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Fast Positioning of Rotating Center Based on Correction of Finite Angle Deviation of CT System

Deyu Duan [1,2], Fahui Zhai [1,2], Yuqin Cao [1], Huaqiong Hou [1], Shuguo Yang [2*]
1. Research Center for Mathematical Modeling, School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao 266061, China
E-mail: daul_0124@163.com
2. Institute of Intelligence Science & Data Technology, School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao 266061, China
*Corresponding author, E-mail: ysg_2005@163.com

Abstract: Note that it is very important to determine accurately the position of the Center of Rotation (COR) to the image reconstruction in the CT scanning system, in this paper, we establish the model of fast determining COR by using the correction of finite angle deviation, moreover apply certain algorithms to achieve the center of rotation calibration. By simulating original signal or the original signal with noise, we obtain that the artifact of the reconstructed image is significantly less and the image quality is also raised, thus the center of rotation can be accurately determined. Compared with the known algorithms, the model in this paper has a small amount of calculation and strong resistance to random noise. Consequently, our model and algorithm are helpful for determining COR.

Key Words: 2D-CT, Finite Angle Deviation, Improved Inverse Transformation of Radon, Least Square Method, White Gaussian Noise

1 Introduction

X-ray computed tomography is an important technical means for nondestructive internal observation, it is widely applied in various fields, for example, medical treatment, industrial nondestructive testing, security check etc. When an industrial CT imaging system is constructed, since the deviation of the system rotation center from the rebuilt rotation center maybe cause artifacts in the CT image, the quality of the CT image will be degraded. In order to reduce error of the reconstruction of CT image, the coordinates of the rotation center position need to be precisely measured.

Because of the importance of rotation center positioning, some research results about the positioning method of the rotation center of CT system have appeared. For instance, O`lander and others use the properties of Radon transform, shift and turn the projection of 180 degrees of visual difference to study the rotation center of CT system, Taylor and Lupton determined the center of rotation is located in the case of best matching. For more information on calculation of the rotational centers, the reader can refer to [1-11] and their references.

Observe that the method in [1] needs to accurately get the projection data with a visual difference of 180 degrees , and the location of the center of rotation is greatly influenced by noise, an improved relative angle method which is based on sine graph was given in [6-8] by Li Baolei et al. In addition, in [10] and [11], for the parallel beam case, the position of the center of rotation can be also measured from the projection data of 60 angles by using the correspondence between the mass center of the object space and the centered of the projection space.

By the above analysis about methods of determination of center of rotation, it is worth studying the determination of center of rotation for the case which is the deflection angle of projection data is unknown. In this paper, we discuss this case, a new method of determination of center of rotation is proposed based on the techniques of plane analytic geometry. This method applies the least square algorithm to estimate the 180 sets of deflection angles, and the improved Radon inverse transform to reconstruct the image from the back projection. In addition, this method also uses coordinate transformation of the original image and reconstructed image to realize the rapid determining the location of the rotation center. By the simulation, it can be seen that the finite angle measurement method overcomes the defects of the existing rotary center positioning method. Moreover, this method is simple, convenient, and more suitable to the case of deflection angle of unknown projection data. Thus, the model and the algorithm introduced in this paper are helpful for determination of center of rotation.

2 Preliminaries

2.1 Receive Information

For a single energy X ray, let I_{in} be the incident intensity and I_{out} the radiation intensity after attenuation. According to Beer-Lambert Law, then

$$I_{out} = I_{in} e^{-\mu l}$$

where is μ attenuation coefficient. Let l be the propagation distance of the ray in the template, then the projection data is

$$\mu l = \ln(I_{in} / I_{out})$$

Consequently, the obtained information is

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$$I = k\mu l \tag{1}$$

where k is the gain coefficient.

2.2 Calibration Template

Take a template, and establish the xoy coordinate system such that the origin of coordinates of coordinate system is the center of elliptical template, see following Fig.1.

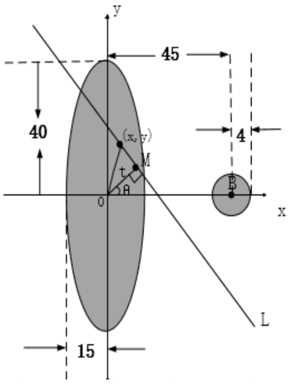

Fig.1: Template schematic diagram
(Specifications: 100mm×100mm)

In this coordinate system, the analytical equations of the elliptical and circular for work are of forms

$$\begin{cases} \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \\ (x-c)^2 + y^2 = r^2 \end{cases} \tag{2}$$

Since computing methods about the propagation distance of the ray passing through the elliptical and the circular template are similar, without loss generality, we take a ray to discuss the propagation distance. As shown in Figure 1. Let t be the length of the vertical line OM of the origin to the ray L, in this paper, positive and negative definitions are defined, it is defined as negative with the X axis on the negative axis, otherwise, it is positive. We denote the θ as the tilt angle of the line OM , and (x, y) is the coordinates of any point on the ray, then

$$x\cos\theta + y\sin\theta = t \tag{3}$$

where t is the inner product of the position vector of any point on the ray L and the unit vector of OM .

Let l_1 and l_2 be the length of the ray through the ellipse template and the circular template, respectively. Hence,

$$\begin{cases} l_1 = \frac{2ab}{T}\sqrt{T - t^2} \\ l_2 = \frac{2\sqrt{\Delta}}{\sin\theta} \end{cases} \tag{4}$$

where

$$T = a^2\cos^2\theta + b^2\sin^2\theta$$

$$\Delta = (c\sin^2\theta + t\cos\theta)^2 + (c^2 - r^2)\sin^2\theta - t^2$$

By above arguments, the length of the rays passing through the template is:

$$l = l_1 + l_2 \tag{5}$$

Consider that the template is of uniform media, then

$$\mu = 1$$

substituting the upper equality into (1) , then

$$I = k(l_1 + l_2) = kl \tag{6}$$

3 Algorithm of Determination of Rotation Centre Based on Finite Angle Deviation Correction

Observe that the received information are data which are obtained from the gain processing of the projection data, and the distance between the detector elements is unknown, so we will firstly calibrate the gain coefficient and the detector cell spacing.

Take circular template, by using techniques in analytic geometry, we discuss the relationship between the detector cell spacing and the length of the ray passing through the template. Under the conventional CT scan layout, the receiving information is processed by binarization. The binarized image is shown in Fig.2.

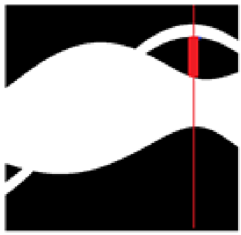

Fig.2: Class sine graph

To analyze the Fig.2, then the receiving information of the circular arc is circular template, moreover, if the area in which the upper right's data is 0, then such area represents the receiving information of the ray that passes through between the elliptical template and the circular template.

Again analyze the three columns with the highest vacancy height, then the gain coefficient k and the detector unit spacing d are determined by the minimum square sum of the error between the theoretical value and the actual value of the received information.

Secondly, use the plane analytic geometry knowledge, we characterize the relationship between the rotation angle and the length of the ray through the template, see the equality (4). In addition, we also determine the rotation angles of the system by the minimum square sum of the error between the theoretical value and the actual value of the received information.

Finally, based on the improved finite angle deviation correction, use an improved Radon inverse transformation to reconstruct image, determine the central coordinates of the ellipse and the circle in the reconstructed image, make the coordinate transformation to the coordinates of the original image and the reconstructed image. Concretely, we firstly make translation transformation to center coordinate of ellipse, then take the rotation transformation to center coordinate of circle. Thus the position of the rotation center in the square pallet is obtained, furthermore, the rotation center location is completed.

Based on the above analysis, the algorithm of COR positioning is in following.

- Use least square method to determine the distance d of the detector unit, and the gain coefficient k .
- Also use least square method to measure rotation angle θ .

- Use an improved Radon inverse transformation to reconstruct the image based on the improved finite angle deviation correction.
- Complete rotation center positioning by using the coordinate transformation of the original image and the reconstructed image.

4 Imeplementation of Algorithm

In this section, we implement the algorithm introduced in section 3.

4.1 The Optimization Model of Calibration Gain Coefficient and Detector Cell Spacing

Let x_0 be in the X-axis of the ray which is closest to the circle and does not intersect with the circle,

$$x_i = x_0 + id \ (i = 1, 2, ..., n)$$

be the transverse coordinates of the i-th ray, thus the length of the X ray in the template is

$$l_i^* = 2\sqrt{r^2 - (x_0 + id)^2} \tag{7}$$

Substituting the equality (7) into (1), the receiving information corresponding to the i-th ray can be obtained:

$$I_i^*(k,d) = 2k\sqrt{r^2 - (x_0 + id)^2} \tag{8}$$

By using least square method, we have the following objective function,

$$\min \sum_{i=1}^{n} \left(I_i^*(k,d) - I_i(k,d) \right)^2 \tag{9}$$

Consider that the number of X rays in the different groups is different in the three sets of data selected, under the assumption which is that n rays pass through in the direction of the normal line and intersect with the circle, we obtain the constraint conditions of d, that is,

$$\frac{2r}{n_{\max} - 1} = d_{\min} \le d \le d_{\max} = \frac{2r}{n_{\min} - 1} \tag{10}$$

Consequently, we can determine the cell spacing and gain coefficient of the detector by following optimization model,

$$\min \sum_{i=1}^{n} \left(I_i^*(k,d) - I_i(k,d) \right)^2$$

$$s.t. \begin{cases} k > 0 \\ \dfrac{2r}{n_{\max} - 1} = d_{\min} \le d \le d_{\max} = \dfrac{2r}{n_{\min} - 1} \\ -r - d_{\max} \le x_0 \le -r \end{cases} \tag{11}$$

4.2 The Optimization Model for Measuring Rotation Angle

Let θ_j be the detector's *j-th* direction angle, d_j be the distance between the leftmost detector unit and the origin. Hence, the distance between the origin and the X ray of the *i-th* unit of the detector is $d_j + id$.

Take $d_j + id$ in the equality (6), we get the receiving information

$$I_{ij}^*(\theta) = k\left(\frac{2ab}{T}\sqrt{T - (d_j + id)^2} + \frac{2\sqrt{\Delta}}{\sin\theta}\right) \tag{12}$$

where

$$T = a^2 \cos^2\theta + b^2 \sin^2\theta$$

$$\Delta = (c\sin^2\theta + (d_j + id)\cos\theta)^2 - (c^2 - r^2)\sin^2\theta - (d_j + id)^2$$

Let $I_{ij}(\theta_j)$ be the actual value of the received information, then we get following objective function

$$\min \sum_{i=1}^{n} \left(I_{ij}^*(\theta_j) - I_{ij}(\theta_j) \right)^2 \tag{13}$$

Moreover, observing the shape of the received information, we can see that for the detector parallel to the X-axis, the distance between the leftmost detector and the origin does not exceed d_1, the detector perpendicular to the x-axis, the leftmost detector the distance from the origin does not exceed d_2, we obtain that d_j satisfies following constraint conditions

$$\begin{cases} \min d_{0j} = \min\{d_1, |512d - d_1|, d_2, |512d - d_2|\} \\ \max d_{0j} = \max\{d_1, |512d - d_1|, d_2, |512d - d_2|\} \end{cases} \tag{14}$$

$$\min d_{0j} \le d_{0j} \le \max d_{0j} \tag{15}$$

In addition, since the 151st group of detectors is approximately parallel to the x-axis, and the detector deflection angle can be obtained by the counter-clockwise rotation of the CT system and the deflection angle is approximately 1, one has the constraint condition of rotation angle θ_{152} is:

$$\text{daul}\,\theta_{151} - 1 \le \theta_{152} \le \theta_{151} + 1 \tag{16}$$

Similarly, the constraint condition on θ_j is

$$\theta_{j-1} - 1 \le \theta_j \le \theta_{j-1} + 1 \tag{17}$$

By above arguments, we get the optimization model of rotation angle is

$$\min \sum_{i=1}^{n} \left(I_{ij}^*(\theta_j) - I_{ij}(\theta_j) \right)^2$$

$$s.t. \begin{cases} \min d_j \le d_j \le \max d_j \\ \theta_{151} - 1 \le \theta_{152} \le \theta_{151} + 1 \\ \theta_{j-1} - 1 \le \theta_j \le \theta_{j-1} + 1 \end{cases} \tag{18}$$

4.3 Rotation Center Positioning Model

1. Reconstruction of Image Based on Improved Radon Inverse Transform

In image reconstruction, we apply Radon inverse to the projection data of cross-sectional images under different projection angles, get following two-dimensional Radon inverse transformation

$$f(x,y)=\frac{1}{2\pi^2}\int_0^{\pi}\int_{-\infty}^{\infty}\frac{1}{r\cos(\phi-\theta)-s}\frac{\partial}{\partial s}p(s\theta)dsd\theta \quad (19)$$

In order to reconstruct the pixel value at that point, we improve the equation (19) by using the projection values in multiple directions of the point. In fact, we adopt a method of changing the definite integral of the angle to a cumulative addition of the limited angle deviation correction, then we reconstruct the image with the improved radon inverse projection based on the limited angle deviation correction.

2. Coordinate Transformation of Original Image and Reconstructed Image

Search for the coordinates of the center of ellipse and the center of the small circle in the reconstructed image.

Let the coordinates of the center as:

Abscissa: $x_i=\dfrac{x_{i(n)}-x_{i(n+1)}}{\max n}$

Ordinate: $x_j=\dfrac{x_{j(n)}-x_{j(n+1)}}{\max n}$

Step 1: Search from the left edge of the reconstructed image, write the first column satisfying summation $c_i\neq 0$ as $i(n)$, and find the value of $x_{i(n)}$.

Step 2: Again search for the column item which satisfies $c_i\to 0$ on the right edge of the ellipse, and write the column of the previous column $c_i\neq 0$ as $i(n+1)$, and find the value of $x_{i(n+1)}$.

Step 3: Search from the top of the reconstructed image, we write the first row satisfying summation $c_j\neq 0$ as $j(n)$, and find the value of $x_{j(n)}$.

Step 4: Again search for the row item which satisfies $c_j\to 0$ on the lower boundary of the ellipse, and write the column of the previous column $c_j\neq 0$ as $j(n+1)$, and find the value of $x_{j(n+1)}$.

Step 5: Use

$$\left(\frac{x_{i(n)}-x_{i(n+1)}}{\max n},\frac{x_{j(n)}-x_{j(n+1)}}{\max n}\right)$$

to calculate coordinates of the center.

Make translation transformation to center coordinate of ellipse, then take the rotation transformation to center coordinate of circle. Consequently, the center of rotation is determined.

5 Experimental Results

In this paper, we use some techniques of plane analytic geometry and the least-squares algorithm to get an optimization model, as well as apply MATLAB soft to solve such optimization model, obtain the detector element spacing and gain coefficient of CT system which are 0.2768 and 1.7725, respectively, where the date which we used are form http://mcm.blyun.com/front/detailTopic.

Again use least-squares optimization model, we get the rotation angles of 180 groups, see Table 1.

Table 1: 180 Sets of Deflection Angle

Serial number	Deflection angle	Serial number	Deflection angle
1	29.6549	151	179.4610
2	31.0083	152	180.7515
⋮	⋮	⋮	⋮
149	177.6134	179	207.6541
150	178.5937	180	208.6445

By Table 1, we easily see that CT system scanned is of non-uniform angular deflections, and are not complete half-circle scans according to the 180 sets of data. Hence, it is difficult to accurately determine the rotation center when we ignore the deviation of the rotation angle of the CT system, these will result in greater systematic errors to reconstruction of image. So we must consider the deviation of the rotation angle, by correcting finite angle deflection solve this model, furthermore get the rotation center is (-9.2635, 6.2725).

As to the effectiveness of the algorithm, compare the quality of images reconstructed by uniform angular rotation with that of images reconstructed by finite angular error correction, we obtain that the images reconstructed by finite angular error correction is higher quality than that of uniform angular rotation, see following Fig.3(a) and (d).

In the following, we analyze the influence of noise to reconstruct the CT image. Add white noise to the original signal, assume the white noise obeys the standard normal distribution, use the known relevant system parameters, we reconstruct the image by using the methods with uniform angular deflection and the finite angular deviation respectively, see Fig.3(b) and (e).

For general case, respectively take Gaussian white noise which obeys $N(0, 1)$, $N(0, 8)$, $N(0, 16)$, $N(0, 24)$ and $N(0, 28)$ to add to the original signal, and get the mean of the five groups of noises with noises, thus we have following Fig.3(c) and (f), which shows that the method of finite angular rotation is superior to that of uniform angular rotation.

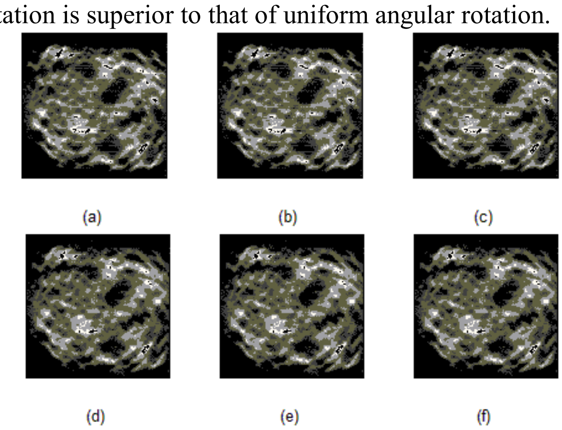

Fig.3: Reconstructed image at a uniform/finite angle
Uniform: (a) No noise (b) Add noise ($\sigma=1$) (c) Mean noise
Finite: (d) No noise (e) Add noise ($\sigma=1$) (f) Mean noise

By above arguments, for the case of addition white noise to original signal, observe that Fig.3 (d), (e) and (f), we see that there is no significant difference for reconstructed images about the different white noise which is of different variance. That is, our model has strong anti-noise performance.

Note that the smoothness of the contour curve can show the superior or inferior of the reconstruction image quality. Corresponding to the above reconstruction image, randomly select a fixed line for profile analysis, we get the corresponding cross-sectional curve in following Fig.4.

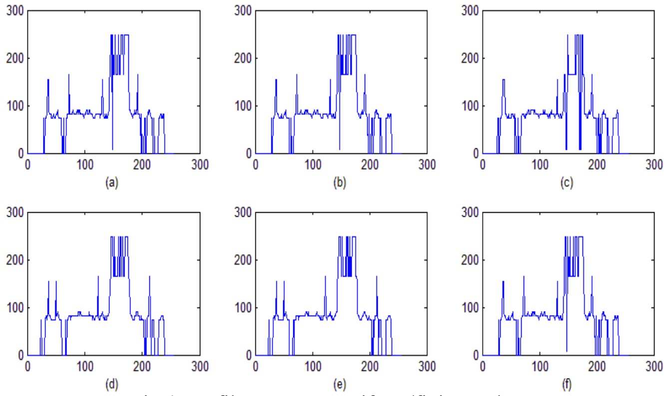

Fig.4: Profile curve at a uniform/finite angle
Uniform: (a)No noise (b) Add noise ($\sigma = 1$) (c) Mean noise
Finite: (d)No noise (e) Add noise ($\sigma = 1$) (f) Mean noise

Observed that the corresponding profile curve, we can see that the profile curves in Fig.4 (d), (e) and (f) are more smooth than that of in Fig.4 (a), (b) and (c). Thus, the images reconstructed by the finite angular deviation correction have a more uniform gray distribution and a higher quality than that of uniform angle whether we add white noise to original signal or don't add white noise to original signal.

6 CONCLUSIONS

In this paper, we introduce a new COR fast calibration algorithm based on finite angle error correction. By Comparing our model and algorithm with known models and algorithms, we see that our algorithm has the following features.

1. The algorithm is more suitable for the case of any geometric parameters unknown, and is of effect of suppression noise. In particular, this algorithm reduce the error caused by geometric parameter deviation of CT system and rotation in homogeneity of the CT system.

2. The determination of rotating center by finite angle deviation is of simplicity and accuracy, furthermore also can be applied to some practical problems in engineering.

In addition, consider that the elliptic circular template with homogeneous medium is simple for the determination of rotating center, we will deal the template with complicated geometric shapes for the question of the determination of rotating center in future works.

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