n = -\kappa_n \zeta_n \quad (47)$$

$$\varpi(t) = \eta(\varphi_n) \psi_n \quad (48)$$

$$\begin{aligned} \psi_n = & \kappa_n z_n + \hat{\theta}_n^T S_{m_n} + \frac{\phi_{n2} v_n}{(k_{cn}^4 - v_n^4)^{\frac{1}{3}}} \\ & + \hat{h}_{n2} \bar{M} \tanh \left( \frac{\bar{M} \frac{v_n^3}{k_{cn}^4 - v_n^4}}{\delta_1} \right) - \dot{x}_{n,c} \end{aligned} \quad (49)$$

$$\dot{\varphi}_n = \frac{v_n^3}{k_{cn}^4 - v_n^4} \psi_n \quad (50)$$

$$\dot{\hat{\theta}}_n = \Gamma_n \left[ \frac{v_n^3}{k_{cn}^4 - v_n^4} S_{m_n} - \sigma_n \hat{\theta}_n \right] \quad (51)$$

$$\dot{\hat{h}}_{n2} = D \left[ \bar{M} \frac{v_n^3}{k_{cn}^4 - v_n^4} \tanh \left( \frac{\bar{M} \frac{v_n^3}{k_{cn}^4 - v_n^4}}{\delta_1} \right) - \rho \hat{h}_{n2} \right] \quad (52)$$

where  $\sigma_n$ ,  $\rho$ ,  $\delta_1$  and  $\kappa_n$  represent positive design constants. The definition of  $\bar{M}$  will be given later.

To save the communication resource, the switching-threshold-based ET mechanism is considered to be built between the controller and the actuator

$$u(t) = \varpi(t_k), \quad \forall t \in [t_k, t_{k+1}) \quad (53)$$

$$t_{k+1} = \begin{cases} \inf \{t \mid |\varrho(t)| \geq M\}, & |u(t)| \geq Q \\ \inf \{t \mid |\varrho(t)| \geq \beta |u(t)| + M_1\}, & |u(t)| < Q \end{cases} \quad (54)$$

where  $t \in R$ ,  $k \in Z^+$ ,  $0 < \beta < 1$ ,  $M$  and  $M_1$  are positive design constants;  $Q$  is a design parameter;  $\bar{M} \geq \max\{\beta Q + M_1, M\}$  is designed as a positive constant.  $\varrho(t) = \varpi(t) - u(t)$  denotes the measurement error.

**Remark 3.1:** Different from the mechanism in the literature (B. H. Wang et al., 2019), the switching-threshold-based ET mechanism is a combination of fixed-threshold-based ET mechanism and relative-threshold-based ET mechanism. Therefore, it offers the advantages of the two ET design strategies and provides greater flexibility in balancing the system performance and conserving network resources.

With the help of (53)–(54), for  $\forall t \in [t_k, t_{k+1})$ , we have

$$u(t) = \varpi(t) - \zeta(t) \bar{\varrho} \quad (55)$$

where  $\zeta(t)$  is a time-varying parameter with  $|\zeta(t)| < 1$ ,  $\bar{\varrho} = \max\{\beta Q + M_1, M\}$ .

Then, combining (46) and (47)–(52), the following inequality holds

$$\begin{aligned} LV_n \leq & -\kappa_n \frac{v_n^4}{k_{cn}^4 - v_n^4} + \sigma_n \tilde{\theta}_n^T \hat{\theta}_n + \varepsilon_{n1} + \varepsilon_{n2} \tau_n^4 \\ & + \rho \tilde{h}_{n2} \dot{\hat{h}}_{n2} + (h_n \eta(\varphi_n) + 1) \dot{\varphi}_n \\ & - \frac{v_n^3}{k_{cn}^4 - v_n^4} h_{n2} \bar{M} \tanh \left( \frac{\bar{M} \frac{v_n^3}{k_{cn}^4 - v_n^4}}{\delta_1} \right) \\ & - \frac{v_n^3}{k_{cn}^4 - v_n^4} h_n \zeta(t) \bar{\varrho} \end{aligned} \quad (56)$$

In the end, by taking a similar approach as in (22)–(24) again, and taking Lemmas 2.2–2.4 into consideration, (56) can be converted to the following form

$$LV_n \leq -c_n V_n^l + \Delta_n + (h_n \eta(\varphi_n) + 1) \dot{\varphi}_n \quad (57)$$

where  $c_n = \min\{4^l \kappa_n, \sigma_n \lambda_{\min}(\Gamma_n), \rho D^l\}$  and  $\Delta_n = \iota^{\frac{l}{1-l}} (1 - \iota) (1 + \sigma_n \lambda_{\min}(\Gamma_n)) + \frac{\sigma_n}{2} \|\theta_n\|^2 + \frac{\rho}{2} h_{n2}^2 + \varepsilon_{n1} + \varepsilon_{n2} \tau_n^4 + 0.2785 \delta_1 h_{n2}$ .

### 3.2 Stability analysis of closed loop system

**Theorem 3.1:** For the controlled stochastic system (1), if Assumptions 2.1 and 2.2 are satisfied. Let the virtual control signals be designed as (17), (34) and (48), the actual control input be selected as (53)–(54) and the update laws be chosen as (20), (37) and (51)–(52), it holds (i) The controlled system (1) realises the SGFSP attribute. (ii) The tracking error will be bounded in a finite

time. (iii) System states do not violate predefined constraints and the Zeno phenomenon does not happen.

**Proof:** (i) Take the BLF  $V = \sum_{i=1}^n V_i$  into account and based on Lemma 2.2, it is easy to obtain that the following inequality holds

$$LV \leq -cV^l + \Delta + (1 + h\eta(\varphi))\dot{\varphi} \quad (58)$$

where  $c = \min\{c_i, i = 1, \dots, n\}$ ,  $\Delta = \sum_{i=1}^n \Delta_i$  and  $(1 + h\eta(\varphi))\dot{\varphi} = \sum_{i=1}^n (1 + h_i(\bar{x}_i)\eta(\varphi_i))\dot{\varphi}_i$ .

Then, similarly to the proof of Wang and Wang (2022), it is easy to conclude that  $(h\eta(\varphi) + 1)\dot{\varphi}$  is bounded, i.e. there exists a constant  $N$  such that  $N > \|(h\eta(\varphi) + 1)\dot{\varphi}\|$ , let  $\bar{\Delta} = \Delta + N$ , we have

$$LV \leq -cV^l + \bar{\Delta} \quad (59)$$

Therefore, according to Lemma 2.5, it is not difficult to conclude that there exists a setting time  $T$  at which the controlled stochastic system (1) realises the SGFSP attribute.

(ii) Take (59) and the certificate process of Theorem 1, Theorem 2 and Theorem 3 in Min et al. (2021) into consideration, it is not hard to obtain

$$E(|v_1|^4) \leq k_{c1}^4 \left( 1 - e^{-4\left(\frac{\bar{\Delta}}{(1-\gamma)c}\right)^{\frac{1}{l}}} \right), \quad \forall t \geq T^* \quad (60)$$

where  $0 < \gamma < 1$  is a constant and  $T^* = \frac{1}{(1-\gamma)\gamma c} \left[ EV^{1-l}(0) - \left(\frac{\bar{\Delta}}{(1-\gamma)c}\right)^{\frac{1-l}{l}} \right]$ .

Then, in view of (8) and (60), one has

$$E(|y - y_r|^4) \leq 8k_{c1}^4 \left( 1 - e^{-4\left(\frac{\bar{\Delta}}{(1-\gamma)c}\right)^{\frac{1}{l}}} \right) + 8E(|\zeta_1|^4), \quad \forall t \geq T^* \quad (61)$$

The boundedness of  $|\zeta_1|$  has been demonstrated in detail in the literature (Yu et al., 2019). As a result, there exists a finite time  $T^*$  which satisfies (61) such that the tracking error is bounded.

(iii) Based on the certificate process of Lemma 2 in Yu et al. (2019), define  $|\zeta_i| < \bar{\zeta}$ . According to  $v_1 = z_1 - \zeta_1$ , then  $|z_1| < k_{c1} + \bar{\zeta}$ . Since  $x_1 = z_1 + y_r$  and  $|y_r| \leq \bar{A}_0$  mentioned in Assumption 2.1, it is easy to know  $|x_1| \leq |z_1| + |y_r| < k_{c1} + \bar{\zeta} + \bar{A}_0$ . Define  $k_{c1} = k_{b1} - \bar{\zeta} - \bar{A}_0$ , then  $|x_1| < k_{b1}$ . Moreover, from (34), it can be realised that  $\alpha_i$  is a function of  $z_i, v_i, \hat{\theta}_i$ . Thus there has to be a constant  $c_1 > 0$  satisfying  $|\alpha_i| < c_1$ . According to Lemma 2.7, we have  $|x_{i,c} - \alpha_i| \leq p$ . It is not difficult to obtain  $|x_{i,c}| < c_1 + p$ . Since  $x_i = v_i + x_{i,c} + \zeta_i$ , let  $k_{ci} = k_{bi} - c_1 - \bar{\zeta} - p$ , one has  $|x_i| < k_{bi}$  for  $i = 2, \dots, n$ . As a result, all the system states do not violate predefined constraints.

Then, we need to prove that for  $\forall k \in \mathbb{Z}^+$ , there exists a time constant  $t^* > 0$  such that  $\{t_{k+1} - t_k\} \geq t^*$ . Based on  $\varrho(t) = \varpi(t) - u(t)$ , we can get  $\frac{d\varrho(t)}{dt} \leq |\dot{\varrho}(t)| \leq |\dot{\varpi}(t)|$ . From (48),  $\dot{\varpi}(t)$  is a continuous function composed of  $\varphi_n$  and  $\psi_n$ . Since

$\varphi_n$  and  $\psi_n$  are both bounded, so there is a constant  $v > 0$  such that  $|\dot{\varpi}(t)| \leq v$ . Besides, from  $\varrho(t_k) = 0$  and  $\lim_{t \rightarrow t_{k+1}} \varrho(t) = M$  or  $\lim_{t \rightarrow t_{k+1}} \varrho(t) = \beta|u(t)| + M_1$ , thus it can further infer that the lower bound of inter-execution intervals  $t^*$  must fulfil  $t^* \geq \frac{M}{v} > 0$  or  $t^* \geq \frac{\beta|u(t)| + M_1}{v} \geq \frac{M_1}{v} > 0$ . Furthermore, the controller excludes Zeno phenomenon.

In summary, Theorem 3.1 is proved. ■

## 4. Simulation study

To verify the merits of the controller designed in this article, a numerical simulation example and a practical simulation example are taken into consideration in the section.

**Example 4.1:** Consider a second-order stochastic system with the following mathematical expression:

$$\begin{cases} dx_1 = (h_1 x_2 - 0.3x_1) dt + 0.1 \cos(x_1) d\omega \\ dx_2 = (h_2 u - 0.5x_2 \cos(x_1^2)) dt + 0.1e^{-x_1^2} \cos(x_1 x_2) d\omega \\ y = x_1 \end{cases} \quad (62)$$

where  $h_1 = -1.1 \cos(x_1^2)$ ,  $h_2 = 3$ .

The command filter is designed as follows:

$$\begin{cases} \dot{\zeta}_{1,1} = \mu_{1,n} \zeta_{1,2} \\ \dot{\zeta}_{1,2} = -2k_1 \mu_{1,n} \zeta_{1,2} - \mu_{1,n} (\zeta_{1,1} - \alpha_1) \end{cases} \quad (63)$$

where  $\zeta_{1,1} = x_{2,c}$ . The corresponding parameters are  $\mu_{1,n} = 0.4$  and  $k_1 = 0.01$ .

In simulation, set the expectation signal  $y_r = 0.5 \sin(t)$ . The initial state values are chosen to be  $x_1(0) = 0$  and  $x_2(0) = 0$ . All design parameters are assigned as  $k_{b1} = 0.6$ ,  $\kappa_1 = 2.5$ ,  $\kappa_2 = 3$ ,  $\sigma_1 = 11$ ,  $\sigma_2 = 10$ ,  $D = 10$ ,  $\bar{M} = 13$ ,  $M = 2$ ,  $M_1 = 0.5$ ,  $\rho = 0.2$ ,  $\beta = 0.1$ ,  $\delta_1 = 20$ ,  $Q = 10$ ,  $\phi_{12} = 2$  and  $\phi_{22} = 0.1$ . Figures 1–3 verify the validity of the designed controller.

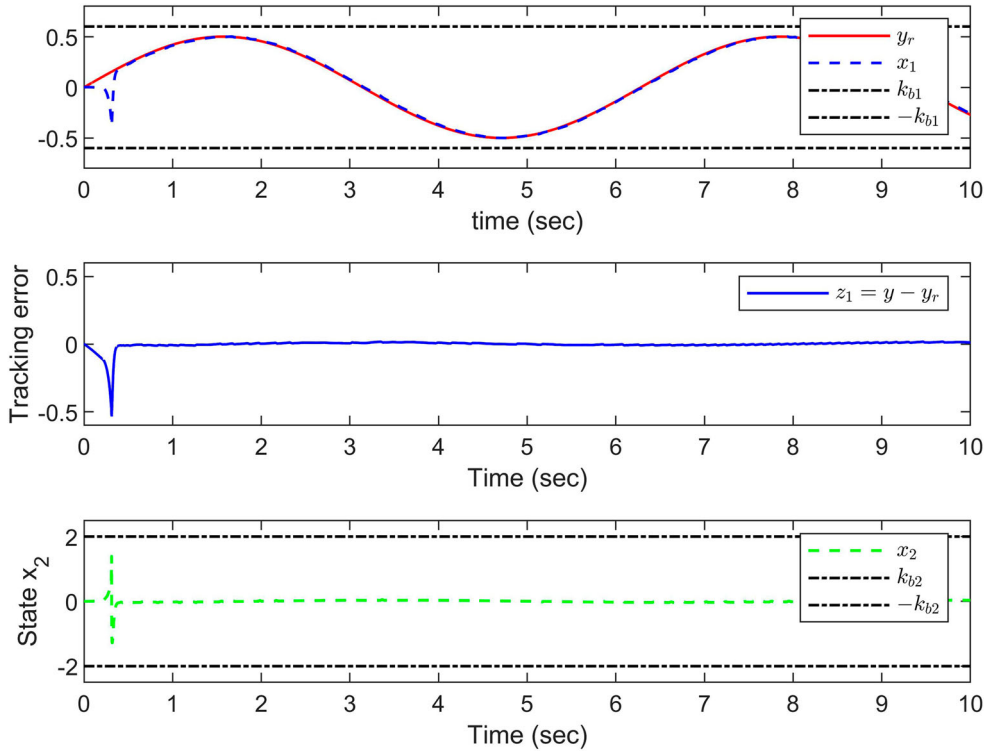
As illustrated in Figure 1, it is clear that all system states are obtained without violating the given constraints, and the tracking error  $z_1$  is boundary and converges to close to the origin, which shows a satisfactory tracking effect. Figure 2 shows a significant reduction in the communication load between the controller and the actuator, which can save network resources. Figure 3 shows that the interval between events is greater than zero, which confirms that Zeno behaviour has been effectively avoided.

**Example 4.2:** To further substantiate the efficacy of the ET adaptive finite-time controller designed, the single-link robot arm system with stochastic white noise in Min et al. (2021) is considered as follows:

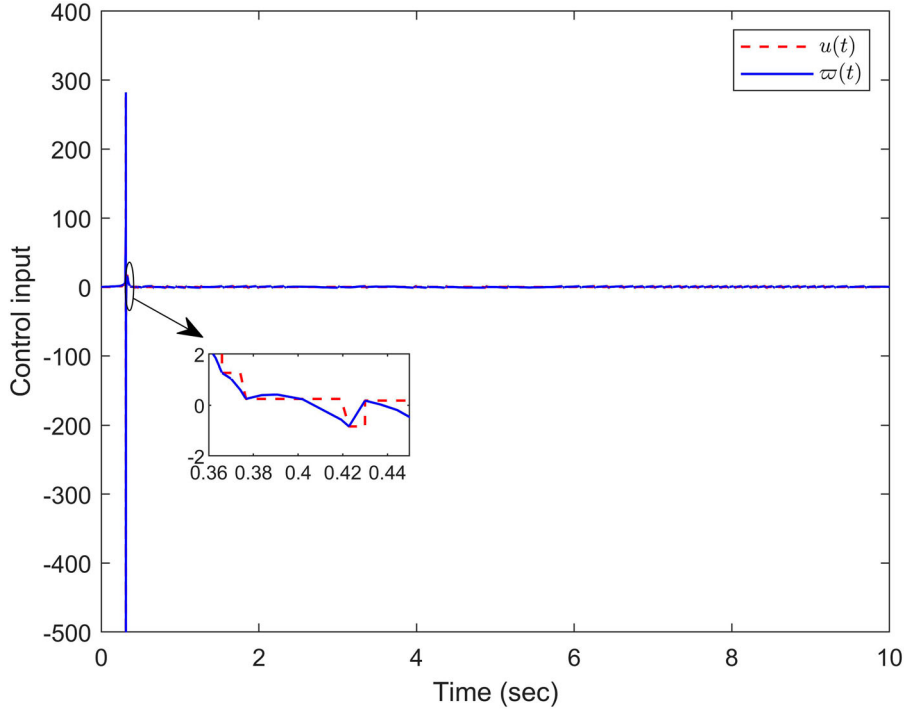
$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = ((1/R)u + cf_2) dt - 0.2x_2 d\omega \\ y = x_1 \end{cases} \quad (64)$$

where  $f_2 = -10.5 \sin(x_1) - 8x_2$ ,  $c = 0.25$  m,  $R = 0.5$  kg · m<sup>2</sup>.

In simulation, set the expectation signal  $y_r = 0.5 \sin(t)$ . The design of the command filter is the same as in Example 4.1.



**Figure 1.** System states and tracking error.



**Figure 2.** The controller  $w(t)$  and actuator  $u(t)$ .

The initial state values are assigned as  $x_1(0) = 0$  and  $x_2(0) = 0$ . All design parameters are assigned as  $\mu_{1,n} = 0.1$ ,  $k_1 = 0.01$ ,  $k_{b1} = 0.6$ ,  $\kappa_1 = 2$ ,  $\kappa_2 = 0.5$ ,  $\sigma_1 = 11$ ,  $\sigma_2 = 10$ ,  $D = 50$ ,  $\bar{M} = 20$ ,  $M = 2$ ,  $M_1 = 0.5$ ,  $\rho = 0.2$ ,  $\beta = 0.1$ ,  $\delta_1 = 40$ ,  $Q = 20$ ,  $\phi_{12} = 1$  and  $\phi_{22} = 2$ . Figures 4–6 verify the validity of the designed controller.

From Figure 4, it is easy to obtain that all system states are obtained without violating the given constraints, and the tracking error  $z_1$  is boundary and converges to near the origin, which shows a satisfactory tracking result. Figure 5 shows a significant reduction in the communication workload between the controller and the actuator, which can save network resources.

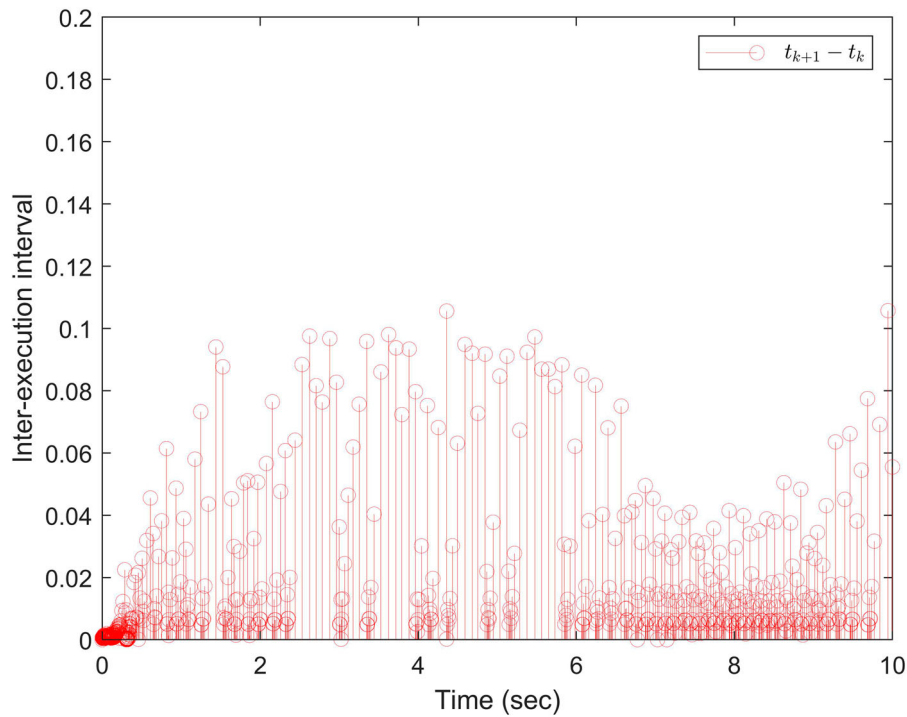


Figure 3. Inter-event times.

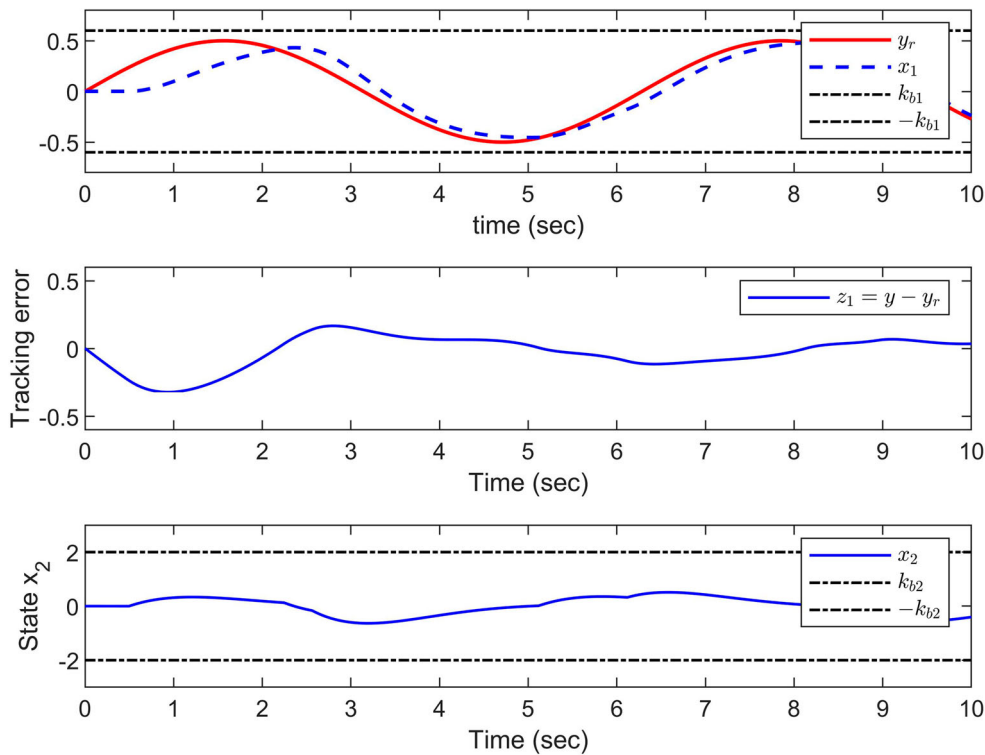
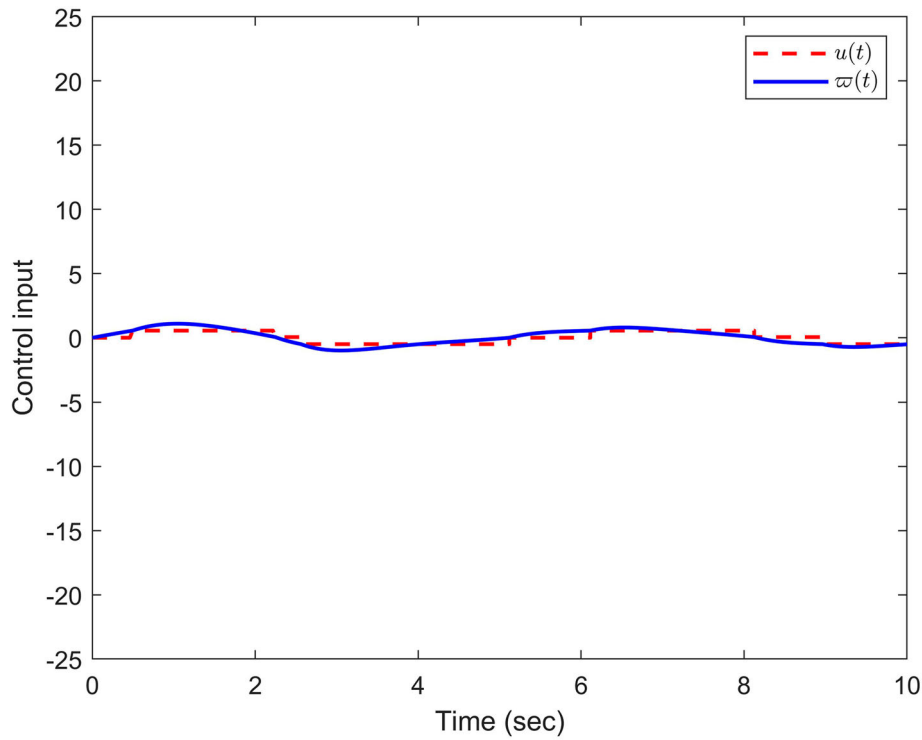


Figure 4. System states and tracking error.

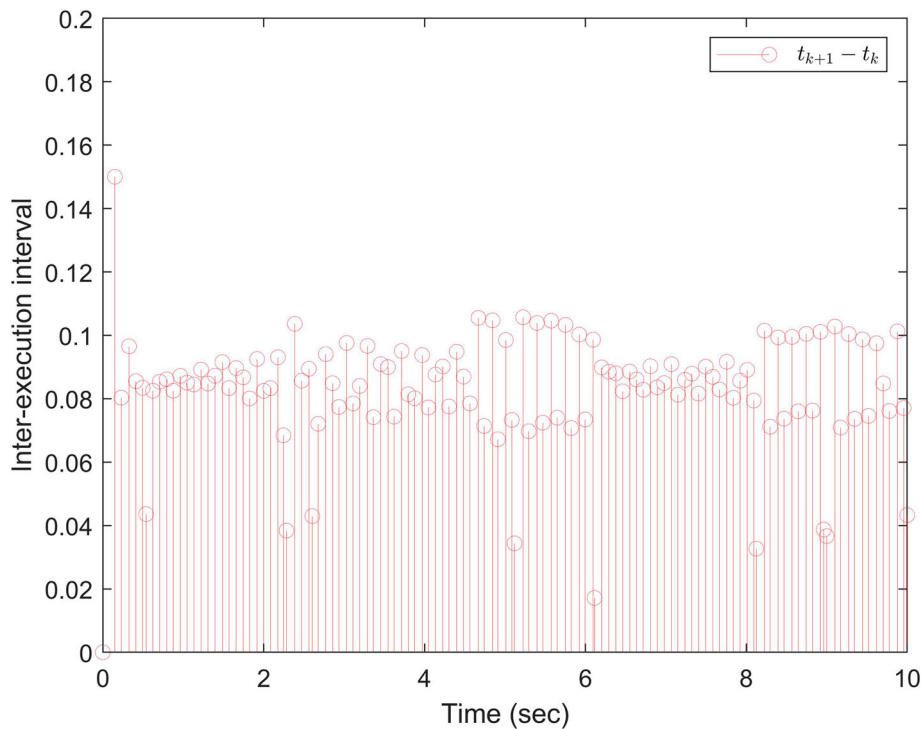
Figure 6 shows that the interval between events is greater than zero, which confirms that Zeno behaviour has been avoided successfully.

**Remark 4.1:** The aforementioned stability analysis and simulation results show that the control performance of system can be influenced by the design parameters chosen. In order

to achieve both tracking control and full-state constraints, the tracking expectation signal  $y_r$  must satisfy  $|y_r| < k_{b1}$ . The tracking performance and convergence speed can be enhanced by selecting the optimal design parameters. Therefore, in order to more successfully meet the specific control objectives, the real engineering system should be carefully adjusted to select the most appropriate parameters.



**Figure 5.** The controller  $w(t)$  and actuator  $u(t)$ .



**Figure 6.** Inter-event times.

## 5. Conclusion

This article has studied the ET adaptive finite-time tracking control for FCSNs with unknown control directions. The adaptive backstepping technique and BLF have been combined to deal with full-state constraints. Meanwhile, the command filter has been used to solve the ‘explosion of complexity’ problem and

the unknown nonlinearities have been approximated with the aid of MTNs. Moreover, the Nussbaum-type functions and a novel adaptive updated law have been used to co-design the controller and the switching-threshold-based ET mechanism, which can ensure the closed-loop system to realise the SGFSP attribute and economise communication resource. In addition, the designed controller can achieve that all the system states

have been restricted to the predefined compact sets and the tracking error was bounded and converged to satisfactory range in a finite time. At last, two examples have been given to substantiate the efficacy of the controller designed.

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