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New Design to Adaptive Neural Asymptotic Tracking Control for a Class of Uncertain Stochastic Nonlinear Systems With Unknown Input Constraints

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ABSTRACT

In this paper, we present a new scheme to design an adaptive neural backstepping tracking controller for a class of stochastic nonlinear systems with unknown input constraints including saturation and dead-zone. The control design is achieved by using certain auxiliary techniques. More specifically, some piecewise injective differential functions are constructed to approximate these nonlinearity constraints with a bounded approximation error. A new dimension reduction inequality related the norm of basis vectors of radial basis functions is established to design the virtual signals. Some negative power exponential functions and the inverse functions of the above injective differentiable functions are introduced to design the adaptive laws and controller, respectively. These techniques can enlarge the nonlinear systems currently studied by backstepping approach and neural networks. Furthermore, it is shown that all the signals in the closed-loop system are semi-globally uniformly, ultimately bounded in probability and the tracking error converges to zero. Meanwhile, the effectiveness of the proposed controller is demonstrated in the simulation study.

1 | Introduction

During the last two decades, many practical systems have been regarded as uncertain nonlinear stochastic systems. The control problem of such systems has always been one of the hot issues due to the existence of uncertainty and randomness in the modern control field. Especially, the topic on adaptive tracking control design for these systems with unknown nonsmooth nonlinear plant input constraints by using backstepping techniques and radial basis function neural networks (RBFNNs) or fuzzy logic systems has attracted many researchers from the control community. Many control schemes for these systems have been developed, such as [1–15], and the references therein. Here, a detailed presentation of prior works, motivations, and advances on this direction of research is omitted. For more information on this topic, the reader can refer to [16–18] for stochastic systems and their stability [19, 20], for adaptive control and backstepping techniques [21–25], for NNs or fuzzy logic systems-based control strategies of uncertain nonlinear systems and [7–15, 26–30] for some techniques to deal with nonsmooth nonlinearity input constraints. Although a number of research results have been obtained in this control

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field, there are two sources that motivate us to carry out this research work.

First, considering the stability of the proposed control schemes for uncertain nonlinear stochastic systems such as [1–15] and [26–28], we see that Itô correction term (high-order Hessian term) is commonly handled by applying Young's inequality to enlarge it. As a result, this method finally generates a nonzero constant term in the right side of the enlarged derivative of Lyapunov function candidate. Such nonzero constant maybe lead to that most developed control schemes only guaranteeing outputs converge to a neighborhood of the desired trajectories, but cannot converging to zero by Lyapunov stability theorems. Noting that it is inevitable that the developed control schemes are of semi-global stability, because the universal approximation property of RBFNNs or fuzzy logic systems is valid only for any given compact set. Therefore, asymptotic stability has been further considered by many researchers. The reader can refer to [31–35] and the references therein for more information on asymptotic tracking control design for uncertain nonlinear systems. To the authors' knowledge, asymptotic tracking control design for nonlinear stochastic system seems to have not been completely solved owing to the fact that Itô correction term makes the controller design much more difficult than that of the uncertain systems in [31–36]. For example, Remark 1 in [10], Remark 8 in [37], and [38] has already explained this issue. Therefore, it is necessary to design effective adaptive neural asymptotical tracking control strategies for stochastic systems in theoretical research and application fields.

Second, based on previous research findings, including [1–15] and the references therein, it can also be found that the most controllers were designed by a similar method. This method expresses the plan input constraints as a combination of a linear term (with time-varying gain) and a disturbance-like term under certain conditions, and then enlarges the derivative of the employed Lyapunov function step by step until the linear term of control input appears for the controller design. Such normal method is concise and ensures that the designed controller guarantees stability of the system in use. However, it is also occasionally seen that the same controllers are designed for the studied system subject to different nonsmooth nonlinear inputs by this method. For example, the controller of the employed system with an actuator hysteresis [14] was selected as the same as that of an actuator dead-zone [14] (Eq. 94). The controller for a system with dead-zone input [15] (Eq. 47) is also the same as that of another system with saturation input [39] (Eq. 43). Maybe such cases should be taken into account during the design phase of an experiment when practitioners in some practical application fields hope that the differences of the designed controller can be reflected if the studied system is subject to different input constraints.

Motivated by the above two sources, we intend to address the following three questions. Specifically, we aim to develop an adaptive tracking control scheme for uncertain stochastic nonlinear systems with unknown plant input constraints. The goal of this scheme is to ensure that the tracking error converges to zero. Additionally, we design the adaptive controllers in such a way that they can reflect differences in the studied system if it has different nonsmooth input constraints.

- *Question 1*: Can a method for dealing with Itô correction term be given so that the proposed adaptive tracking scheme can ensure the semi-global asymptotic stability of the studied stochastic system with input constraints when backstepping approach and RBFNNs or fuzzy logic systems are used to design the control scheme?
- *Question 2*: Can a method for designing controller be complemented so that the designed controllers can reflect their differences when the studied system is subject to the different nonsmooth input constraints?
- *Question 3*: Compared with common methods, can a design method be provided so that it is valid for more complex uncertain stochastic nonlinear systems under mild conditions when backstepping approach and RBFNNs or fuzzy logic systems are used in adaptive tracking control design?

For the above questions, we come up with following auxiliary techniques (I), (II) and (III) to discuss them, which can be regarded as the main contributions of this paper.

(I) For *Question 1*, the following two techniques are used to overcome the difficulties caused by Itô correction term that influence asymptotic tracking control design from the 1th step to $(n-1)$th step of iterative backstepping processes. (i) The right side of the enlarged differential operator associated with the studied system is expressed as a quadratic virtual error multiplied by an unknown continuous function. (ii) A negative power exponential function is introduced to design adaptive laws. In the nth step, the obstacle caused by Itô correction term in asymptotic tracking design is also handled by addition or subtraction terms and dimensionality reduction techniques. Based on these works, a new adaptive neural asymptotic tracking scheme is finally developed under mild conditions.

(II) For *Question 2*, according to the idea of horizontal and oblique asymptotes, certain injective differentiable function models are constructed to approximate some nonsmooth nonlinearity inputs including saturation, dead-zone, and saturation & dead-zone nonlinearity with bounded errors. These constructed models are further designed in the backstepping design process. And the controllers are finally selected as the inverse functions of the designed models. The proposed technique is a valuable complement to the common method to design controller, which can avoid designing the same controller for the studied system with different input constraints and also reflects more features of the plant input nonsmooth nonlinearity.

(III) For *Question 3*, a new inequality related to the norm of Gaussian functions vector in RBFNNs is proposed by the aid of Lemma 1 in [40] or Corollary 4.2 in [41], which plays a role of dimensionality reduction and is led into the design of virtual control function. By combining this inequality with a technique for adding and subtracting appropriate state variables in backstepping design process, the differentiable virtual control functions and adaptation laws are directly designed under very relaxed assumptions. These methods not only ensure that the backstepping approach can be implemented owing to the fact that backstepping approach requires all functions to be differentiable but also are valid for more complex systems under mild conditions.

With the help of the above auxiliary techniques (I), (II), and (III), a new adaptive semi-global asymptotic tracking control design scheme is developed for a class of uncertain stochastic systems with some unknown system input constraints by using backstepping approach and RBFNNs.

In order to describe the development of this control scheme in detail, the remainder of this paper is organized as follows. Section 2 presents the problem statement and some preliminaries including certain injective differentiable function models related to some constraints nonlinearity, universal approximation of RBFNNs, and a stability theorem for stochastic system. Section 3 develops an adaptive asymptotic tracking control design scheme for uncertain stochastic systems based on backstepping approach, RBFNNs and the aforementioned auxiliary techniques (I), (II), and (III). In Section 4, a class of second-order stochastic nonlinear system with any saturation constraint or dead-zone is introduced to illustrate the effectiveness of these method developed in this paper. Finally, some conclusions are presented in Section 5.

2 | Problem Statement and Preliminaries

Throughout this paper, let $\mathscr{R}$ and $\mathscr{R}^n$ be the real number field and the dimensional real vector space, respectively.

2.1 | System Description and Control Objective

Consider the following uncertain stochastic nonlinear system

$$\begin{cases} d\zeta_i = \left(f_i\left(\overline{\zeta}_{i+1}\right) + d_i\left(t, \overline{\zeta}_n\right)\right)dt + g_i^{\mathrm{T}}\left(\overline{\zeta}_{i+1}\right)d\omega \\ \quad i = 1, 2, \ldots, n-1 \\ d\zeta_n = \left(f_n\left(\overline{\zeta}_n\right) + d_n\left(t, \overline{\zeta}_n\right) + h\left(\overline{\zeta}_n\right)\, u\right)dt + g_n^{\mathrm{T}}\left(\overline{\zeta}_n\right)d\omega \\ \quad y = \zeta_1, \\ \quad u = u(\upsilon(t)) \end{cases} \tag{1}$$

where $\zeta_i \in \mathscr{R}$ are state variables, and $\overline{\zeta}_i = [\zeta_1, \zeta_2, \ldots, \zeta_i]^{\mathrm{T}} \in \mathscr{R}^i$ are state vectors. ω is an r-dimensional variable introduced as standard Brownian motion defined on a complete probability space. $f_i : \mathscr{R}^{i+1} \to \mathscr{R}$, $g_i : \mathscr{R}^{i+1} \to \mathscr{R}^r$ and $h : \mathscr{R}^n \to \mathscr{R}$ are unknown nonlinear differentiable functions which satisfy locally Lipschitz condition and $f_i(0) = 0$, $g_i(0) = 0$. d_i are unknown time-varying disturbances and also satisfy locally Lipschitz condition. $y \in \mathscr{R}$ and $u \in \mathscr{R}$ represent the input and output of the system, respectively.

In this paper, $u = u(\upsilon(t))$ can be taken as the following two nonlinear constraints, including the saturation nonlinearity $u_s(\upsilon(t), k)$, and the dead-zone $u_d\left(\upsilon(t), k_1, k_2\right)$, where

$$u_s(\upsilon(t), k) = \begin{cases} ka, & \upsilon(t) \le a \\ k\ \upsilon(t), & a \le \upsilon(t) \le b \\ kb, & b \le \upsilon(t) \end{cases} \tag{2}$$

$$u_d\left(\upsilon(t), k_1, k_2\right) = \begin{cases} k_1(\upsilon(t) - a), & \upsilon(t) \le a \\ 0, & a \le \upsilon(t) \le b \\ k_2(\upsilon(t) - b), & b \le \upsilon(t) \end{cases} \tag{3}$$

and $c < a < 0$, $0 < b < d$, $k > 0$, $k_1 > 0$ and $k_2 > 0$.

Remark 1. It is crucial to underscore that the aforementioned system (1) can encapsulate a wide range of practical systems, including the Brusselator model [26, 42], the one-link robot arm system [28], and the pendulum system equipped with a motor. Consequently, the investigation of stochastic nonlinear systems described as (1) holds immense significance.

Remark 2. In fact, the system (1) can be seen as a class of nonstrict feedback stochastic systems with unknown control gain, unknown disturbances and input nonlinearities. Notice that the plant input u can represent any of the three input nonlinearities including input saturation, dead-zone and saturation & dead-zone, and the system (1) is thus more general than the known systems in the literatures such as [1, 11, 38, 43, 44]. Furthermore, it is well known that many practical systems are often subjected to unknown disturbances and input constraints [11, 43, 45]. Therefore, the investigation of the system (1) is also meaningful.

The control objective is to develop adaptive tracking control scheme such that the designed scheme guarantees that all the signals in the closed-loop system are of semi-global asymptotic tracking stability for the system (1) with plant input constraint (2) or (3).

In order to complete the control objective, the following assumptions are introduced.

Assumption 1. The function d_i are smooth and bounded. Therefore, we can further assume that there exist unknown positive constants M_{d_i} such that $\left|d_i\left(t, \overline{\zeta}_n(t)\right)\right| < M_{d_i}$ for $i = 1, 2, \ldots, n$.

Assumption 2 ([46]). The desired trajectory y_d and its ith order derivatives are assumed to be continuous and bounded, denoted by $\left|y_d^{(i)}(t)\right| \le M_{y_d}$ for $1 \le i \le n$ and $M_{y_d} > 0$.

Assumption 3. The nonlinear function h is unknown, but its sign is known, and there exist positive constants a_m and a_M such that $a_m < \left|h\left(\overline{\zeta}_n\right)\right| < a_M$. Without loss of generality, it is further assumed that $h\left(\overline{\zeta}_n\right) > 0$.

Assumption 4 ([11]). If the system (1) with the input dead-zone (3), then there exist unknown positive constants k_{12} and k_{21}, such that $0 < k_{12} < k_1 < k_{21}$, $0 < k_{12} < k_2 < k_{21}$.

Remark 3. Assumptions 1 and 2 are standard conditions for the external disturbances and desired trajectory of the system, which have been widely used in [46, 47]. Assumptions 3 and 4 delineate the viable prerequisites for the control method proposed in this paper, which are standard assumptions in existing literature. Assumption 3 is supported by the evidence presented in [42], whereas Assumption 4 is corroborated by the findings in [11].

2.2 | Injective Differentiable Function to Approximate (2) and (3)

According to the ideas of the horizontal and oblique asymptote of function, the following function $u_{sm}(\upsilon(t))$ and $u_{dm}\left(\upsilon(t), k_1, k_2\right)$ are established to approximate $u_s(\upsilon(t))$ and $u_d\left(\upsilon(t), k_1, k_2\right)$, respectively.

$$u_{sm}(\upsilon(t)) = \begin{cases} r_a + \frac{\upsilon(t)-r_a}{1+r_a-\upsilon(t)}, & \upsilon(t) \le r_a \\ \upsilon(t), & r_a \le \upsilon(t) \le r_b \\ r_b + \frac{\upsilon(t)-r_b}{1-r_b+\upsilon(t)}, & r_b \le \upsilon(t) \end{cases} \tag{4}$$

$$u_{dm}\left(\upsilon(t), k_1, k_2\right) = \begin{cases} k_1\left[-r_a^2 - 2r_a(\upsilon(t) - r_a)\right], & \upsilon(t) \le r_a \\ -k_1\upsilon^2(t), & r_a \le \upsilon(t) \le 0 \\ k_2\upsilon^2(t), & 0 \le \upsilon(t) \le r_b \\ k_2\left[r_b^2 + 2r_b(\upsilon(t) - r_b)\right], & r_b \le \upsilon(t) \end{cases} \tag{5}$$

where r_a, r_b, r_c, r_d are constants and satisfy $r_c < r_a < 0$ and $0 < r_b < r_d$.

According to the definition of derivative of function, the following Lemma 1 is clearly true, so its proof is omitted.

Lemma 1. *Let $u_{sm}(\upsilon(t))$ and $u_{dm}(\upsilon(t))$are injective differentiable functions, respectively. Furthermore, there exist positive constants M_s and M_d, such that*

$$\begin{cases} u_s(\upsilon(t)) = k_{12}u_{sm}(\upsilon(t)) + \delta_s(\upsilon(t)) \\ u_d\left(\upsilon(t), k_1, k_2\right) = u_{dm}\left(\upsilon(t), k_1, k_2\right) + \delta_d(\upsilon(t)) \end{cases}$$

where $\left|\delta_s(\upsilon(t))\right| \le M_s$ and $\left|\delta_d(\upsilon(t))\right| \le M_d$.

Remark 4. To the authors' knowledge, the hyperbolic tangent function or Gaussian function is commonly used to approximate the saturation nonlinearity [48, 49]. The function formed by a linear term of control input with a bounded time-varying gain plus a disturbance-like term is used to approximate the dead-zone nonlinearity [10–12], as well as saturation & dead-zone nonlinearity [44, 50] under certain conditions. The functions (4) and (5) can be used to approximate any given input saturation (2) and dead-zone (3), respectively. Thus Lemma 1 is a useful complement to approximate models of the saturation, dead-zone, as well as saturation & dead-zone nonlinearity. In addition, the reader may refer to the monograph [51] and the references therein for a detailed understanding of the approximate model related backlash or hysteresis nonlinearity.

Remark 5. In this paper, the functions $u_{sm}(\upsilon(t))$ and $u_{dm}\left(\upsilon(t), k_1, k_2\right)$ can be used to substitute (2) and (3) in nth step of backstepping design process, respectively. Therefore, these functions will be designed by combining Lyapunov stability theorem with the enlarged derivative of Lyapunov function candidate, respectively. Subsequently, the corresponding controller can be further obtained by finding inverse function of the designed $u_{sm}(\upsilon(t))$, $u_{dm}(\upsilon(t))$ or $u_{sdm}(\upsilon(t))$. Based on these designed functions, such designed controller can reflect a distinguishing feature if the system (1) has the input (2) or (3).

Remark 6. Consider the stochastic nonlinear system (1) with the unknown dead-zone input (3). The unknown constants k_1 and k_2 in the dead-zone nonlinearity (3) are unable to use to design controller, so the following auxiliary result is provided to design controller $\upsilon(t)$ via the design of $u_{dm}(\upsilon(t)) = u_{dm}(\upsilon(t), 1, 1)$.

Lemma 2. *If the system (1) with the unknown dead-zone input (3), and a function $\varsigma(t)$ satisfies $\varsigma(t)\ u_{dm}(\upsilon(t)) \le 0$, then*

$$\varsigma(t)u_{dm}\left(\upsilon(t), k_1, k_2\right) \le k_{12}\ \varsigma(t)u_{dm}(\upsilon(t)) \tag{6}$$

where $k_{12} > 0$ is a constant.

Proof. In view of Assumption 4, (5) and $\varsigma_n(t)\ u_{dm}(\upsilon(t)) \le 0$, it is not difficult to get $\varsigma(t)\left(u_{dm}(\upsilon(t), k_1, k_2\right) - k_{12}u_{dm}(\upsilon(t)) \le 0$. Thus, the conclusion is valid. □

2.3 | Universal Approximation of RBFNN

Lemma 3 ([49]). *Let $f(Z)\ : \mathscr{R}^n \to \mathscr{R}$ be a continuous function over the compact set $\Omega \subset \mathscr{R}^n$. Let $\varepsilon > 0$ be any given accuracy. Then, there is a RBFNN $W^{*\mathrm{T}}S(Z)$ such that*

$$f(\boldsymbol{Z}) = \boldsymbol{W}^{*\mathrm{T}}S(\boldsymbol{Z}) + \delta(\boldsymbol{Z}) \tag{7}$$

where $\boldsymbol{Z} \in \Omega_Z \subset \mathscr{R}^q$ with $q \le n$ is the input vector, and $\delta(Z)$ denotes the smallest approximation error satisfying $|\delta(Z)| < \varepsilon$. $\boldsymbol{W}^* = \left[w_1^*, w_2^*, \dots, w_l^*\right]^{\mathrm{T}} \in \mathscr{R}^l$ with $l > 1$ means the ideal weight vector which is defined as

$$\boldsymbol{W}^* = \underset{\boldsymbol{W}\in\mathscr{R}^l}{\arg\min}\left\{ \sup_{Z\in\Omega_Z} \left| f(\boldsymbol{Z}) - \boldsymbol{W}^{\mathrm{T}}S(\boldsymbol{Z}) \right| \right\}$$

where $W = \left[\omega_1, \omega_2, \dots, \omega_l\right]^{\mathrm{T}} \in \mathscr{R}^l$ denotes the updated weight vector. l is the number of NN nodes and $S(\boldsymbol{Z}) = \left[s_1(\boldsymbol{Z}),\ s_2(\boldsymbol{Z}), \dots, s_l(\boldsymbol{Z})\right]^{\mathrm{T}} \in \mathscr{R}^l$ is the basis function vector with $s_i(\boldsymbol{Z})$ being the form of Gaussian function as follows

$$s_i(\boldsymbol{Z}) = \exp\left[-\frac{\left(\boldsymbol{Z} - \boldsymbol{c}_i\right)^{\mathrm{T}}\left(\boldsymbol{Z} - \boldsymbol{c}_i\right)}{\eta_i^2}\right], i = 1, 2, \dots, l$$

where $\boldsymbol{c}_i = \left[c_{i1},\ c_{i2}, \dots, c_{in}\right]^{\mathrm{T}} \in \mathscr{R}^n$ and $\eta_i > 0$ are the center and width of the neural cell of the i-th hidden layer, respectively.

Assumption 5. The ideal weight $\boldsymbol{W}^*$ related to f in Lemma 3 is unknown and bounded.

Remark 7. Assumption 5 typically appears in the design of NN backstepping controllers such as [49, 52, 53] and the references therein, which is only used for stability analysis and is not required when designing the virtual signals and adaptive control laws. Thus, the ideal weight in Lemma 3 can be unknown bounded constants.

According to [40] and [41], let s_Z^* denotes the upper bound on the 2-norm of vector $S(Z)$ introduced in Lemma 1 in [40], noting that $1 \leq s^*_{\overline{Z}_i}\left(\left\|S\left(\overline{Z}_i\right)\right\|\right)^{-1}$ holds from Lemma 1 in [40], where $Z = \left[\check{Z}_1, \check{Z}_2, \ldots, \check{Z}_q\right]^{\mathrm{T}}$ and $\overline{Z}_i = \left[\check{Z}_1, \check{Z}_2, \ldots, \check{Z}_i\right]^{\mathrm{T}}$, $i = 1, 2, \ldots, q$. Thus, the following Lemma 4 is valid.

Lemma 4. *For a given RBFNN, then we have*

$$\|S(Z)\| \leq s^*_{\overline{Z}_i} s^*_Z \left(\left\|S\left(\overline{Z}_i\right)\right\|\right)^{-1} \tag{8}$$

Remark 8. It is clear that Lemma 4 plays a role to reduce the dimension of state vector. Combining a method of adding and subtracting appropriate state variable with Lemma 4, the differentiable virtual control functions and adaptive laws can be easily designed, which ensure backstepping approach can be repeated until the nth step in backstepping process. Furthermore, the studied systems (1) also can be extended to more complex systems by using such techniques under mild conditions. In addition, noting that $\left\|S\left(\overline{Z}_i\right)\right\| \leq 1$ for any vector $\overline{Z}_i$, then $\|S(Z)\| \leq \left\|S\left(\overline{Z}_i\right)\right\|$. Such inequality has been used to design NN adaptive controllers such as Lemma 3 in [38] and Lemma 3 in [54]. Therefore, Lemma 4 also provides a new method for adaptive tracking controller design.

2.4 | Stability Theorem

Let $(\Omega, \Gamma, \mathscr{P})$ denote a probability space, where Ω, Γ, and $\mathscr{P}$ are the set of samples, the designated collection of subsets of Ω, and the probability measure, respectively. For a time-varying stochastic nonlinear system,

$$d\zeta = f(t, \zeta)dt + h(t, \zeta)dw \tag{9}$$

where $\zeta = \left[\zeta_1, \zeta_2, \ldots, \zeta_n\right]^{\mathrm{T}} \in \mathscr{R}^n$ is the state vector, and w is a r-dimensional independent standard Wiener process. f and g are vector-value or matrix-value functions, respectively.

Definition 1 ([26]). The solution process $\{\zeta(t),\ t \geq 0\}$ of the system (9) is said to be bounded in probability if $\lim_{c\to\infty} \sup\limits_{0\leq t<\infty} P\{\|\zeta(t)\| > c\} = 0$, where $P(A)$ denotes the probability of event A.

Definition 2 ([26]). Let $V(t,\ \zeta)$ be a function acting on $\mathscr{R} \times \mathscr{R}^n$, then $V(t,\ \zeta)$ is said to be radially unbounded if that $V(t,\ \zeta) \to \infty$ as $|\zeta| \to \infty$ for $t \geq 0$.

Definition 3 ([43]). If $V(t,\ \zeta)$ is a positive definite, radially unbounded and continuously once differentiable in t and twice in ζ, then $\mathcal{L}V(t, \zeta)$, a differential operator associated with (9), is defined as

$$\mathcal{L}V(t, \zeta) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2}\mathrm{Tr}\left(h^{\mathrm{T}} \frac{\partial^2 V}{\partial x^2} h\right) \tag{10}$$

where $\mathrm{Tr}(A)$ is the trace of a matrix A.

Remark 9. As stated in [26, 37], the term $\frac{1}{2}\mathrm{Tr}\left(h^{\mathrm{T}} \frac{\partial^2 V}{\partial x^2} h\right)$ is called Itô correction term or high-order Hessian term. It is common knowledge that this term is treated with Young's inequality in the process of adaptive controller design, such as [1–15], and their related literature. However, this method usually generates a nonzero constant term in the right end of the enlarged derivative of Lyapunov function of the studied system, which leads to the most developed control schemes that only guarantee outputs converge to a neighborhood of the desired trajectories, but cannot converge to zero. How to deal with the Itô correction term so that the adaptive tracking control scheme based on backstepping approach and RBFNNs or fuzzy logic systems is of semi-global asymptotic stability for the stochastic system with input constraints is still a noteworthy issue in the modern control field.

In order to analyze the stability of control design, a stability theorem of nonlinear stochastic system is also listed as Lemma 5, which is a corollary of Theorem 2.1 in [17].

Lemma 5 ([17]). *Let $t \geq 0$, $W(\zeta) : \mathscr{R}^n \to \mathscr{R}$ is a continuous and nonnegative function, for the stochastic nonlinear system (9), if there exists positive constants a_0 and b_0, such that the differential operator $\mathcal{L}V(t, \zeta)$ associated with (9) satisfies that*

$$\mathcal{L}V(t, \zeta) \leq -a_0 W(\zeta) + b_0 e^{-t} \tag{11}$$

Then, (i) the system (9) has a unique solution surely; (ii) the system (9) is bounded in probability; and (iii) the system (9) is asymptotically stable in the large.

3 | Adaptive Neural Asymptotic Tracking Control Design

In this section, an adaptive asymptotic tracking control scheme based on backstepping approach and RBFNNs is developed for the system (1) with unknown input constraints. The following definitions and symbols are given for the convenience of discussion.

Let $\hat{\theta}_i$ be the estimation of the unknown constants θ_i which is caused by using Lemma 3 to approximation some unknown functions and defined as $\theta_i = \left\|W_i\right\|^2$, $i = 1, 2, \ldots, n$, where W_i represents the weight vector of the ith RBFNN, and its value will be specified later. $S_i\left(Z_i\right)$ and $S_i\left(\check{\zeta}_i\right)$ denote the given basis vector functions corresponding to vectors Z_i and $\check{\zeta}_i$ in the ith RBFNN, respectively.

Similar to traditional backstepping design procedure, the following coordinate transformation is needed:

$$\begin{cases} z_1 = \zeta_1 - y_d \\ z_i = \zeta_i - \alpha_{i-1}\left(\overline{\zeta}_i^{\mathrm{T}}, \left(\overline{y}_d^{(i)}\right)^{\mathrm{T}}, \overline{\hat{\theta}}_{i-1}^{\mathrm{T}}\right) \end{cases} \tag{12}$$

where $\overline{y}_d^{(i)} = \left[y_d, \dot{y}_d, \ldots, y_d^{(i)}\right]^{\mathrm{T}}$, $\overline{\hat{\theta}}_i = \left[\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_i\right]^{\mathrm{T}}$. $z_i (i = 1, 2, \ldots, n)$ is the virtual error, and α_{i-1} is the virtual control signal which need to be designed later.

3.1 | Controller Design

Step 1: From (1) and (12), it is easy to be obtained that

$$dz_1 = \left(f_1\left(\overline{\zeta}_2\right) + d_1\left(t, \overline{\zeta}_n(t)\right) - \dot{y}_d\right)dt + g_1^{\mathrm{T}}\left(\overline{\zeta}_2\right)dw \tag{13}$$

Consider a Lyapunov function candidate as

$$V_1 = \frac{1}{4k_{12}} z_1^4 + \frac{1}{2}\tilde{\theta}_1^2 \tag{14}$$

where k_{12} is a positive constant, and $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$, $\hat{\theta}_1$ is the estimation of the unknown constants θ_1, which will be defined later.

According to Assumption 1 and Young's inequality, it is clear that

$$z_1 d_1\left(t, \overline{\zeta}_n(t)\right) \le M_{d_1}|z_1| \tag{15}$$

$$z_1^3 z_2 \le \frac{3}{4}z_1^4 + \frac{1}{4}z_2^4 \le z_1^4 + z_2^4 \tag{16}$$

where M_{d_1} is an unknown bounded positive constant.

Let $f_{11} = \frac{1}{k_{12}}\left(f_1\left(\overline{\zeta}_2\right) - \dot{y}_d\right)$ and $g_{11} = \frac{3}{2k_{12}} g_1^{\mathrm{T}}\left(\overline{\zeta}_2\right) g_1\left(\overline{\zeta}_2\right)$, noticing that $\zeta_2 = z_2 + \alpha_1$, from (10), (15) and (16), one has

$$\begin{aligned}\mathcal{L}V_1 &= z_1^2\left(z_1\left(f_{11} - \zeta_2 + \zeta_2\right) + \frac{1}{k_{12}} z_1 d_1\left(t, \overline{\zeta}_n(t)\right) + g_{11}\right) - \tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &\le z_1^2 f_{12} + z_1^3\alpha_1 - \tilde{\theta}_1\dot{\hat{\theta}}_1 + z_2^4 \end{aligned}\tag{17}$$

where $f_{12} = z_1\left(f_{11} + \frac{1}{k_{12}} M_{d_1}\operatorname{sgn}\left(z_1\right) - \zeta_2\right) + g_{11} + z_1^2$.

For the subsequent theoretical analysis, introducing the function P_1 as follows

$$P_1 = e^{-z_1^N}\left\|S_1(\breve{\zeta}_1)\right\|^{-2}\int_0^t z_1^2\left\|S_1(\breve{\zeta}_1)\right\|^{-2} d\tau$$

where $\breve{\zeta}_1 = \left[\zeta_1, y_d, \dot{y}_d\right]^{\mathrm{T}}$, and N is any positive integer.

By adding and subtracting P_1 and $\left(s^*_{Z_1} s^*_{\breve{\zeta}_1}\right)^2 + M_{y_d}$ to f_{12}, inequality (17) can be rewritten as

$$\mathcal{L}V_1 \le z_1^2\left(f_{13} - \left(s^*_{Z_1} s^*_{\breve{\zeta}_1}\right)^2 - M_{y_d}\right) + z_1^3\alpha_1 - z_1^2 P_1 - \tilde{\theta}_1\dot{\hat{\theta}}_1 + z_2^4 \tag{18}$$

where $f_{13} = f_{12} + P_1 + \left(s^*_{Z_1} s^*_{\breve{\zeta}_1}\right)^2 + M_{y_d}$.

Clearly, f_{13} is a continuous unknown function with variable Z_1. Then, according to Lemma 3, for any given $0 < \varepsilon < M_{y_d}$, we have

$$f_{13} = W_1^{*T} S_1\left(Z_1\right) + \delta_1\left(Z_1\right) \tag{19}$$

where $Z_1 = \left[\zeta_1,\ \zeta_2, y_d,\ \dot{y}_d\right]^{\mathrm{T}}$, and approximation error $\delta_1\left(Z_1\right)$ satisfies $\left|\delta_1\left(Z_1\right)\right| < \varepsilon$.

Applying Lemma 4 to $\left\|S_1\left(Z_1\right)\right\|$ in (19) yields

$$\begin{aligned} f_{13} &\le \|W_1^*\|\left\|S_1\left(Z_1\right)\right\| + \varepsilon \\ &\le \|W_1^*\|\left\|S_1(\breve{\zeta}_1)\right\|^{-1} s^*_{Z_1} s^*_{\breve{\zeta}_1} + \varepsilon \\ &\le \|W_1^*\|^2\left\|S_1(\breve{\zeta}_1)\right\|^{-2} + \left(s^*_{Z_1} s^*_{\breve{\zeta}_1}\right)^2 + \varepsilon \end{aligned}\tag{20}$$

Combining (18) and (20), and noting that $0 < \varepsilon < M_{y_d}$, we have

$$\mathcal{L}V_1 \le z_1^2\|W_1^*\|^2\left\|S_1(\breve{\zeta}_1)\right\|^{-2} + z_1^3\alpha_1 - z_1^2 P_1 - \tilde{\theta}_1\dot{\hat{\theta}}_1 + z_2^4 \tag{21}$$

Since $\|W_1^*\|^2 = \theta_1$ and $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$, it follows (21) that

$$\begin{aligned}\mathcal{L}V_1 &\le z_1^2\theta_1\left\|S_1(\breve{\zeta}_1)\right\|^{-2} + z_1^3\alpha_1 - z_1^2 P_1 - \tilde{\theta}_1\dot{\hat{\theta}}_1 + z_2^4 \\ &= z_1^2(\tilde{\theta}_1 + \hat{\theta}_1)\left\|S_1\left(\breve{\zeta}_1\right)\right\|^{-2} + z_1^3\alpha_1 - z_1^2 P_1 - \tilde{\theta}_1\dot{\hat{\theta}}_1 + z_2^4 \\ &= z_1^2\hat{\theta}_1\left\|S_1\left(\breve{\zeta}_1\right)\right\|^{-2} + z_1^3\alpha_1 - z_1^2 P_1 \\ &\quad + \tilde{\theta}_1\left(z_1^2\left\|S_1\left(\breve{\zeta}\right)\right\|^{-2} - \dot{\hat{\theta}}_1\right) + z_2^4 \end{aligned}\tag{22}$$

Considering the conditions of Lemma 5, design the following virtual control α_1 and adaptive law $\hat{\theta}_1$ as follows

$$\alpha_1 = \begin{cases} -\mu_1 z_1 - \frac{1-e^{-z_1^{2N}}}{z_1}\hat{\theta}_1\left\|S_1(\breve{\zeta}_1)\right\|^{-2}, & z_1 \ne 0 \\ 0, & z_1 = 0 \end{cases}\tag{23}$$

$$\dot{\hat{\theta}}_1 = z_1^2\left\|S_1(\breve{\zeta}_1)\right\|^{-2} - 2\gamma_1 e^{-t}\hat{\theta}_1 \tag{24}$$

where $\hat{\theta}_1\left(t_0\right) = 0$, μ_1 and γ_1 are positive design parameters, respectively.

Apparently, the solution of the Equation (24) with the initial value $\hat{\theta}_1\left(t_0\right) = 0$ can be described by

$$\hat{\theta}_1(t) = \int_{t_0}^t\left[z_1^2\left\|S_1(\breve{\zeta}_1)\right\|^{-2}\exp\left(\int_\tau^t - 2\gamma_1 e^{-s} ds\right)\right] d\tau \tag{25}$$

Obviously, $\exp\left(\int_\tau^t - 2\gamma_1 e^{-s} ds\right) < 1$. According to the properties of definite integral, the following inequality holds

$$\hat{\theta}_1(t) \le \int_{t_0}^t z_1^2\left\|S_1(\breve{\zeta}_1)\right\|^{-2} d\tau \tag{26}$$

Substituting P_1 into the term $z_1^2\hat{\theta}_1\left\|S_1\left(\breve{\zeta}_1\right)\right\|^{-2} - z_1^2 P_1$, and using the inequality (26), we have

$$\begin{aligned} & z_1^2 e^{-z_1^{2N}}\hat{\theta}_1\left\|S_1(\breve{\zeta}_1)\right\|^{-2} - z_1^2 P_1 \\ &= e^{-z_1^{2N}}\left\|S_1(\breve{\zeta}_1)\right\|^{-2}\left(\hat{\theta}_1 - \int_{t_0}^t z_1^2\left\|S_1(\breve{\zeta}_1)\right\|^{-2} d\tau\right) \le 0 \end{aligned}\tag{27}$$

Substituting (23) and (24) into (22), using the inequality (27), we obtain that

$$\mathcal{L}V_1 \le -\mu_1 z_1^4 + 2\gamma_1 e^{-t}\tilde{\theta}_1\hat{\theta}_1 + z_2^4 \tag{28}$$

Furthermore, noting that

$$\tilde{\theta}_1\hat{\theta}_1 = \tilde{\theta}_1\theta_1 - \tilde{\theta}_1^2 \le -\frac{1}{2}\tilde{\theta}_1^2 + \frac{1}{2}\theta_1^2 \tag{29}$$

Then, substituting (29) into (28) gives

$$\mathcal{L}V_1 \le -\mu_1 z_1^4 - \gamma_1 e^{-t}\tilde{\theta}_1^2 + \gamma_1 e^{-t}\theta_1^2 + z_2^4 \le -\mu_1 z_1^4 + \gamma_1 e^{-t}\theta_1^2 + z_2^4 \tag{30}$$

Step $i(1 < i \le n-1)$: For $j = 1, 2, \ldots, i-1$, we suppose that virtual controls α_j and adaptive laws $\hat{\theta}_j$ have been designed in the following forms.

$$\alpha_j = \begin{cases} -\mu_j z_j - \frac{1-e^{-z_j^{2N}}}{z_j}\hat{\theta}_j \left\| S_j(\check{\zeta}_j)\right\|^{-2}, & z_j \neq 0 \\ 0, & z_j = 0 \end{cases} \tag{31}$$

$$\dot{\hat{\theta}}_j = z_j^2 \left\| S_j(\check{\zeta}_j)\right\|^{-2} - 2\gamma_j e^{-t}\hat{\theta}_j \tag{32}$$

where $\check{\zeta}_j = \left[\bar{\zeta}_j, \quad y_d, \ \dot{y}_d, \ldots, y_d^{(j)}, \hat{\theta}_1, \quad \ldots, \hat{\theta}_{j-1}\right]^{\mathrm{T}}$. μ_j and γ_j are positive parameters, respectively.

Obviously, (31) and (32) hold for $i-1 = 1$. In the following, we only need to give the design process of virtual control α_i and adaptive law $\hat{\theta}_i$.

According to (1), (10) and (12), it follows that

$$dz_i = \left(f_i\left(\bar{\zeta}_{i+1}\right) + d_i\left(t, \bar{\zeta}_n(t)\right) - \mathcal{L}\alpha_{i-1}\right) dt + g_{i1}^{\mathrm{T}}\, dw \tag{33}$$

where $\mathcal{L}\alpha_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\zeta_j}\left(f_i\left(\bar{\zeta}_i, \zeta_{i+1}\right) + d_i\left(t, \bar{\zeta}_n(t)\right)\right) + \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial\zeta_j\partial\zeta_k} g_j^{\mathrm{T}} g_k + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k-1)}} y_d^{(k+1)}$ and $g_{i1}^{\mathrm{T}} = g_i^{\mathrm{T}}\left(\bar{\zeta}_{i+1}\right) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\zeta_j} g_j^{\mathrm{T}}\left(\bar{\zeta}_{j+1}\right)$.

Choose the following Lyapunov function candidate

$$V_i = \frac{1}{4k_{12}} z_i^4 + \frac{1}{2}\tilde{\theta}_i^2 + V_{i-1} \tag{34}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, $\hat{\theta}_i$ is the estimation of the unknown constant θ_i, which will defined later.

According to Lemma 3 in [10], if the given initial value $\hat{\theta}_j(t_0) \ge 0$ for $j = 1, 2, \ldots, i-1$, then $\hat{\theta}_j(t) \ge 0$ for $\forall t \ge t_0$. Therefore, it follows from (32) that

$$\dot{\hat{\theta}}_j \le z_j^2 \left\| S_j(\check{\zeta}_j)\right\|^{-2} + 2\gamma_j \hat{\theta}_j \tag{35}$$

Furthermore, from inequality (35), $\mathcal{L}\alpha_{i-1}$ can be enlarged as a continuous function with variable $\boldsymbol{Z}_i = \left[\bar{\zeta}_{i+1}^T, \left(\bar{\boldsymbol{y}}_d^{(i)}\right)^{\mathrm{T}}, \bar{\hat{\theta}}_{i-1}^{\mathrm{T}}\right]^{\mathrm{T}}$. Based on this, repeating the similar method used in (15), (16), (17) of Step 1, we can imply that there exists a continuous function f_{i2} with variable $\boldsymbol{Z}_i \in \Omega_{Z_i} \subset \mathscr{R}^{3i+1}$ such that

$$\mathcal{L}V_i \le z_i^2 f_{i2} + z_i^3\alpha_i - \tilde{\theta}_i\dot{\hat{\theta}}_i + z_{i+1}^4 + \sum_{j=1}^{i-1}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\right) \tag{36}$$

Following the similar procedure as (18), (36) can be rewritten as

$$\mathcal{L}V_i \le z_i^2\left(f_{i3} - \left(s_{Z_i}^* s_{\check{\zeta}_i}^*\right)^2 - M_d\right) + z_i^3\alpha_i - z_i^2 P_i - \tilde{\theta}_i\dot{\hat{\theta}}_i + z_{i+1}^4 \tag{37}$$

where $P_i = e^{-z_i^{2N}}\left\|S_i(\check{\zeta}_i)\right\|^{-2}\int_{t_0}^t z_i^2\left\|S_i(\check{\zeta}_i)\right\|^{-2} d\tau$, $\check{\zeta}_i = \left[\bar{\zeta}_i^{\mathrm{T}}, \left(\bar{\boldsymbol{y}}_d^{(i)}\right)^{\mathrm{T}}, \bar{\hat{\theta}}_{i-1}^{\mathrm{T}}\right]^{\mathrm{T}}$ and $f_{i3} = f_{i2} + P_i + \left(s_{Z_i}^* s_{\check{\zeta}_i}^*\right)^2 + M_{y_d}$.

Similar to (19), (20), (21), and (22), according to Lemma 3, for any given $0 < \varepsilon < M_{y_d}$, the unknown continuous function f_{i3} can be approximated as

$$\mathcal{L}V_i \le z_i^2\hat{\theta}_i\left\|S_i(\check{\zeta}_i)\right\|^{-2} + z_i^3\alpha_i - z_i^2 P_i + \tilde{\theta}_i\left(z_i^2\left\|S_i(\check{\zeta}_i)\right\|^{-2} - \dot{\hat{\theta}}_i\right) + z_{i+1}^4 + \sum_{j=1}^{i-1}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\right) \tag{38}$$

Based on (38), design the virtual control signal α_i and the adaptation law $\hat{\theta}_i$ in the following forms

$$\alpha_i = \begin{cases} -\mu_i z_i - \frac{1-e^{-z_i^{2N}}}{z_i}\hat{\theta}_i \left\| S_i(\check{\zeta}_i)\right\|^{-2}, & z_i \neq 0 \\ 0, & z_i = 0 \end{cases} \tag{39}$$

$$\dot{\hat{\theta}}_i = z_i^2 \left\| S_i(\check{\zeta}_i)\right\|^{-2} - 2\gamma_i e^{-t}\hat{\theta}_i \tag{40}$$

where μ_i and γ_i are positive design parameters, respectively.

Substituting (39) and (40) into (38), and following the similar method used in (28), (29) and (30), we have

$$\mathcal{L}V_i \le \sum_{j=1}^{i}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\right) + z_{i+1}^4 \tag{41}$$

Step n: By (1), (10) and (12), it follows that

$$dz_n = \left(f_n\left(\bar{\zeta}_{i+1}\right) + d_n\left(t, \bar{\zeta}_n(t)\right) - \mathcal{L}\alpha_{n-1}\right) dt + g_{n1}^{\mathrm{T}}\, dw \tag{42}$$

where $\mathcal{L}\alpha_{n-1} = \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\zeta_j}\left(f_i\left(\bar{\zeta}_i, \zeta_{i+1}\right) + d_i\left(t, \bar{\zeta}_n(t)\right)\right) + \frac{1}{2}\sum_{j,k=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial\zeta_j\partial\zeta_k} g_j^{\mathrm{T}} g_k + \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k-1)}} y_d^{(k+1)}$ and $g_{n1}^{\mathrm{T}} = g_n^{\mathrm{T}}\left(\bar{\zeta}_{i+1}\right) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\zeta_j} g_j^{\mathrm{T}}\left(\bar{\zeta}_{j+1}\right)$.

Taking the Lyapunov function candidate V_n as

$$V_n = \frac{1}{4k_{12}} z_n^4 + \frac{1}{2}\tilde{\theta}_n^2 + V_{n-1} \tag{43}$$

where $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$, $\hat{\theta}_n$ is the estimation of the unknown constant θ_n, which will defined later.

Similar to step i, $\mathcal{L}\alpha_{n-1}$ is also can be enlarged as a continuous function with variable $\check{\zeta}_n = \left[\bar{\zeta}_n^{\mathrm{T}}, \left(\bar{\boldsymbol{y}}_d^{(n)}\right)^{\mathrm{T}}, \bar{\hat{\theta}}_{n-1}^{\mathrm{T}}\right]^{\mathrm{T}}$, which is also integrated into an unknown function and approximated by using Lemma 3. That is to say, there exist continuous functions f_{n1} and g_{n1} such that

$$\mathcal{L}V_n = z_n^2\left(z_n f_{n1} + g_{n1}\right) + z_n^3 h\left(\bar{\zeta}_n\right) k_{12}^{-1}\, u - \tilde{\theta}_n\dot{\hat{\theta}}_n + \mathcal{L}V_{n-1} \tag{44}$$

Considering that the control objective of this paper, based on Lemma 1, we further assume that there exists a positive constant $M_u > 0$, such that

$$u(v(t)) = \rho(v(t)) + \delta(v(t)) \tag{45}$$

where $\rho(\upsilon(t))$ is a differentiable function and $|\delta(\upsilon(t))| < M_u$.

Substituting (41) with $i = n-1$ and (45) into (44), we have

$$\begin{aligned}\mathcal{L}V_n &\leq z_n^2\left(z_n f_{n1} + g_{n1}\right) + z_n^3 h\left(\overline{x}_n\right)k_{12}^{-1}\left(\rho(\upsilon(t)) + \delta(\upsilon(t))\right) - \tilde{\theta}_n\dot{\hat{\theta}}_n \\ &\quad + \sum_{j=1}^{n-1}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\right) + z_n^4 \\ &= z_n^2\left(z_n f_{n1} + g_{n1} + z_n h\left(\overline{x}_n\right)k_{12}^{-1}\delta(\upsilon(t)) + z_n^2\right) \\ &\quad + h\left(\overline{x}_n\right)z_n^3 k_{12}^{-1}\rho(\upsilon(t)) - \tilde{\theta}_n\dot{\hat{\theta}}_n + \sum_{j=1}^{n-1}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\right) \end{aligned} \tag{46}$$

where $f_{n2} = f_{n1} + h\left(\overline{x}_n\right)k_{12}^{-1}\delta(\upsilon(t)) + z_n$.

In the following, a detailed deduction process for designing $\rho(\upsilon(t))$ will be provided, as it is a key step in designing an asymptotic tracking controller.

Similar to the functions P_1 and P_i, define the function P_n as follows

$$P_n = e^{-z_n^{2N}}\left\|S_n(\breve{\zeta}_n)\right\|^{-2} h\left(\overline{\zeta}_n\right)a_m^{-1}\int_{t_0}^{t} z_n^2\left(S_n(\breve{\zeta}_n)\right)^{-2} d\tau \tag{47}$$

By adding and subtracting $z_n^2 P_n$ in (46), we have

$$\begin{aligned}\mathcal{L}V_n &\leq z_n^2\left(f_{n2} + P_n\right) - z_n^2 P_n + h\left(\overline{\zeta}_n\right)z_n^3 k_{12}^{-1}\ \rho(\upsilon(t)) - \tilde{\theta}_n\dot{\hat{\theta}}_n \\ &\quad + \sum_{j=1}^{n-1}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\right)\end{aligned} \tag{48}$$

Similar to (18), we can get

$$\begin{aligned}\mathcal{L}V_n &\leq z_n^2\left(f_{n3} - \left(s^*_{Z_n} s^*_{\breve{\zeta}_n}\right)^2 - M_{y_d}\right) - z_n^2 P_n - \tilde{\theta}_n\dot{\hat{\theta}}_n \\ &\quad + h\left(\overline{\zeta}_n\right)z_n^3 k_{12}^{-1}\ \rho(\upsilon(t)) + \sum_{j=1}^{n-1}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\right)\end{aligned} \tag{49}$$

where $f_{n3} = f_{n2} + P_n + (s^*_{Z_n} s^*_{\breve{\zeta}_n})^2 + M_{y_d}$, $\boldsymbol{Z}_n = \left[\overline{\zeta}_n^{\mathrm{T}}, \left(\overline{\boldsymbol{y}}_d^{(n)}\right)^{\mathrm{T}}, \overline{\hat{\theta}}_n^{\mathrm{T}}\right]^{\mathrm{T}}$.

Again, applying Lemma 3 to the unknown function f_{n3} over a sufficient large compact set $\Omega_{Z_n} \subset \mathscr{R}^{3n+1}$ and repeating the similar method used in (19), (20), (21) and (22), we can get that

$$\begin{aligned}\mathcal{L}V_n &\leq z_n^2\hat{\theta}_n\left\|S_n(\breve{\zeta}_n)\right\|^{-2} - z_n^2 P_n + h\left(\overline{\zeta}_n\right)z_n^3 k_{12}^{-1}\ \rho(\upsilon(t)) \\ &\quad + \tilde{\theta}_n\left(z_n^2||S_n(\breve{\zeta}_n)||^{-2} - \dot{\hat{\theta}}_n\right) + \sum_{j=1}^{n-1}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\ \right)\end{aligned} \tag{50}$$

Based on (50), define the function $\rho(\upsilon(t))$ and the adaptive law $\dot{\hat{\theta}}_n$ as follows

$$\rho(\upsilon(t)) = k_{12}a_m^{-1}\ \varphi(z_n, \breve{\zeta}_n) \tag{51}$$

$$\dot{\hat{\theta}}_n = z_n^2\left\|S_n(\breve{\zeta}_n)\right\|^{-2} - 2\gamma_n e^{-t}\hat{\theta}_n \tag{52}$$

with

$$\varphi(z_n, \breve{\zeta}_n) = \begin{cases} -\mu_n z_n - \frac{1-e^{-z_n^{2N}}}{z_n}\hat{\theta}_n\left\|S_n(\breve{\zeta}_n)\right\|^{-2}, & z_n \neq 0 \\ 0, & z_n = 0 \end{cases} \tag{53}$$

where $\mu_n > 0$ is a positive parameter, and $\hat{\theta}_n$ satisfies an initial condition $\hat{\theta}_n(t_0) = 0$, and γ_n is a positive parameter.

Clearly, $z_n(t)\rho(\upsilon(t)) \leq 0$. By the Equation (53) with the initial value $\hat{\theta}_n(t_0) = 0$, it follows that

$$\begin{aligned}\hat{\theta}_n(t) &= \int_{t_0}^{t}\left[z_n^2||S_n(\breve{\zeta}_n)||^{-2}\exp\left(\int_{\tau}^{t} -2\gamma_n e^{-s}ds\right)\right]d\tau \\ &\leq \int_{t_0}^{t} z_n^2\left\|S_n(\breve{\zeta}_n)\right\|^{-2}d\tau\end{aligned} \tag{54}$$

From (47) and (54), we have

$$-z_n^2 P_n + a_m^{-1}h\left(\overline{\zeta}_n\right)z_n^2 e^{-z_n^{2N}}\hat{\theta}_n\left\|S_n(\breve{\zeta}_n)\right\|^{-2} \leq 0 \tag{55}$$

Furthermore, for $z_n = 0$, it follows that $h\left(\overline{\zeta}_n\right)z_n^3\rho(\upsilon(t)) = 0$. For $z_n \neq 0$, from (51), we have

$$\begin{aligned}&h\left(\overline{\zeta}_n\right)z_n^3 k_{12}^{-1}\ \rho(\upsilon(t)) \\ &= a_m^{-1}h\left(\overline{\zeta}_n\right)z_n^3\ \varphi(z_n, \breve{\zeta}_n) \\ &= -\mu_n a_m^{-1}h\left(\overline{\zeta}_n\right)z_n^4 - a_m^{-1}h\left(\overline{\zeta}_n\right)z_n^2\ \hat{\theta}_n\left\|S_n(\breve{\zeta}_n)\right\|^{-2} \\ &\quad + a_m^{-1}h\left(\overline{\zeta}_n\right)e^{-z_n^{2N}}\hat{\theta}_n\left\|S_n(\breve{\zeta}_n)\right\|^{-2}\end{aligned} \tag{56}$$

Substituting (51) and (52) into (50), by (55) and (56), (50) can be further enlarged as

$$\mathcal{L}V_n \leq -\mu_n z_n^4 + 2\gamma_n e^{-t}\tilde{\theta}_n\hat{\theta}_n + \sum_{j=1}^{n-1}\left(-\mu_j z_j^4 + \gamma_j e^{-t}\theta_j^2\right) \tag{57}$$

Repeating the similar method used in (28), (29), and (30), we have

$$\mathcal{L}V_n \leq -\sum_{j=1}^{n}\mu_j z_j^4 + e^{-t}\sum_{j=1}^{n}\gamma_j\theta_j^2 \tag{58}$$

Based on the above analysis, the input $u(\upsilon(t))$ in different situations can be designed as follows.

Case 1: If it is only needed to design the plant input $u(\upsilon(t))$ for the system (1) without input constraints, according to (45) and (51), we can take $\delta(\upsilon(t)) = 0$, then

$$u(\upsilon(t)) = \rho(\upsilon(t)) \tag{59}$$

Case 2: If the system (1) is a controlled system with unknown nonsmmoth nonlinearity constraint $u(\upsilon(t))$, it is necessary to further design the controller for the system (1). For this case, the controller $\upsilon(t)$ can be designed by solving the Equation (51) if $\rho(\upsilon(t))$ can ensure the Equation (51) is solvable.

Considering the system (1) with the saturation input constraint (2), according to (45) and Lemma 1, we can take

$\rho(\upsilon(t)) = k_{12}u_{sm}(\upsilon(t))$ in (45). Obviously, $\rho(\upsilon(t))$ is injective. Therefore, the controller $\upsilon(t)$ can be designed as

$$\upsilon(t) = u_{sm}^{-1}\left(a_m^{-1}\ \varphi\left(z_n, \check{\xi}_n\right)\right)$$

Considering the system (1) with dead-zone input (3), in view of (45) and Lemma 1, we take $\rho(\upsilon(t)) = u_{dm}\left(\upsilon(t), k_1, k_2\right)$ in (45). Noting that k_1, k_2 are unknown and cannot be used for controller design, we again take $u_{dm}(\upsilon(t)) = k_{12}a_m^{-1}\ \varphi\left(z_n, \check{\xi}_n\right)$. Then, $z_n(t)u_{dm}(\upsilon(t)) \le 0$. By Lemma 2, we can enlarge $h\left(\overline{\zeta}_n\right)z_n^3k_{12}^{-1}\rho(\upsilon(t))$ in (46), (48), (49) and (50) as $h\left(\overline{\zeta}_n\right)z_n^3u_{dm}(\upsilon(t))$. Furthermore, notice that $u_{dm}(\upsilon(t))$ is injective. Therefore, the dead-zone controller $\upsilon(t)$ can be designed as

$$\upsilon(t) = u_{dm}^{-1}\left(a_m^{-1}\ \varphi\left(z_n, \check{\xi}_n\right)\right)$$

Based on the analysis above, the controller $\upsilon(t)$ of the system (1) with the input constraint (2) or (3) can be designed as follow

$$\upsilon(t) = u_{\kappa m}^{-1}\left(a_m^{-1}\ \varphi\left(z_n, \check{\xi}_n\right)\right) \tag{60}$$

where $\kappa \in \{s, d\}$.

Remark 10. The non-smooth nonlinearities of the actual control system are usually unknown and time-varying, which will limit the system performance. The ideas and techniques such as Lemma 1, Lemma 4, and (51) in this paper can provide the effective control schemes to deal with such nonsmooth nonlinearities, so that the developed schemes are able to accommodate the uncertainties. In particular, by virtue of (45) and (51), the problem of designing adaptive asymptotic controllers for a class of uncertain systems with input constraint converts to how to find a differentiable function to approximate such input constraints with bounded approximation errors. If the established differential function satisfies (45), it is easy to design a controller for the system (1) with any input constraint based on (51).

3.2 | Stability Analysis

Based on Lemma 5 and the above discussions, the control objectives are achieved. We summarized the stability result of the above control design in the following theorem and corollary.

Theorem 1. *For the system (1), based on Assumptions 1–5, backstepping approach and RBFNNs, the designed virtual signals (23), (31) and (39), adaptive laws (24), (32), (40) and (52), and the system input (59) guarantee that all the signals in the closed-loop control system remain ultimately bounded in probability and the tracking error converges in to zero. Furthermore, if the plant input $u(\upsilon(t))$ is unknown nonsmooth nonlinearity satisfying the expression (45), and the Equation (51) is solvable, then a controller designed by selecting a solution of the Equation (51), as well as virtual signals (23), (31) and (39), adaptive laws (24), (32), (40), and (52) guarantees the semi-global asymptotic tracking stability of the system (1) with input constraints $u(\upsilon(t))$.*

Proof. According to Lemma 5 and (58), the conclusion of Theorem 1 is valid. □

Before concluding this section, we present the following corollary.

Corollary 1. *Based on Theorem 1, it is evident that we can draw a conclusion that the virtual signals (23), (31) and (39), adaptive laws (24), (32), (40), and (52), as well as controller (60) guarantees the semi-global asymptotic tracking stability of the system (1) with plant input (2) or (3).*

4 | Controller Design and Simulation Examples

By Corollary 1, it is clear that we can design the semi-global asymptotic tracking controller for the system (1) with the input constraints (2) or (3).

Consider the following second-order stochastic nonlinear system with input saturation

$$\begin{cases} d\zeta_1 = \left(0.5\zeta_1 + \left(1 + 0.1\zeta_1^2\right)\zeta_2 + 0.5\sin t\right)dt + 0.03\zeta_1 dw \\ d\zeta_2 = \left(\zeta_1\zeta_2 + 0.5\cos t + \left(2 + \cos\ \left(\zeta_1\zeta_2\right)\right)u\right)dt + 0.04\zeta_2 dw \\ \quad y = \zeta_1 \end{cases} \tag{61}$$

where ζ_1 and ζ_2 represent the system states and the initial condition satisfy $\zeta_1(0) = 0$ and $\zeta_2(0) = 0$. y and u represent the input and output of the system, respectively. The desired signal is chosen as $y_d(t) = 0.5\sin t$.

Example 1. Consider the system (61) with the following input saturation

$$u = u(\upsilon) = \begin{cases} -2.5, & \upsilon(t) \le -2.5 \\ \upsilon(t), & -2.5 < \upsilon(t) < 1.5 \\ 1.5, & \upsilon(t) \ge 1.5 \end{cases} \tag{62}$$

For the purpose of simplicity, let $\varphi = \varphi\left(z_n, \check{\xi}_n\right)$. Noting that $u_{sm}(\upsilon(t))$ is injective, and from (60), it follows that

$$\upsilon(t) = u_{sm}^{-1}\left(a_m^{-1}\ \varphi\right) \tag{63}$$

From (63), the designed controller $\upsilon(t)$ can be expressed as

$$\upsilon(t) = \begin{cases} r_a + \frac{\varphi - a_m r_a}{a_m - a_m r_a + \varphi}, & a_m(r_a - 1) < \varphi \le a_m r \\ \frac{\varphi}{a_m}, & a_m r_a \le \varphi \le a_m r_b \\ r_b + \frac{\varphi - a_m r_b}{a_m + a_m r_b -\ \varphi}, & a_m r_b \le \varphi < a_m(r_b + 1) \end{cases} \tag{64}$$

It is apparent that this system satisfies Assumptions 1 and 3. Based on the control design process, the designed controller can be expressed as (64), and the virtual control α_1, the adaptive laws $\hat{\theta}_i$ are constructed, respectively, as

$$\alpha_1 = \begin{cases} -\mu_1 z_1 - \frac{1 - e^{-z_1^{2N}}}{z_1}\hat{\theta}_1 \|S_1(\check{\xi}_1)\|^{-2}, & z_1 \ne 0 \\ 0, & z_1 = 0 \end{cases} \tag{65}$$

$$\dot{\hat{\theta}}_i = z_i^2\left\|S_i\left(\check{\xi}_i\right)\right\|^{-2} - 2\gamma_i e^{-t}\hat{\theta}_i, \quad i = 1, 2 \tag{66}$$

where $z_1 = \zeta_1 - y_d$, $z_2 = \zeta_2 - \alpha_1$, $\check{\xi}_1 = \left[\zeta_1, y_d, \dot{y}_d\right]^{\mathrm{T}}$, and $\check{\xi}_2 = \left[\overline{\zeta}_2^{\mathrm{T}}, \left(\overline{y}_d^{(2)}\right)^{\mathrm{T}}, \hat{\theta}_1\right]^{\mathrm{T}}$.

In the simulation, the correlative design parameters are chosen as: $\mu_1 = 80, \mu_2 = 50, N = 1, a_m = 2, r_a = -30,\quad r_b = 20,$

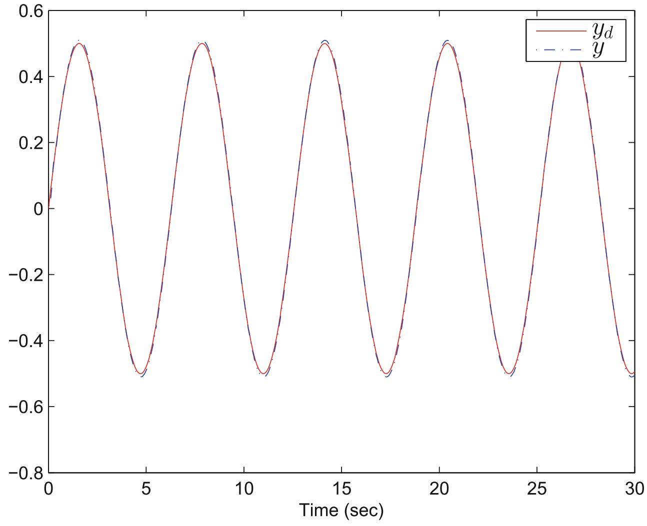

FIGURE 1 | The trajectories of y and y_d of Example 1.

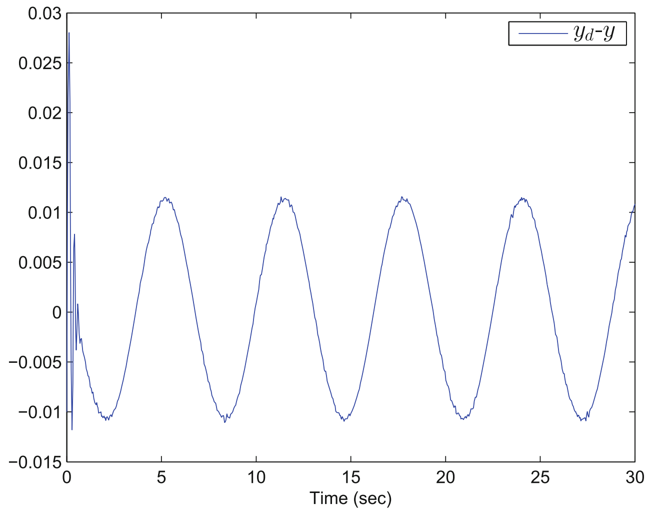

FIGURE 2 | The trajectory of the tracking error $y - y_d$ of Example 1.

$\gamma_1 = \gamma_2 = 1$. The simulation is run with the initial conditions $\left[\zeta_1(0), \zeta_2(0)\right]^{\mathrm{T}} = [0.01, 0.01]^{\mathrm{T}}$, and $\left[\hat{\theta}_1(0), \hat{\theta}_2(0)\right]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$. The simulation results are shown in Figures 1–4. Figure 1 exhibits the time trajectories of the desired signal y_d and system output y. It can be seen that the output y can almost surely track the given reference signal y_d asymptotically. Figure 2 shows the trajectory of tracking error $y_d - y$. It appears that the tracking error is always converges in to zero and the effect of asymptotic tracking is achieved. Figure 3 depicts the trajectories of v and u. Figure 4 shows the time trajectories of the system state ζ_2.

As a result, the simulation results demonstrate the effectiveness of the proposed controller in handling input saturation and the good asymptotic tracking performance of the control scheme.

Example 2. Consider the system (61) with the following input dead-zone

$$u = u(v) = \begin{cases} v + 1.5, & v \leq -1.5 \\ 0, & -1.5 < v < 1.5 \\ v - 1.5, & v \geq 1.5 \end{cases} \tag{67}$$

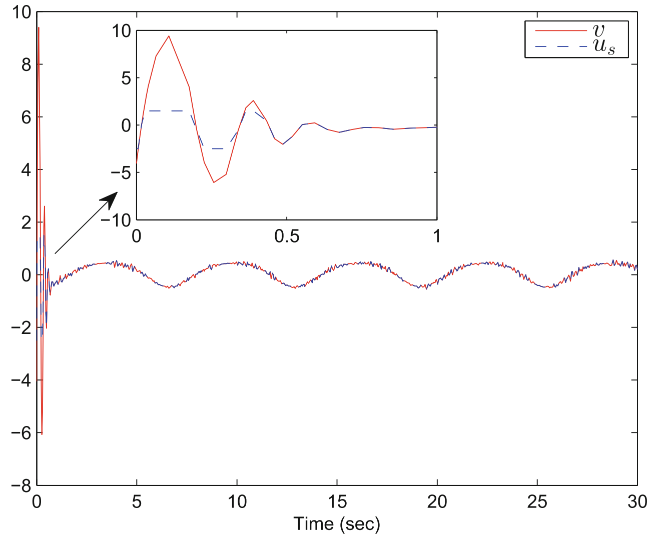

FIGURE 3 | The trajectories of v and u of Example 1.

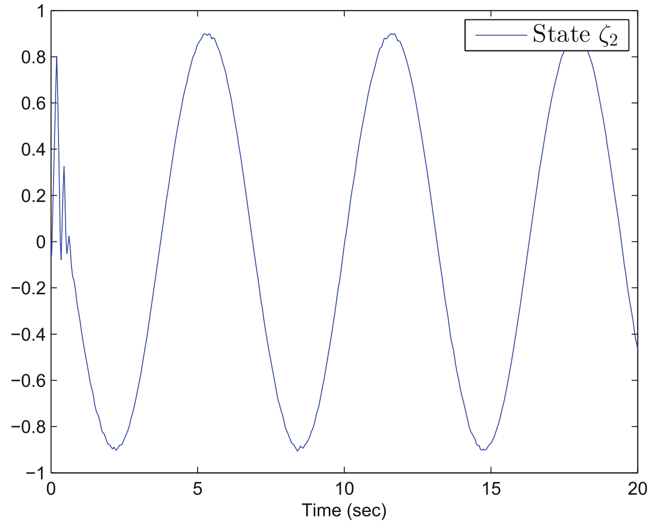

FIGURE 4 | The trajectory of system state ζ_2 of Example 1.

Noting that $u_{dm}(v(t)) = u_{dm}(v(t), 1, 1)$ is also injective, by (60), it follows that

$$v(t) = u_{dm}^{-1}\left(a_m^{-1}\ \varphi\right) \tag{68}$$

According to (5) and (68), the controller $v(t)$ is designed as

$$v(t) = \begin{cases} \frac{1}{2}\left(r_a - \frac{\varphi}{a_m r_a}\right), \ \varphi \leq -a_m r_a^2 \\ -\sqrt{-\frac{\varphi}{a_m}}, -a_m r_a^2 \leq \varphi \leq 0 \\ \sqrt{\frac{\varphi}{a_m}}, 0 \leq \varphi \leq a_m r_b^2 \\ \frac{1}{2}\left(r_b + \frac{\varphi}{a_m r_b}\right), \varphi \geq a_m r_b^2 \end{cases} \tag{69}$$

The virtual control law α_1, the adaptive laws $\dot{\hat{\theta}}_i$ are defined in (65) and (66), respectively.

The simulation is carried out with $\left[\zeta_1(0), \zeta_2(0)\right]^{\mathrm{T}} = [0.01, 0.01]^{\mathrm{T}}$, $\left[\hat{\theta}_1(0), \hat{\theta}_2(0)\right]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, $\mu_1 = 80$, $\mu_2 = 30$, $N = 1$, $a_m = 1$,

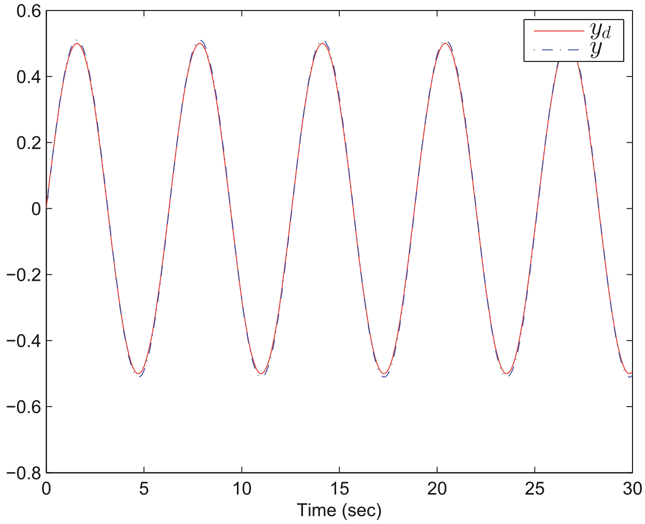

FIGURE 5 | The trajectories of y and y_d of Example 2.

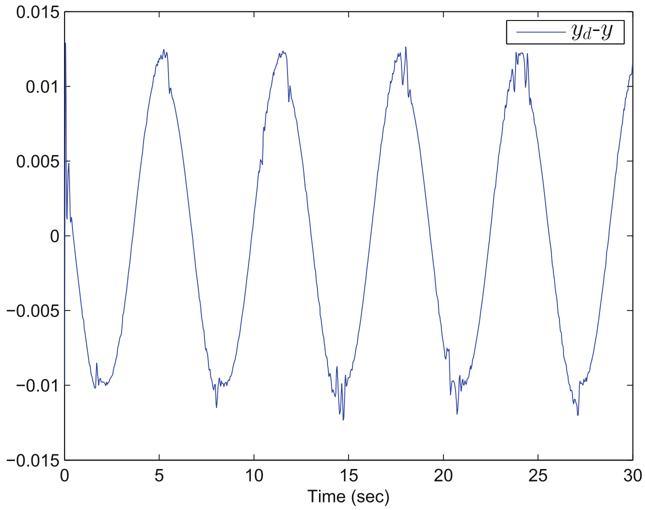

FIGURE 6 | The trajectory of tracking error $y - y_d$ of Example 2.

$r_a = -1.5,\ r_b = 1.5,\ \gamma_1 = 1,\ \gamma_2 = 1$. The simulation results are shown in Figures 5–8.

From above simulation results, it can clearly verify that the good asymptotic tracking performance is achieved and the proposed control scheme is also effective for the studied system (61) with dead-zone (67).

From the above simulation results, it can clearly be verified that good asymptotic tracking performance is achieved and that the proposed control scheme is also effective for the studied system (61) with the dead-zone (67).

Remark 11. Note that the systems (61) studied in Examples 1 and 2 are the same, but the input constraints (62) and (67) of the system are different, and then two different controllers are designed, respectively. Therefore, the methods proposed in this paper can avoid designing the same controller for the same system with different input constraints and reflects more features of the plant input nonsmooth nonlinearity.

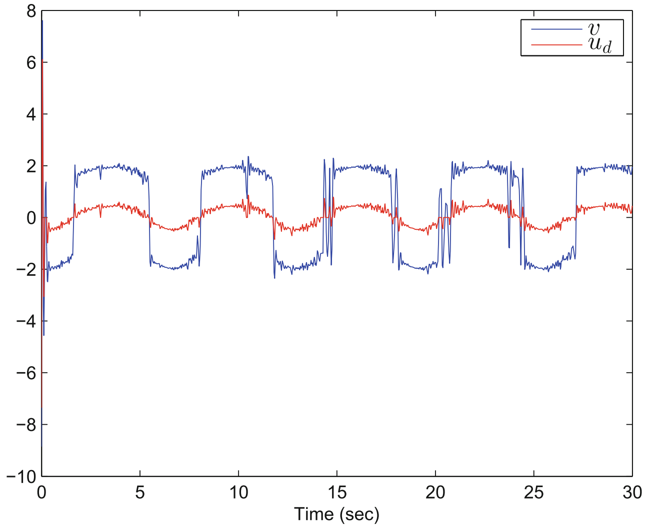

FIGURE 7 | The trajectories of v and u of Example 2.

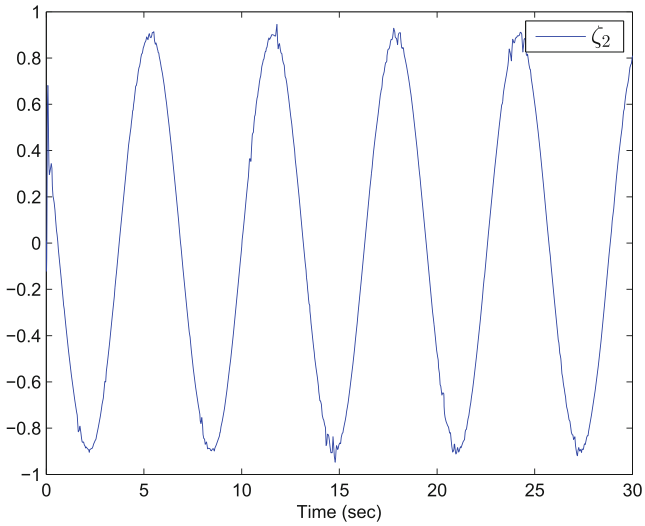

FIGURE 8 | The trajectory of system state ζ_2 of Example 2.

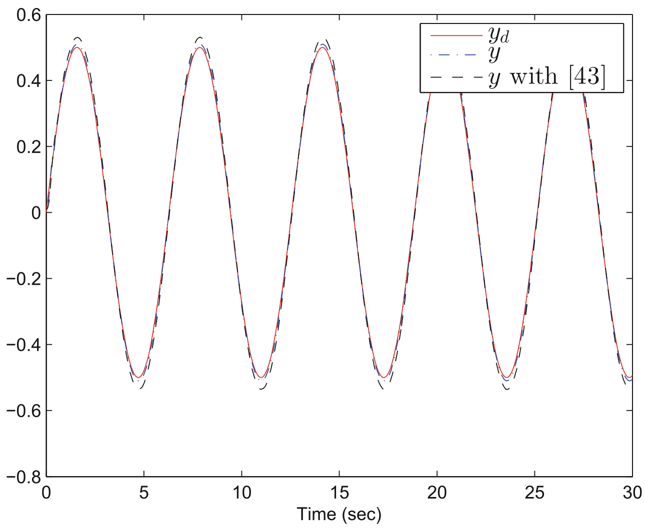

FIGURE 9 | The trajectories of y and y_d under different control methods.

Example 3. To further underscore the superiority of our proposed control scheme, a comparative experiment is conducted based on system (61) with input saturation (62), pitting the method proposed in this paper against the method in [42]. The simulation comparison outcomes are depicted in Figure 9.

As illustrated in Figure 9, both the method introduced in this paper and the method presented in [42] exhibit the capability to achieve tracking control, with virtually indistinguishable tracking effects. This further underscores the effectiveness of the control strategy proposed in this paper.

5 | Conclusion

In this paper, a new adaptive neural backstepping asymptotic tracking control scheme is proposed for a class of uncertain stochastic nonlinear systems with unknown input constraints under loose conditions. Some novel auxiliary techniques such as differentiable approximation function and dimension reduction inequality are provided to avoid the possible obstacles which result in the tracking error can't converge zero in the process of NN controller design. Two simulation examples with different input nonlinearity are applied to elucidate the validity of the designed scheme. Furthermore, the methods used in this paper are also valid for the asymptotic tracking controller design of more complex uncertain nonlinear systems with unknown nonsmooth nonlinear input. Although these methods have some clear advantages, this study still has its limitations and leaves some questions for further exploration. For example, our results only demonstrate the local stability of the studied system because they depend on the given compact set. And we also don't give a method to determine the sufficient large compact set which heavily influences the choice and design of RBFNNs and fuzzy logic systems. These limitations will be discussed in the follow-up research work.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

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