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Multi-dimensional Taylor network-based adaptive fixed-time tracking control for a class of nonlinear systems with input delay

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ABSTRACT

For the nonlinear systems with input delay, an adaptive fixed-time tracking control strategy based on multi-dimensional Taylor network (MTN) is proposed for the first time. First, to address the main challenge brought by input delay, an auxiliary system is constructed, which transforms the input delay systems into non-delay systems. Second, the MTNs are introduced into the backstepping design process, which reduce the design difficulty by approximating unknown nonlinear functions. Thus, a control scheme subject to low complexity is obtained. Third, aside from ensuring that the tracking error converges within a small neighbourhood of the origin within a fixed-time, the designed control scheme not only makes all signals in the closed-loop system remain bounded, but also avoids the problem of finite-time control relying on initial state. Finally, three simulations are given to prove the feasibility and superiority of the proposed control scheme.

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Fixed time; input delay; multi-dimensional Taylor network; backstepping technique; nonlinear system

1. Introduction

Due to the complexity of actual systems and the increasing demand for control performance and accuracy in industrial production, scholars have proposed many control methods for nonlinear systems, such as adaptive backstepping control (Chiang & Fu, 2014; Hua et al., 2016), H_∞ control (Yang et al., 2022), and sliding mode control (Li et al., 2016; Mei & Ding, 2021). Among them, adaptive backstepping control is an effective control method, which can online compensate the uncertain constant or function in the system. However, this method is difficult to deal with the control problem of systems with complex and uncertain nonlinear structures. To overcome the above problem, neural networks (NNs) and fuzzy logic systems (FLSs) are extended to the field of nonlinear system control. Up to now, a variety of adaptive control methods based on NNs or FLSs have been developed, and a lot of research results have been achieved (Li et al., 2014; Wang et al., 2014a; Wang et al., 2018). In recent years, as a new approximation method, multi-dimensional Taylor network (MTN) has attracted wide attention and been applied to various systems, such as nonlinear systems (Han, 2021; Han et al., 2021), stochastic nonlinear systems (Han, 2018; Han & Yan, 2018), switched nonlinear systems (He et al., 2023a; Zhu et al., 2020) and large-scale nonlinear systems (Chu et al., 2022). Although many achievements have been made in the research of nonlinear systems based on MTN, the above achievements do not consider both input constraints and control efficiency. Therefore, it is important to further study the fixed-time control of nonlinear systems with input delay based on MTN.

On the one hand, with the intelligence of industrial production processes, the physical constraints of the actual control systems are growing, while the demands for safety performance are also escalating. Therefore, the input delay, one of the most common physical limitations, has become one of the research hotspots. To avoid the degradation of system control performance and the damage of transient performance, a large number of control methods have been proposed for the input time-delay systems, such as predictor-based control method (Krstic, 2010), proportional integral differential (Obuz et al., 2017), Padé approximation (Li et al., 2017; Zhou et al., 2021), and constructing auxiliary systems (Ma et al., 2018; Wang et al., 2020a). Among them, Padé approximation and constructing auxiliary systems are the two most popular methods for dealing with input delay. However, due to the fact that Padé approximation is based on approximation thinking to handle input delay, it has the limitation of only dealing with small delay. Therefore, this article will compensate for the impact of input delay by constructing an auxiliary system. This approach effectively solves the awkward situation where relatively large time delay cannot be disposed by Padé approximation.

On the other hand, in industrial production processes, to improve control efficiency and achieve real-time control performance, it is usually necessary for the controlled systems to meet a certain degree of rapidity. Based on this, finite time control has been developed in controlled systems (Cui et al., 2021; Mao et al., 2023; Sui & Tong, 2023). However, the convergence time of finite-time control depends on the initial state of the system. This makes it difficult for controlled systems with unknown or

varying initial states to achieve transient performance through finite time control. The concept of fixed-time stability was first put forward in Polyakov et al., (2015), which brings hope to solve the above difficulty. Based on this, Zuo (2015a, 2015b) studied non-singular fixed-time control problem and provided a fixed-time convergence controller. After that, it is widely applied to various systems, such as nonlinear systems (Chen & Sun, 2018; Sui et al., 2023), stochastic systems (Tao et al., 2022; Yu et al., 2019), multi-agent systems (Li et al., 2022; Liu et al., 2022), switched systems (Xiang & Cui, 2023) and uncertain systems (Lai et al., 2023; Pan et al., 2020). However, as far as the author knows, there are few results on the fixed-time control of nonlinear systems with input delay under the unified framework, which also promotes the research of this paper.

Based on the above discussion, the adaptive fixed-time control problem for a class of nonlinear systems with input delay is studied in this article. The influence of input delay on the system is compensated by constructing an auxiliary system. An adaptive fixed-time control strategy is proposed via MTN approach. By comparing with existing literatures, the main contributions of this article are as follows:

- (1) For the control problem of nonlinear systems with input delay, an adaptive fixed-time control scheme based on MTN is proposed for the first time. This scheme has a simple structure, low computational complexity, and good control performance.
- (2) To transform the nonlinear system with input delay into nonlinear system without delay, a suitable auxiliary system is constructed. Different from Li et al., (2017) and Yang et al., (2020), the auxiliary system adopted in this paper can not only deal with hour delay but also deal with relatively large time delay.
- (3) This article is devoted to the fixed-time control of nonlinear systems, which has the advantages of fast convergence speed, short convergence time, high convergence accuracy, and strong anti-interference. Compared with He et al. (2023a), the control scheme does not have the defect of relying on the initial state, which indicates that the proposed scheme in this paper is more practical.

The remainder of this paper is organised as follows. Section 2 is problem formulation and preliminaries. Section 3 is the main result, which includes the process of the controller design and stability analysis. The simulation analysis of this article is presented in Section 4. This section contains three simulations, which are numerical simulation, actual simulation, and initial state comparison simulation. Section 5 summarises the whole paper and gives the future research direction.

2. Problem formulation and preliminaries

2.1 Problem formulation

The nonlinear system subject to input delay is considered, whose mathematical model is shown below

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + w_i(t) \\ i = 1, 2, \dots, n-1 \\ \dot{x}_n = u(t-\tau) + f_n(\bar{x}_n) + w_n(t) \\ y = x_1 \end{cases} \quad (1)$$

where $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i, i = 1, 2, \dots, n$ and $y \in R$ denote the state vectors and the system output, respectively. $f_i(\cdot), i = 1, 2, \dots, n$ stand for the unknown smooth nonlinear functions. $w_i(t), i = 1, 2, \dots, n$ are defined as the unknown interferences, $u(t-\tau)$ indicates the system input and τ is the input delay time.

This paper aims to design an adaptive fixed-time control strategy for nonlinear system (1) to achieve the following two objectives:

- (1) All signals in the closed-loop controlled system remain bounded.
- (2) The system output y can track the reference signal y_r , and the tracking error $y - y_r$ converges to an arbitrary small neighbourhood of the origin within a fixed time.

2.2 Definition and lemma of fixed time control

To ensure the steady-state performance of the closed-loop system, this section provides the following definition and lemma. For convenience, consider the following nonlinear system:

$$\dot{x}(t) = f(x(t)), x(0) = x_0 \quad (2)$$

in which $x(t) \in R^n$ denotes the state vector in the system and $f(\cdot)$ is defined as the nonlinear function with $f(0) = 0$.

Definition 2.1 (Zuo, Tian, et al., 2018): The system (2) is said to be fixed-time stable if it has the following properties:

- (i) the nonlinear system (2) is stable at the origin;
- (ii) there exists a finite convergence time $T_s(x_0)$, such that $x(t) = 0$, for all $t \geq T_s(x_0)$;
- (iii) the settling-time function $T_s(x_0)$ has a supremum.

Lemma 2.1 (Zuo, Han, et al., 2018): For system (2), if there are the positive real numbers $\mu_1 > 0, \mu_2 > 0, 0 < \chi < 1$ and $v > 1$, the time derivative of a continuous radially unbounded and positive definite function $V(x)$ satisfies the following inequality:

$$\dot{V}(x) \leq -\mu_1 V^\chi(x) - \mu_2 V^v(x) \quad (3)$$

The system (2) is said to be fixed time stable, and its convergence time is

$$T_s \leq \frac{1}{\mu_1(1-\chi)} + \frac{1}{\mu_2(v-1)} \quad (4)$$

Furthermore, if $0 < \chi\rho < 1, 1 < v\rho < \infty$ and $0 < \Xi < \infty$ are selected, the following inequality holds:

$$\dot{V}(x) \leq -(\mu_1 V^\chi(x) + \mu_2 V^v(x))^\rho + \Xi \quad (5)$$

Then the system (2) is practical fixed time stable.

Remark 2.1: To facilitate the derivation of the proof process, in the subsequent design process, the parameters in Lemma 2.1 will be changed $\rho = 1, \chi = 0.75, v = 2$.

2.3 Other relevant theories

To better scale the formula in the derivation process, so as to reduce the difficulty of derivation, the following lemmas are given.

Lemma 2.2 (Zhu et al., 2022): On compact set Ω_Z , for arbitrary constant $\varepsilon > 0$, continuous nonlinear function $f(Z)$ satisfies the following equation:

$$f(Z) = \theta^T P_{m_n}(Z) + \delta(Z), |\delta(Z)| < \varepsilon \quad (6)$$

where $Z = [z_1, z_2, \dots, z_n]^T \in R^n$ indicates the input vector of MTN, $\theta = [\zeta_1, \zeta_2, \dots, \zeta_l]^T \in R^l$ represents the weight vector. $P_{m_n}(Z) \in R^l$ is defined as the middle layer of MTN, its mathematical expression is as follows:

$$P_{m_n}(Z) = [z_1, z_2, \dots, z_n, z_1^2, z_1 z_2, \dots, z_n^2, \dots, z_1^m, \dots, z_n^m]^T$$

Remark 2.2: It is of interest to note that more details of MTN have been obtained in our recent work (He et al., 2023b; Li et al., 2023; Wang et al., 2023). In a nutshell, MTN can be regarded as a radial basis function neural network (RBFNN) with a special structure, and its middle layer is composed of an array of polynomials, which contains only addition and multiplication, which can effectively simplify the structure of MTN, shorten the training time, and then improve the convergence speed, and reduce the computational complexity.

Lemma 2.3 (Wang & Lin, 2015): For $\lambda \in R$ and any constant $\vartheta > 0$, one has

$$0 \leq |\lambda| - \frac{\lambda^2}{\sqrt{\lambda^2 + \vartheta^2}} < \vartheta \quad (7)$$

Lemma 2.4 (Zhu et al., 2011): For $\beta_i \in R, i = 1, 2, \dots, n$, the following inequality holds:

$$\left(\sum_{k=1}^n |\beta_k| \right)^\iota \leq \sum_{k=1}^n |\beta_k|^\iota \quad (8)$$

where $\iota \in (0, 1]$.

Lemma 2.5 (Xu, 2019): For $\varphi_i \geq 0$, the following inequality is true:

$$\left(\sum_{k=1}^m \varphi_k \right)^2 \leq m \sum_{k=1}^m \varphi_k^2 \quad (9)$$

Lemma 2.6 (Wang et al., 2021b): For $o_1 > 0, o_2 > 0, o_3 > 0, \gamma_1 \geq 0, \gamma_2 \geq 0$ and $\gamma_3 \geq 0$, the following relation holds:

$$\begin{aligned} \gamma_1^{o_1} \gamma_2^{o_2} \gamma_3 &\leq o_3 \gamma_1^{o_1+o_2} + \frac{o_2}{o_1+o_2} \\ &\times \left[\frac{o_1}{o_3(o_1+o_2)} \right]^{\frac{o_1}{o_2}} \gamma_2^{o_1+o_2} \gamma_3^{\frac{o_1+o_2}{o_2}} \end{aligned} \quad (10)$$

Lemma 2.7 (Wang et al., 2014b): For any $\xi > 0$, the lower formula holds

$$\omega_1 \omega_2 \leq \frac{\xi^p}{p} |\omega_1|^p + \frac{1}{q\xi^q} |\omega_2|^q \quad (11)$$

where $p > 1, q > 1$ and $(p-1)(q-1) = 1$.

2.4 Assumptions and auxiliary system

In this part, the assumptions are given and the auxiliary system is constructed for the input delay problem.

Assumption 2.1: The tracking target y_r and its time derivative $\dot{y}_r^{(i)}, i = 1, 2, \dots, n$ are continuously bounded.

Assumption 2.2: The interferences $w_i(t), i = 1, 2, \dots, n$ in system (1) are bounded, that is to say, there are the positive constants d_i such that

$$|w_i(t)| < d_i \quad (12)$$

To compensate the influence of input delay on the nonlinear system (1), the following auxiliary system is introduced in this paper

$$\begin{cases} \dot{\Lambda}_i = \Lambda_{i+1} - p_i \Lambda_i, & i = 1, 2, \dots, n-1 \\ \dot{\Lambda}_n = -p_n \Lambda_n + u(t-\tau) - u(t) \end{cases} \quad (13)$$

where $p_1 > \frac{1}{2}$ and $p_i > 1, i = 2, \dots, n$ are the design parameters, while $[\Lambda_1(0), \Lambda_2(0), \dots, \Lambda_n(0)]^T = [0, 0, \dots, 0]^T$ holds.

Remark 2.3: If system (13) is a system without input delay, i.e. $\tau = 0$, then the variables $\Lambda_i, i = 1, 2, \dots, n$ remain 0 in the auxiliary system whose initial state satisfies $[\Lambda_1(0), \Lambda_2(0), \dots, \Lambda_n(0)]^T = [0, 0, \dots, 0]^T$.

3. Main results

3.1 The process of controller design

First, perform the following coordinate transformation:

$$z_i = x_i - \alpha_{i-1} - \Lambda_i, i = 1, 2, \dots, n \quad (14)$$

where $\alpha_{i-1}, i = 2, \dots, n$ stand for the virtual controllers and $\alpha_0 = y_r$.

Step 1: According to (1) and (14), there is

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r - \dot{\Lambda}_1 = z_2 + \alpha_1 + f_1 + w_1 + p_1 \Lambda_1 - \dot{y}_r \quad (15)$$

For the first subsystem, the candidate Lyapunov function is constructed as follows:

$$V_1 = \frac{z_1^2}{2} + \frac{1}{2r_1} \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1 \quad (16)$$

where $\hat{\theta}_1$ denotes the estimate of θ_1 , $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ is defined as the estimate error. $r_1 > 0$ indicates a design parameter and $\Gamma_1^T = \Gamma_1 > 0$ represents a symmetric positive definite constant matrix.

Combining the above equation, the time derivative of V_1 can be expressed as

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 - \frac{1}{r_1} \tilde{\theta}_1^T \Gamma_1^{-1} \dot{\tilde{\theta}}_1 \\ &= z_1 (z_2 + \alpha_1 + \bar{f}_1 + w_1 + p_1 \Lambda_1) - z_1^2 - \frac{1}{r_1} \tilde{\theta}_1^T \Gamma_1^{-1} \dot{\tilde{\theta}}_1 \end{aligned} \quad (17)$$

where $\bar{f}_1 = f_1 - \dot{y}_r + z_1$ is an unknown nonlinear function.

In line with Lemma 2.2, for $\forall \varepsilon_1 > 0$, there is an MTN $\theta_1^T P_{m_1}(Z_1)$ with bounded error $|\delta_1(Z_1)| \leq \varepsilon_1$, such that function \bar{f}_1 satisfies the following equation:

$$\bar{f}_1 = \theta_1^T P_{m_1}(Z_1) + \delta_1(Z_1), |\delta_1(Z_1)| \leq \varepsilon_1 \quad (18)$$

On the basis of Lemma 2.7, the following inequality holds:

$$z_1 \bar{f}_1 = z_1 (\theta_1^T P_{m_1}(Z_1) + \delta_1(Z_1)) \leq z_1 \theta_1^T P_{m_1} + \frac{z_1^2}{2} + \frac{\varepsilon_1^2}{2} \quad (19)$$

From Lemma 2.7 and Assumption 2.2, it is obtained that

$$|z_1 w_1| \leq \frac{z_1^2}{2} + \frac{d_1^2}{2} \quad (20)$$

Substituting (19) and (20) into (17) to produce

$$\begin{aligned} \dot{V}_1 &\leq z_1(z_2 + \alpha_1 + p_1 \Lambda_1) + z_1 \theta_1^T P_{m_1} \\ &\quad + \frac{d_1^2}{2} + \frac{\varepsilon_1^2}{2} - \frac{1}{r_1} \tilde{\theta}_1^T \Gamma_1^{-1} \dot{\hat{\theta}}_1 \end{aligned} \quad (21)$$

Based on (21), the adaptive law α_1 and virtual controller $\hat{\theta}_1$ are designed as

$$\dot{\hat{\theta}}_1 = r_1 \Gamma_1 P_{m_1} z_1 - \Gamma_1 \eta_1 \hat{\theta}_1 - \frac{\zeta_1}{r_1} \Gamma_1 \hat{\theta}_1^3 \quad (22)$$

$$\alpha_1 = -\frac{z_1 \tilde{\alpha}_1^2}{\sqrt{z_1^2 \tilde{\alpha}_1^2 + \vartheta_1^2}} - p_1 \Lambda_1 \quad (23)$$

where $\tilde{\alpha}_1 = -\Upsilon_{11} \left(\frac{1}{2}\right)^{0.75} \frac{\Psi_{z_1}}{z_1} - \Upsilon_{12} \left(\frac{1}{2}\right)^2 z_1^3 - \hat{\theta}_1^T P_{m_1} \cdot \eta_1 > 0$, $\zeta_1 > 0$, $\vartheta_1 > 0$, $\Upsilon_{11} > 0$ and $\Upsilon_{12} > 0$ are design parameters. Ψ_{z_1} is defined as follows:

$$\Psi_{z_1} = \begin{cases} (z_1^2)^{0.75}, & |z_1| \geq \vartheta_{10} \\ \sum_{j=1}^n C_j (z_1^2)^j (\vartheta_{10}^2)^{-j+0.75}, & |z_1| < \vartheta_{10} \end{cases}$$

where $\vartheta_{10} > 0$ is a small parameter, and the coefficient $C_j, j = 1, 2, \dots, n$ can be calculated by the following equation:

$$\begin{bmatrix} 1 & 1 & \dots & 1 & 1 \\ 1 & 2 & \dots & n-1 & n \\ 0 & 2 \times 1 & \dots & (n-1)(n-2) & n(n-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \prod_{j=1}^{n-2} (n-1-j) & \prod_{j=0}^{n-2} (n-j) \end{bmatrix} \times \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{4} \left(\frac{3}{4} - 1\right) \\ \vdots \\ \prod_{j=0}^{n-2} \left(\frac{3}{4} - j\right) \end{bmatrix}$$

From Lemma 2.3, it is obtained that

$$-\frac{z_1^2 \tilde{\alpha}_1^2}{\sqrt{z_1^2 \tilde{\alpha}_1^2 + \vartheta_1^2}} < \vartheta_1 + z_1 \tilde{\alpha}_1 \quad (24)$$

According to (23) and (24), $z_1 \alpha_1$ can be reduced to the following form:

$$\begin{aligned} z_1 \alpha_1 &= -\frac{z_1^2 \tilde{\alpha}_1^2}{\sqrt{z_1^2 \tilde{\alpha}_1^2 + \vartheta_1^2}} - p_1 \Lambda_1 z_1 < \vartheta_1 + z_1 \tilde{\alpha}_1 \\ &\quad - p_1 \Lambda_1 z_1 \end{aligned} \quad (25)$$

Combining (21), (25) and $\tilde{\alpha}_1$, \dot{V}_1 can be further written as

$$\begin{aligned} \dot{V}_1 &\leq z_1(z_2 + p_1 \Lambda_1) + z_1 \hat{\theta}_1^T P_{m_1} + \frac{d_1^2}{2} + \frac{\varepsilon_1^2}{2} + \frac{1}{r_1} \tilde{\theta}_1^T \eta_1 \hat{\theta}_1 \\ &\quad + \frac{\zeta_1}{r_1^2} \tilde{\theta}_1^T \hat{\theta}_1^3 + \vartheta_1 + z_1 \tilde{\alpha}_1 - p_1 \Lambda_1 z_1 \\ &= -\Upsilon_{11} \left(\frac{1}{2}\right)^{0.75} \Psi_{z_1} - \Upsilon_{12} \left(\frac{1}{2}\right)^2 z_1^4 + z_1 z_2 + \frac{1}{r_1} \tilde{\theta}_1^T \eta_1 \hat{\theta}_1 \\ &\quad + \frac{\zeta_1}{r_1^2} \tilde{\theta}_1^T \hat{\theta}_1^3 + \sigma_1 \end{aligned} \quad (26)$$

Case 3.1. When $|z_1| \geq \vartheta_{10} \geq 0$, (26) can be transformed into the following form:

$$\begin{aligned} \dot{V}_1 &\leq -\Upsilon_{11} \left(\frac{z_1^2}{2}\right)^{0.75} - \Upsilon_{12} \left(\frac{z_1^2}{2}\right)^2 + z_1 z_2 \\ &\quad + \frac{1}{r_1} \tilde{\theta}_1^T \eta_1 \hat{\theta}_1 + \frac{\zeta_1}{r_1^2} \tilde{\theta}_1^T \hat{\theta}_1^3 + \sigma_1 \end{aligned} \quad (27)$$

where $\sigma_1 = \frac{d_1^2}{2} + \frac{\varepsilon_1^2}{2} + \vartheta_1 > 0$.

Case 3.1: When $|z_1| < \vartheta_{10}$, $\Psi_{z_1} = \sum_{j=1}^n C_j (z_1^2)^j (\vartheta_{10}^2)^{-j+0.75}$ is

substituted into (26), and it is found that there is a bounded additional term compared with (27). The bounded additional term can be seen as a small increment of σ_1 . Therefore, in the following process, only Case 3.1 will be discussed.

Step i ($2 \leq i \leq n-1$): The derivative of z_i can be obtained as follows:

$$\dot{z}_i = \dot{x}_i - \dot{\alpha}_{i-1} - \dot{\Lambda}_i = z_{i+1} + \alpha_i + f_i + w_i + p_i \Lambda_i - \dot{\alpha}_{i-1} \quad (28)$$

On the basis of V_{i-1} , the candidate Lyapunov function V_i is constructed as

$$V_i = V_{i-1} + \frac{z_i^2}{2} + \frac{1}{2r_i} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \quad (29)$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ is defined as the estimate error. $r_i > 0$ indicates a design parameter and $\Gamma_i^T = \Gamma_i > 0$ represents a symmetric positive definite constant matrix.

Combining the above equation, the time derivative of V_i can be expressed as

$$\begin{aligned} \dot{V}_i \leq & - \sum_{j=1}^{i-1} \Upsilon_{j1} \left(\frac{z_j^2}{2} \right)^{0.75} - \sum_{j=1}^{i-1} \Upsilon_{j2} \left(\frac{z_j^2}{2} \right)^2 + z_{i-1} z_i \\ & + \sum_{j=1}^{i-1} \frac{1}{r_j} \tilde{\theta}_j^T \eta_j \hat{\theta}_j + \sum_{j=1}^{i-1} \frac{\xi_j}{r_j^2} \tilde{\theta}_j^T \hat{\theta}_j^3 + \sigma_{i-1} + z_i \dot{z}_i \\ & - \frac{1}{r_i} \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\hat{\theta}}_i \end{aligned} \quad (30)$$

Substituting (28) into (30), the following inequality is correct:

$$\begin{aligned} \dot{V}_i \leq & - \sum_{j=1}^{i-1} \Upsilon_{j1} \left(\frac{z_j^2}{2} \right)^{0.75} - \sum_{j=1}^{i-1} \Upsilon_{j2} \left(\frac{z_j^2}{2} \right)^2 + \sum_{j=1}^{i-1} \frac{1}{r_j} \tilde{\theta}_j^T \eta_j \hat{\theta}_j \\ & + \sum_{j=1}^{i-1} \frac{\xi_j}{r_j^2} \tilde{\theta}_j^T \hat{\theta}_j^3 + \sigma_{i-1} + z_i (z_{i+1} + \alpha_i + \bar{f}_i + p_i \Lambda_i + w_i) \\ & - \frac{1}{r_i} \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\hat{\theta}}_i - z_i^2 \end{aligned} \quad (31)$$

where $\bar{f}_i = f_i - \dot{\alpha}_{i-1} + z_i + z_{i-1}$ stands for an unknown nonlinear function.

In line with Lemma 2.2, for $\forall \varepsilon_i > 0$, there is an MTN $\theta_i^T P_{m_i}(Z_i)$ with bounded error $|\delta_i(Z_i)| \leq \varepsilon_i$, such that function \bar{f}_i satisfies the following equation:

$$\bar{f}_i = \theta_i^T P_{m_i}(Z_i) + \delta_i(Z_i), |\delta_i(Z_i)| \leq \varepsilon_i \quad (32)$$

From Lemma 2.7, the following inequality is true:

$$z_i \bar{f}_i = z_i (\theta_i^T P_{m_i}(Z_i) + \delta_i(Z_i)) \leq z_i \theta_i^T P_{m_i} + \frac{z_i^2}{2} + \frac{\varepsilon_i^2}{2} \quad (33)$$

With the help of Lemma 2.7 and Assumption 2.2, it can be concluded that

$$|z_i w_i| \leq \frac{z_i^2}{2} + \frac{d_i^2}{2} \quad (34)$$

The adaptive law $\dot{\hat{\theta}}_i$ and virtual controller α_i are designed as follows:

$$\dot{\hat{\theta}}_i = r_i \Gamma_i P_{m_i} z_i - \Gamma_i \eta_i \hat{\theta}_i - \frac{\xi_i}{r_i} \Gamma_i \hat{\theta}_i^3 \quad (35)$$

$$\alpha_i = - \frac{z_i \tilde{\alpha}_i^2}{\sqrt{z_i^2 \tilde{\alpha}_i^2 + \vartheta_i^2}} - p_i \Lambda_i \quad (36)$$

where $\tilde{\alpha}_i = -\Upsilon_{i1} \left(\frac{1}{2} \right)^{0.75} \frac{\Psi_{z_i}}{z_i} - \Upsilon_{i2} \left(\frac{1}{2} \right)^2 z_i^3 - \hat{\theta}_i^T P_{m_i} \eta_i > 0$, $\xi_i > 0$, $\vartheta_i > 0$, $\Upsilon_{i1} > 0$ and $\Upsilon_{i2} > 0$ are design parameters, and Ψ_{z_i} is defined as follows:

$$\Psi_{z_i} = \begin{cases} (z_i^2)^{0.75}, & |z_i| \geq \vartheta_{i0} \\ \sum_{j=1}^n C_j (z_i^2)^j (\vartheta_{i0}^2)^{-j+0.75}, & |z_i| < \vartheta_{i0} \end{cases}$$

where $\vartheta_{i0} > 0$ is a small parameter, and the value of parameter C_j is the same as (23).

According to Lemma 2.3 and (36), it can be obtained as

$$z_i \alpha_i = - \frac{z_i^2 \tilde{\alpha}_i^2}{\sqrt{z_i^2 \tilde{\alpha}_i^2 + \vartheta_i^2}} - p_i \Lambda_i z_i < \vartheta_i + z_i \tilde{\alpha}_i - p_i \Lambda_i z_i \quad (37)$$

Substituting (33)–(35), (37) and $\tilde{\alpha}_i$ into (31), \dot{V}_i can be reduced to the following form:

$$\begin{aligned} \dot{V}_i \leq & - \sum_{j=1}^i \Upsilon_{j1} \left(\frac{z_j^2}{2} \right)^{0.75} - \sum_{j=1}^i \Upsilon_{j2} \left(\frac{z_j^2}{2} \right)^2 + \sum_{j=1}^i \frac{1}{r_j} \tilde{\theta}_j^T \eta_j \hat{\theta}_j \\ & + \sum_{j=1}^i \frac{\xi_j}{r_j^2} \tilde{\theta}_j^T \hat{\theta}_j^3 + \sigma_i + z_i z_{i+1} \end{aligned} \quad (38)$$

where $\sigma_i = \sigma_{i-1} + \frac{d_i^2}{2} + \frac{\varepsilon_i^2}{2} + \vartheta_i > 0$.

Step n : Let $z_n = x_n - \alpha_{n-1} - \Lambda_n$ to get the derivative of z_n as follows:

$$\begin{aligned} \dot{z}_n &= u(t - \tau) + f_n + w_n - \dot{\alpha}_{n-1} + p_n \Lambda_n - u(t - \tau) + u(t) \\ &= f_n + w_n - \dot{\alpha}_{n-1} + p_n \Lambda_n + u(t) \end{aligned} \quad (39)$$

With the help of V_{n-1} , we can construct the following candidate Lyapunov function:

$$V_n = V_{n-1} + \frac{z_n^2}{2} + \frac{1}{2r_n} \tilde{\theta}_n^T \Gamma_n^{-1} \tilde{\theta}_n \quad (40)$$

where $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$ is defined as the estimate error. $r_n > 0$ indicates a design parameter and $\Gamma_n^T = \Gamma_n > 0$ represents a symmetric positive definite constant matrix.

By combining the derivative of (40) along with (39), we can obtain the following equation:

$$\begin{aligned} \dot{V}_n \leq & - \sum_{j=1}^{n-1} \Upsilon_{j1} \left(\frac{z_j^2}{2} \right)^{0.75} - \sum_{j=1}^{n-1} \Upsilon_{j2} \left(\frac{z_j^2}{2} \right)^2 + \sum_{j=1}^{n-1} \frac{1}{r_j} \tilde{\theta}_j^T \eta_j \hat{\theta}_j \\ & + \sum_{j=1}^{n-1} \frac{\xi_j}{r_j^2} \tilde{\theta}_j^T \hat{\theta}_j^3 + \sigma_{n-1} + z_n (\bar{f}_n + w_n + p_n \Lambda_n + u(t)) \\ & - \frac{1}{r_n} \tilde{\theta}_n^T \Gamma_n^{-1} \dot{\hat{\theta}}_n - z_n^2 \end{aligned} \quad (41)$$

where $\bar{f}_n = f_n - \dot{\alpha}_{n-1} + z_n + z_{n-1}$ is an unknown nonlinear function.

In line with Lemma 2.2, for $\forall \varepsilon_n > 0$, there is an MTN $\theta_n^T P_{m_n}(Z_n)$ with bounded error $|\delta_n(Z_n)| \leq \varepsilon_n$, such that function \bar{f}_n satisfies the following equation:

$$\bar{f}_n = \theta_n^T P_{m_n}(Z_n) + \delta_n(Z_n), |\delta_n(Z_n)| \leq \varepsilon_n \quad (42)$$

According to Lemma 2.7, the following inequality holds:

$$z_n \bar{f}_n = z_n (\theta_n^T P_{m_n}(Z_n) + \delta_n(Z_n)) \leq z_n \theta_n^T P_{m_n} + \frac{z_n^2}{2} + \frac{\varepsilon_n^2}{2} \quad (43)$$

According to Lemma 2.7 and Assumption 2.2, it is obtained that

$$|z_n w_n| \leq \frac{z_n^2}{2} + \frac{d_n^2}{2} \quad (44)$$

Adaptive law and fixed time tracking controller are constructed as follows:

$$\dot{\hat{\theta}}_n = r_n \Gamma_n P_{m_n} z_n - \Gamma_n \eta_n \hat{\theta}_n - \frac{\zeta_n}{r_n} \Gamma_n \hat{\theta}_n^3 \quad (45)$$

$$u = -\frac{z_n \tilde{\alpha}_n^2}{\sqrt{z_n^2 \tilde{\alpha}_n^2 + \vartheta_n^2}} - p_n \Lambda_n \quad (46)$$

where $\tilde{\alpha}_n = -\Upsilon_{n1} \left(\frac{1}{2}\right)^{0.75} \frac{\Psi_{z_n}}{z_n} - \Upsilon_{n2} \left(\frac{1}{2}\right)^2 z_n^3 - \hat{\theta}_n^T P_{m_n} \eta_n > 0$, $\zeta_n > \gamma_2 = \sum_{j=1}^n \frac{\eta_j \tilde{\theta}_j^2}{2r_j}$ and $\gamma_3 = 1$ in Lemma 2.6, we can get the following inequality:

$$\Psi_{z_n} = \begin{cases} (z_n^2)^{0.75}, & |z_n| \geq \vartheta_{n0} \\ \sum_{j=1}^n C_j (z_i^2)^j (\vartheta_{n0}^2)^{-j+0.75}, & |z_n| < \vartheta_{n0} \end{cases}$$

where $\vartheta_{n0} > 0$ is a small parameter, and the value of parameter C_j is the same as (23).

According to Lemma 2.3 and (46), it can be obtained as

$$z_n u \leq \vartheta_n + z_n \tilde{\alpha}_n - z_n p_n \Lambda_n \quad (47)$$

Substituting (43)–(45), (47) and $\tilde{\alpha}_n$ into (41), \dot{V}_n can be reduced to the following form:

$$\begin{aligned} \dot{V}_n \leq & -\sum_{j=1}^n \Upsilon_{j1} \left(\frac{z_j^2}{2}\right)^{0.75} - \sum_{j=1}^n \Upsilon_{j2} \left(\frac{z_j^2}{2}\right)^2 + \sum_{j=1}^n \frac{1}{r_j} \tilde{\theta}_j^T \eta_j \hat{\theta}_j \\ & + \sum_{j=1}^n \frac{\zeta_j}{r_j^2} \tilde{\theta}_j^T \hat{\theta}_j^3 + \sigma_n \end{aligned} \quad (48)$$

where $\sigma_n = \sigma_{n-1} + \frac{\varepsilon_n^2}{2} + \frac{d_n^2}{2} + \vartheta_n > 0$.

Let $\bar{\mu}_1 = \min\{\Upsilon_{11}, \Upsilon_{21}, \dots, \Upsilon_{n1}\}$ and $\bar{\mu}_2 = \min\{\Upsilon_{12}, \Upsilon_{22}, \dots, \Upsilon_{n2}\}$, from Lemma 2.4 and Lemma 2.5, we get

$$-\sum_{j=1}^n \Upsilon_{j1} \left(\frac{z_j^2}{2}\right)^{0.75} \leq -\bar{\mu}_1 \sum_{j=1}^n \left(\frac{z_j^2}{2}\right)^{0.75} \leq -\bar{\mu}_1 \left(\sum_{j=1}^n \frac{z_j^2}{2}\right)^{0.75} \quad (49)$$

$$-\sum_{j=1}^n \Upsilon_{j2} \left(\frac{z_j^2}{2}\right)^2 \leq -\bar{\mu}_2 \sum_{j=1}^n \left(\frac{z_j^2}{2}\right)^2 \leq -\frac{\bar{\mu}_2}{n} \left(\sum_{j=1}^n \frac{z_j^2}{2}\right)^2 \quad (50)$$

According to Lemma 2.7, it is obtained that

$$\sum_{j=1}^n \frac{\eta_j}{r_j} \tilde{\theta}_j^T \hat{\theta}_j \leq \sum_{j=1}^n \frac{\eta_j}{2r_j} \theta_j^2 - \sum_{j=1}^n \frac{\eta_j}{2r_j} \tilde{\theta}_j^2 \quad (51)$$

Substituting (49)–(51) into (48), the following inequality is reasonable:

$$\dot{V}_n \leq -\bar{\mu}_1 \left(\sum_{j=1}^n \frac{z_j^2}{2}\right)^{0.75} - \left(\sum_{j=1}^n \frac{\eta_j \tilde{\theta}_j^2}{2r_j}\right)^{0.75} - \frac{\bar{\mu}_2}{n} \left(\sum_{j=1}^n \frac{z_j^2}{2}\right)^2$$

$$\begin{aligned} & + \left(\sum_{j=1}^n \frac{\eta_j \tilde{\theta}_j^2}{2r_j}\right)^{0.75} + \sum_{j=1}^n \frac{\eta_j}{2r_j} \theta_j^2 - \sum_{j=1}^n \frac{\eta_j}{2r_j} \tilde{\theta}_j^2 \\ & + \sum_{j=1}^n \frac{\zeta_j}{r_j^2} \tilde{\theta}_j^T \hat{\theta}_j^3 + \sigma_n \end{aligned} \quad (52)$$

Denoting $o_1 = 1 - o_2 = 0.25$, $o_2 = 0.75$, $o_3 = o_1 o_2^3$, $\gamma_1 = 1$, $\gamma_2 = \sum_{j=1}^n \frac{\eta_j \tilde{\theta}_j^2}{2r_j}$ and $\gamma_3 = 1$ in Lemma 2.6, we can get the following inequality:

$$\left(\sum_{j=1}^n \frac{\eta_j \tilde{\theta}_j^2}{2r_j}\right)^{0.75} \leq o_3 + \sum_{j=1}^n \frac{\eta_j \tilde{\theta}_j^2}{2r_j} \quad (53)$$

By using the complete cubic formula, it can be obtained that

$$\begin{aligned} \tilde{\theta}_j^T \hat{\theta}_j^3 &= \tilde{\theta}_j^T (\theta_j - \tilde{\theta}_j)^3 \\ &= \tilde{\theta}_j^T (\theta_j^3 - \tilde{\theta}_j^3 + 3\theta_j \tilde{\theta}_j^2 - 3\tilde{\theta}_j^2 \theta_j) \\ &= \tilde{\theta}_j^T \theta_j^3 - \tilde{\theta}_j^4 + 3\theta_j \tilde{\theta}_j^3 - 3\tilde{\theta}_j^2 \theta_j^2 \end{aligned} \quad (54)$$

By combining (52)–(54), (52) can be rewritten as

$$\begin{aligned} \dot{V}_n \leq & -\bar{\mu}_1 \left(\sum_{j=1}^n \frac{z_j^2}{2}\right)^{0.75} - \left(\sum_{j=1}^n \frac{\eta_j \tilde{\theta}_j^2}{2r_j}\right)^{0.75} - \frac{\bar{\mu}_2}{n} \left(\sum_{j=1}^n \frac{z_j^2}{2}\right)^2 \\ & + \sum_{j=1}^n \frac{\eta_j}{2r_j} \theta_j^2 + \sum_{j=1}^n \frac{\zeta_j \tilde{\theta}_j^3}{r_j^2} - \sum_{j=1}^n \frac{\zeta_j \tilde{\theta}_j^4}{r_j^2} + \sum_{j=1}^n \frac{3\zeta_j \theta_j \tilde{\theta}_j^3}{r_j^2} \\ & - \sum_{j=1}^n \frac{3\zeta_j \theta_j^2 \tilde{\theta}_j^2}{r_j^2} + \sigma_n + o_3 \end{aligned} \quad (55)$$

Applying Lemma 2.7, we have

$$\sum_{j=1}^n \frac{\zeta_j \tilde{\theta}_j^3}{r_j^2} \leq \sum_{j=1}^n \frac{3\zeta_j \theta_j^2 \tilde{\theta}_j^2}{r_j^2} + \sum_{j=1}^n \frac{\zeta_j \theta_j^4}{12r_j^2} \quad (56)$$

$$\sum_{j=1}^n \frac{3\zeta_j \theta_j \tilde{\theta}_j^3}{r_j^2} \leq \sum_{j=1}^n \frac{3\zeta_j \theta_j^4}{4\xi^4 r_j^2} + \sum_{j=1}^n \frac{9\zeta_j \xi^{\frac{4}{3}} \tilde{\theta}_j^4}{4r_j^2} \quad (57)$$

Substituting the above two inequalities into (55), the following inequality can be derived as

$$\begin{aligned} \dot{V}_n \leq & -\bar{\mu}_1 \left(\sum_{j=1}^n \frac{z_j^2}{2}\right)^{0.75} - \left(\sum_{j=1}^n \frac{\eta_j \tilde{\theta}_j^2}{2r_j}\right)^{0.75} - \frac{\bar{\mu}_2}{n} \left(\sum_{j=1}^n \frac{z_j^2}{2}\right)^2 \\ & + \sum_{j=1}^n \frac{\eta_j}{2r_j} \theta_j^2 + \sum_{j=1}^n \frac{\zeta_j \theta_j^4}{12r_j^2} - \sum_{j=1}^n \frac{\zeta_j \tilde{\theta}_j^4}{r_j^2} + \sum_{j=1}^n \frac{3\zeta_j \theta_j^4}{4\xi^4 r_j^2} \\ & + \sum_{j=1}^n \frac{9\zeta_j \xi^{\frac{4}{3}} \tilde{\theta}_j^4}{4r_j^2} + \sigma_n + o_3 \end{aligned}$$

$$\begin{aligned} &\leq -\bar{\mu}_1 \left(\sum_{j=1}^n \frac{z_j^2}{2} \right)^{0.75} - \bar{\mu}_1 \left(\sum_{j=1}^n \frac{\tilde{\theta}_j^2}{2r_j} \right)^{0.75} - \frac{\bar{\mu}_2}{n} \left(\sum_{j=1}^n \frac{z_j^2}{2} \right)^2 \\ &\quad - \bar{\mu}_2 \sum_{j=1}^n \frac{\tilde{\theta}_j^4}{4r_j^2} + \Xi \end{aligned} \quad (58)$$

where $\bar{\mu}_1 = (\min\{\eta_j\})^{0.75}$, $\bar{\mu}_2 = \min\left\{\left(4 - 9\xi^{\frac{4}{3}}\right)\zeta_j\right\}$, $\Xi = \sum_{j=1}^n \frac{\eta_j}{2r_j} \theta_j^2 + \sum_{j=1}^n \frac{\zeta_j \theta_j^4}{12r_j^2} + \sum_{j=1}^n \frac{3\zeta_j \theta_j^4}{4\xi^4 r_j^2} + \sigma_n + o_3$.

According to Lemma 2.5 and (58), it can be obtained that

$$\begin{aligned} \dot{V}_n &\leq -\bar{\mu}_1 \left(\sum_{j=1}^n \frac{z_j^2}{2} \right)^{0.75} - \hat{\mu}_1 \left(\sum_{j=1}^n \frac{\tilde{\theta}_j^T \Gamma_j^{-1} \tilde{\theta}_j}{2r_j} \right)^{0.75} \\ &\quad - \frac{\bar{\mu}_2}{n} \left(\sum_{j=1}^n \frac{z_j^2}{2} \right)^2 - \frac{\hat{\mu}_2}{n} \left(\sum_{j=1}^n \frac{\tilde{\theta}_j^T \Gamma_j^{-1} \tilde{\theta}_j}{2r_j} \right)^2 + \Xi \end{aligned} \quad (59)$$

where $\hat{\mu}_1 = \frac{\bar{\mu}_1}{[\max\{\lambda_{\max}(\Gamma_j^{-1})\}]^{0.75}}$, $\hat{\mu}_2 = \frac{\bar{\mu}_2}{[\max\{\lambda_{\max}(\Gamma_j^{-1})\}]^2}$.

If we make $\mu_1 = \min\{\bar{\mu}_1, \hat{\mu}_1\}$ and $\hat{\mu}_2 = \min\left\{\frac{\bar{\mu}_2}{n}, \frac{\hat{\mu}_2}{n}\right\}$, according to Lemma 2.4 and Lemma 2.5, the following equation is reasonable:

$$\begin{aligned} \dot{V}_n &\leq -\mu_1 \left[\left(\sum_{j=1}^n \frac{z_j^2}{2} \right)^{0.75} + \left(\sum_{j=1}^n \frac{\tilde{\theta}_j^T \Gamma_j^{-1} \tilde{\theta}_j}{2r_j} \right)^{0.75} \right] \\ &\quad - \hat{\mu}_2 \left[\left(\sum_{j=1}^n \frac{z_j^2}{2} \right)^2 + \left(\sum_{j=1}^n \frac{\tilde{\theta}_j^T \Gamma_j^{-1} \tilde{\theta}_j}{2r_j} \right)^2 \right] + \Xi \\ &\leq -\mu_1 V_n^{0.75} - \mu_2 V_n^2 + \Xi \end{aligned} \quad (60)$$

where $\mu_2 = \frac{\hat{\mu}_2}{2n}$.

Remark 3.1: Compared with the work of Wang et al. (2021a), this paper uses MTN to approximate nonlinear functions, which has a simpler structure and higher control accuracy. Additionally, although Wang et al. (2021a) studied the input signal quantisation, it does not consider the problem of input delay. In response to its shortcoming, the object of this study is more general.

At this point, the design process of the control policy is complete, as shown in Figure 1.

3.2 Stability analysis

Theorem 3.1: Considering the nonlinear system (1), if the intermediate virtual controller (23), (36), the adaptive fixed-time controller (46) and the adaptive law (22), (35), (45) are chosen under Assumptions 2.1–2.2, it can be ensured that all signals in the closed-loop system remain bounded and the tracking error converges to a small neighbourhood of the origin in a fixed time.

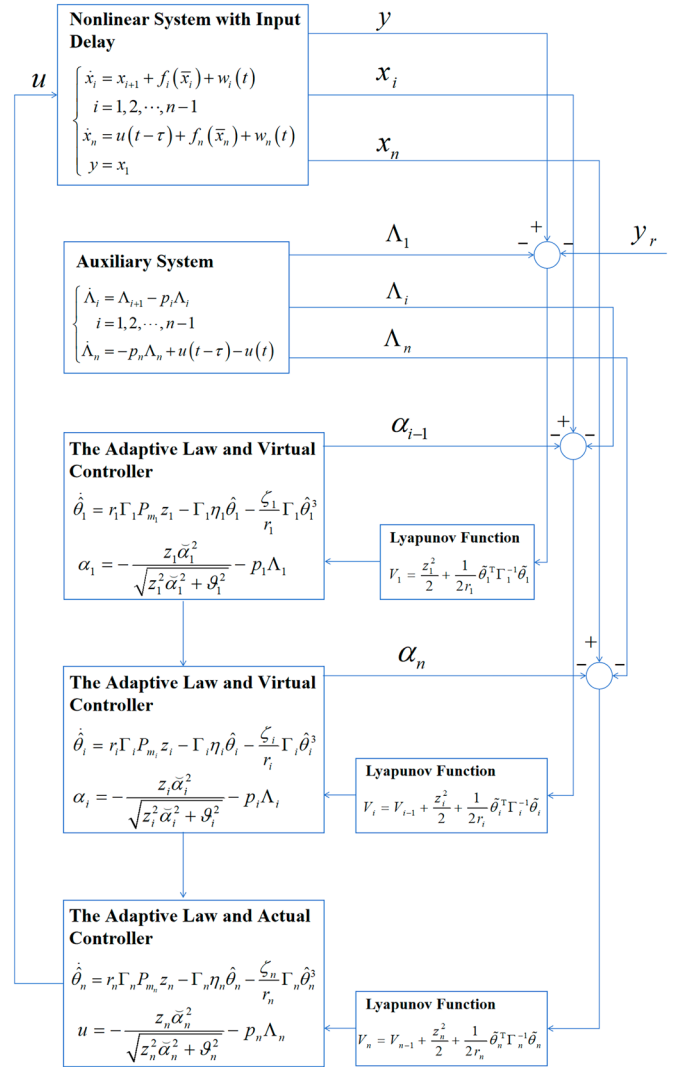


Figure 1. Control system block diagram.

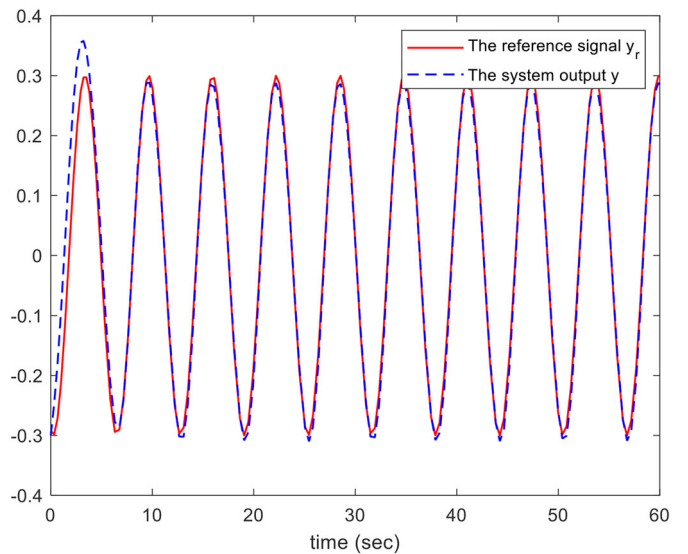


Figure 2. The reference signal y_r and the system output y .

Proof: Choosing $V = V_n$ as the Lyapunov function for the closed-loop system, then the following inequality are reasonable:

$$\dot{V} \leq -\mu_1 V^{0.75} - \mu_2 V^2 + \Xi \quad (61)$$

From (61), it can be concluded that V is bounded, since for $V^2 \geq \frac{\Xi}{\mu_2}$, $\dot{V} \leq -\mu_1 V^{0.75} - \mu_2 V^2 + \Xi \leq 0$. Then it can be inferred that z_i and $\tilde{\theta}_i$ are bounded. Since $\tilde{\theta}_i$ and θ_i are bounded, the boundedness of $\hat{\theta}_i$ can be obtained by $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. Meanwhile, according to Wang et al. (2020b), the state variables Λ_i of the auxiliary system (13) remain bounded. Based on the boundedness of Λ_i , z_i and $\hat{\theta}_i$, it can be seen from the expressions of

the virtual controllers and the actual controller constructed in this paper that both are bounded. Additionally, because $z_i = x_i - \alpha_{i-1} - \Lambda_i$ holds, x_i are bounded. In summary, all signals in the closed-loop system are bounded.

On the basis of (61), further derivation can lead to

$$\dot{V} \leq -\mu_1 V^{0.75} - \mu_2 V^2 + \varpi \mu_2 V^2 - \varpi \mu_2 V^2 + \Xi \quad (62)$$

where $0 < \varpi < 1$.

When $\Xi \leq \varpi \mu_2 V^2$, the following equation holds:

$$\dot{V} \leq -\mu_1 V^{0.75} - \mu_2(1 - \varpi)V^2 \quad (63)$$

According to Lemma 2.1, the convergence time can be determined as follows:

$$T_s \leq \frac{1}{0.25\mu_1} + \frac{1}{\mu_2(1 - \varpi)} \quad (64)$$

It can be seen that the tracking error converges to a small neighbourhood of the origin in a fixed time. ■

4. Simulation example

On the basis of the above theoretical analysis, this part further verifies the feasibility and superiority of the control scheme designed in this paper through the following three simulation examples.

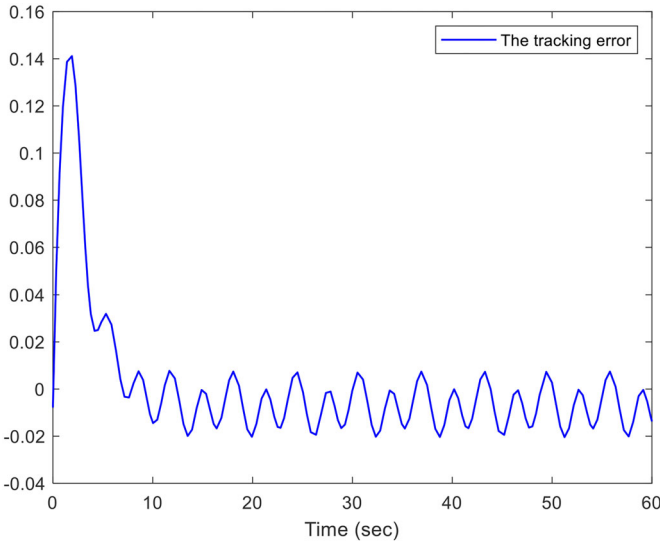


Figure 3. The tracking error $y - y_r$.

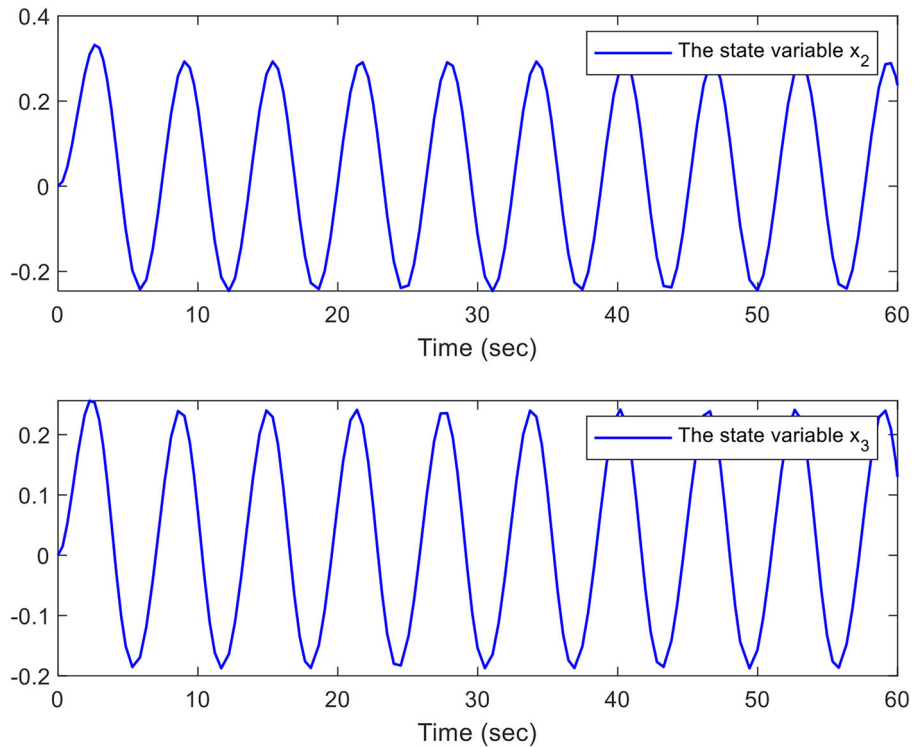


Figure 4. The state variables x_2 and x_3 .

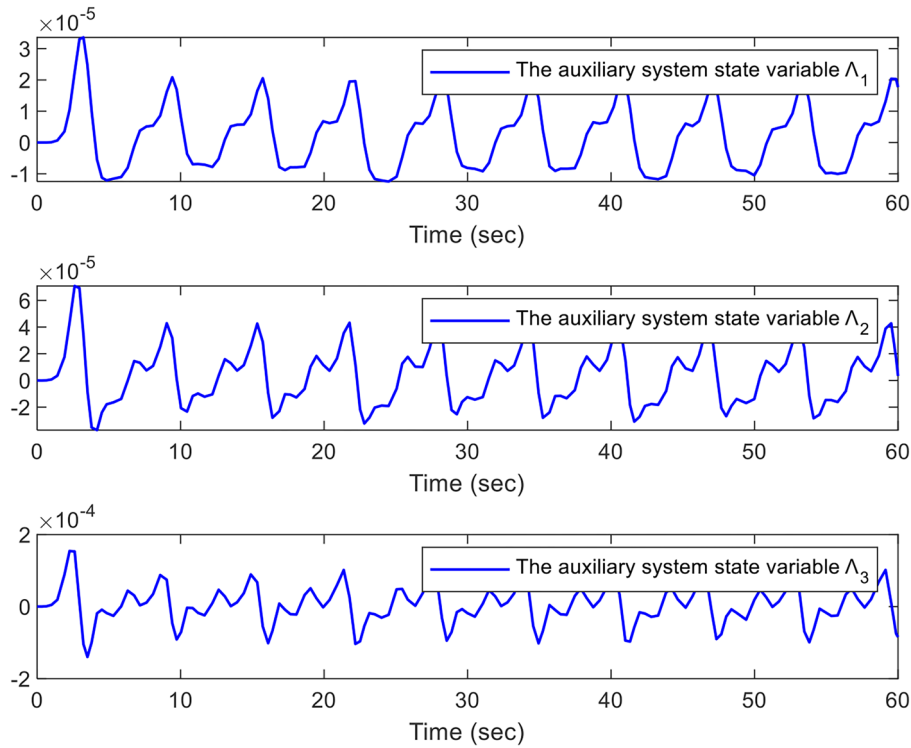


Figure 5. The auxiliary system state variables Λ_1 , Λ_2 and Λ_3 .

4.1 Numerical example

A third-order nonlinear system with input delay is considered and described mathematically as follows:

$$\begin{cases} \dot{x}_1 = x_2 - x_1 e^{x_1 - 0.5} + 0.15 \sin t \\ \dot{x}_2 = x_3 - x_2 + 0.2 \sin t \\ \dot{x}_3 = u(t - 0.1) - x_3 e^{-x_3} + 0.3 \sin t \\ y = x_1 \end{cases}$$

where the initial condition is $[x_1(0), x_2(0), x_3(0)]^T = [-0.3, 0, 0]^T$.

The following auxiliary system is introduced for this nonlin-

ear system

$$\begin{cases} \dot{\Lambda}_1 = \Lambda_2 - 1.5\Lambda_1 \\ \dot{\Lambda}_2 = \Lambda_3 - 1.5\Lambda_2 \\ \dot{\Lambda}_3 = -1.5\Lambda_3 + u(t - 0.1) - u(t) \end{cases}$$

The desired trajectory is $y_r = 0.3 \sin(t - 1.8)$. The control strategy of the nonlinear system is designed based on Theorem 3.1.

Since the selection of parameters has a great influence on the tracking effect, the following parameters are obtained through repeated attempts $r_1 = r_2 = r_3 = 1$, $\eta_1 = \eta_2 = \eta_3 =$

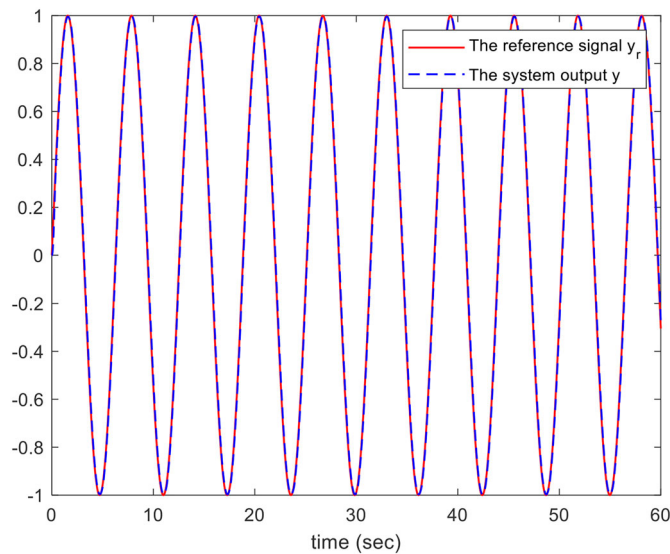


Figure 6. The reference signal y_r and the system output y .

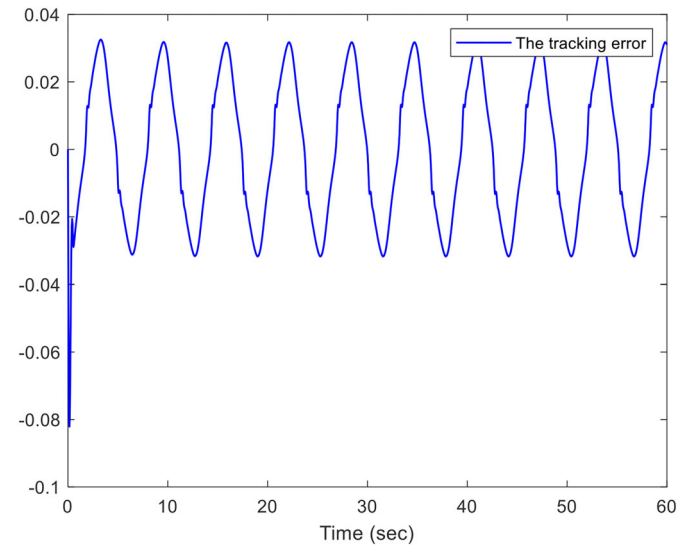


Figure 7. The tracking error $y - y_r$.

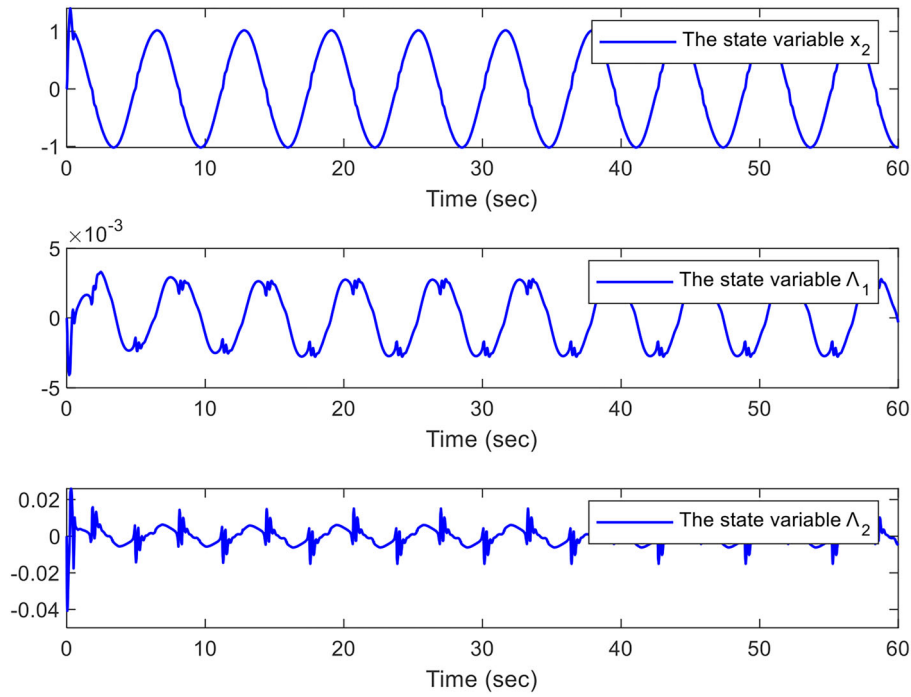


Figure 8. The state variable x_2 and the auxiliary system state variables Λ_1 and Λ_2 .

$1, \zeta_1 = \zeta_2 = \zeta_3 = 1, \Gamma_1 = I_5, \Gamma_2 = I_9, \Gamma_3 = I_{19}, \Upsilon_{11} = \Upsilon_{12} = 10, \Upsilon_{21} = \Upsilon_{22} = 1, \Upsilon_{31} = 0.1, \Upsilon_{32} = 1, \vartheta_1 = 0.01, \vartheta_2 = \vartheta_3 = 1$.

Figures 2–5 show the simulation results of the numerical simulation. Figure 2 displays the reference signal and the system output curve; Figure 3 illustrates the tracking error curve, which shows that the tracking error of the system converges to a small neighbourhood of the origin within a fixed time; Figures 4 and 5 show that the state variables $x_2, x_3, \Lambda_1, \Lambda_2$ and Λ_3 are both bounded.

4.2 Practical example

The actual inverted pendulum system can be described in the following form:

$$\begin{cases} \dot{x}_1 = x_2 - 0.2 \sin x_1 \\ \dot{x}_2 = 0.8 \sin x_1 + u(t - 0.005) \\ y = x_1 \end{cases}$$

where the initial condition means $[x_1(0), x_2(0)]^T = [0, 0]^T$.

The following auxiliary system is constructed as

$$\begin{cases} \dot{\Lambda}_1 = \Lambda_2 - 1.5\Lambda_1 \\ \dot{\Lambda}_2 = -1.5\Lambda_2 + u(t - 0.005) - u(t) \end{cases}$$

The desired trajectory is $y_r = \sin t$. The control strategy of the inverted pendulum system is designed based on Theorem 3.1.

The parameters are as follows:

$$r_1 = r_2 = \eta_1 = \eta_2 = \zeta_1 = \zeta_2 = 1,$$

$\Gamma_1 = I_5, \Gamma_2 = I_9, \Upsilon_{11} = \Upsilon_{12} = \Upsilon_{21} = \Upsilon_{22} = 10, \vartheta_1 = 0.01, \vartheta_2 = 0.001$. Figures 6–8 show the simulation results of actual

simulation. As we can see from Figure 6, the trace scheme developed in this article is quite superior. From Figure 7, it can be found out that system's tracking error converges to a small neighbourhood of the origin within a fixed time. Figure 8 illustrates the status of the system and auxiliary system, which shows that they are bounded.

4.3 Initial state comparison simulation

To demonstrate that the convergence time of fixed-time control is independent of the initial state, based on numerical simulation, the following comparative experiments is presented.

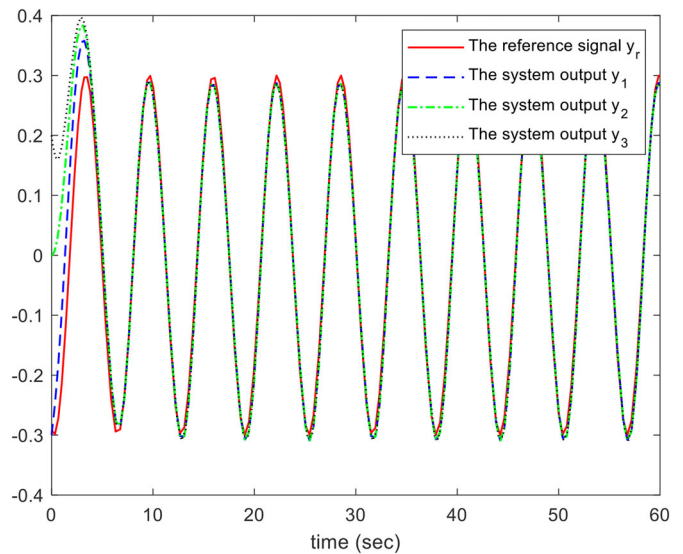


Figure 9. Tracking trajectories with three different initial states.

For the system in the numerical simulation, y_1 represents the system output with the initial state of $[-0.3, 0, 0]^T$. y_2 stands for the system output with the initial state of $[0, 0, 0]^T$. y_3 indicates the system output with the initial state of $[0.2, 0, 0]^T$. As can be seen from Figure 9, the initial state does not affect the convergence time, and the simulation effects of the three different initial states are consistent.

Remark 4.1: From the simulation results, it can be concluded that the proposed adaptive fixed-time controller has the following three advantages. (1) The controller is constructed based on MTN, with a simple structure, fast convergence speed for parameter learning, short learning time, and good real-time performance. (2) The impact of input delay is compensated by the auxiliary system, which does not compromise tracking performance. (3) The convergence time is not affected by the initial state.

5. Conclusion

The tracking control problem of nonlinear systems with input delay was studied for the first time based on MTN under a fixed-time control framework. The auxiliary system is constructed to compensate for the influence of input delay on the system, and the MTNs are used to approximate the unknown nonlinear functions. At the same time, it has been proven that the closed-loop controlled system is fixed time stable. Finally, the simulation results presented in this article also validate the superiority of the proposed scheme. It is worth noting that the results of this paper are obtained under the assumption that the output of the system is unconstrained, but many real systems have output constraints. Therefore, the future research direction is fixed time control of nonlinear systems subject to input and output constraints.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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