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This paper is concerned with the problem of multi-dimensional Taylor network (MTN) tracking control for a class of nonlinear systems with unknown control direction. By combining MTN' universal approximation ability and adaptive control method in the backstepping recursive design, a new adaptive MTN control approach is presented for the nonlinear system. The computation complexity of the designed controller has significantly reduced thanks to the simple structure of MTN. It is shown that the designed controller can guarantee all the signals in the closed-loop system remain bounded and the tracking error finally converges to a small neighborhood of the origin. Finally, simulation results demonstrate the effectiveness of the proposed design approach.

Keywords: multi-dimensional Taylor network, adaptive tracking control, nonlinear systems, backstepping technique

1 Introduction

In recent years, the investigation on stability analysis and control design of nonlinear systems has attracted increasing attention and many interesting control strategies are reported, such as adaptive backstepping design (Kanellakopoulos, Petar, & Morse, 1991; Liu, Gao, Tong, & Li, 2016), sliding mode control (Hsu, 2016; Lin, Hsu, & Chen, 2012; Zhao, Yang, Xia, & Wang, 2017), intelligent control (Gang & Zheng, 2009; Xie, Yue, Zhang, & Xue, 2016), neural networks (Ge, Hong, & Lee, 2004; Liu, Chen, Wen, & Tong, 2011; Tong, Li, Li, & Liu, 2011), and so on. Among them, backstepping has become a popular nonlinear control design technique. By combining the adaptive control approach with the backstepping technique, many interesting results have been achieved for uncertain nonlinear systems (Xu & Huang, 2010), full state constrained nonlinear systems (Liu & Tong, 2017), nonlinear time-varying state constraint systems (Liu, Lu, Li, & Tong, 2017), large-scale stochastic nonlinear systems (Liu, Zhang, & Jiang, 2007), interconnected nonlinear systems (Wang, Liu, Qiu, & Liu, 2018), stochastic high-order nonlinear systems (Li, Xie, & Zhang, 2011) and so on. However, most of the aforementioned approaches are too complex for proper designing of the controllers. In fact, it is still a meaningful work to study the stability of nonlinear systems.

On the other hand, adaptive neural networks (NNs) or fuzzy logic systems (FLSs) control approaches are widely adopted for nonlinear systems identification and control, and many significant results have been obtained for single-input and single-output (SISO) nonlinear systems (Chen, Jiao, Li, & Li, 2010; Du, Shao, & Yao, 2006; Han, 2018a; Liu, Tong, Wang, Li, & Chen, 2011; Wang, Chen, & Lin, 2014; Wang, Liu, & Niu, 2018; Zhao, Zheng, Niu, & Liu, 2015), and multiple-input and multiple-output (MIMO) nonlinear systems (Ge & Wang, 2004; Li, Chen, & Li, 2011; Wang, Liu, & Liu, 2016; Wang, Liu, Qiu, & Liu, 2018; Zhang & Ge, 2007; Zhou, Shi, Liu, & Xu, 2012). In these developed control schemes, adaptive neural or fuzzy control approaches are constructed recursively in the framework of the backstepping approach, and NNs or FLSs are utilized to approximate unknown nonlinear functions. Despite that many interesting results have been achieved for nonlinear systems, due to their own shortcomings, the approximation-based adaptive neural (or fuzzy) control schemes haven't been widely used in control areas. For NNs, the reasons may be given as following (Han, 2018b): (a) there is a trade-off between NN complexity and its good approximation performance; (b) most neural networks can't be suitable for actual dynastic systems due to its neurons only have stated function; (c) the upper bounds of the NN approximation errors and the ideal NN weights are fictitious and unknown constants for a given NN, which further make control design difficult (Ge & Ren, 2007). For FLSs, the reasons may be given as following: (a) fuzzy control has the disadvantage of long computational and bad ability of convergent; (b) the lack of standard methods for transforming human knowledge into a rule base [35]. Therefore, it is still a meaningful and challenging task to construct a simple but effective adaptive control algorithm for the nonlinear systems, which motivates our research.

To handle the above the problems, Zhou and Yan (2013) first put forward the concept of multi-dimensional Taylor network (MTN). Subsequently, MTN has been widely used in nonlinear systems identification and control (Lin, Yan, & Zhou, 2014; Zhou & Yan, 2014a, 2014b). Compared with the NN, MTN has the following merits

(Han & Yan, 2018): (i) MTN has a much simpler structure than most NNs. (ii) MTN possesses better approximation ability than NN in the nonlinear system identification and prediction. (iii) MTN is of more desirable real-time performance. Based on the characteristics of MTN, MTN-approximation-based control schemes have been found to be particularly useful for the control of uncertain, complex, and stochastic systems, and many interesting results have been reported. For example, based on discrete MTN controller, Yan and Kang (2017) dealt with the problem of asymptotic tracking and dynamic regulation of SISO nonlinear systems. Later, Kang and Yan (2018) investigated the stability and dynamic regulation of the MTN controller for SISO nonlinear systems with time-varying delay. Yan, Sun, and Zhou (2018) proposed an MTN-based optimal control method for the real-time optimal tracking control of the SISO nonlinear constant system with modeling errors. Yan, Han, and Sun (2018) proposed a new MTN-based controller for SISO stochastic nonlinear systems. Han and Yan (2018) investigated the problem of adaptive MTN control for SISO uncertain stochastic nonlinear systems. Han (2018b) developed an adaptive MTN control scheme to solve the tracking control problem for the stochastic nonlinear system with immeasurable states. Although some achievements have been obtained, to the best of the authors' knowledge, the research of nonlinear systems control based on MTN is still at the initial stage, related theories and methods are not mature, especially for nonlinear systems with unknown control direction. In fact, it is still a challenging task to construct MTN control algorithm for nonlinear systems with unknown control direction.

Motivated by the above observation, this paper focuses on the problem of adaptive MTN tracking control for a class of nonlinear systems with unknown control direction. In the controller design, MTNs are used to approximate the unstructured uncertainties of systems, and then a new MTN-based adaptive tracking control scheme is developed via backstepping technique. It is shown that all the signals of the closed-loop remain bounded and the tracking error finally converges to a small neighborhood of the origin. The main contributions of this paper are summarised as follows:

(i) A novel adaptive MTN-based control approach is established for a class of nonlinear systems with unknown control direction, which means its system structure is more general than some existing systems, such as those in (Ren, Ge, Tee, & Lee, 2010; Wang & Huang, 2005; Zhou, Shi, Lu, & Xu, 2011).

(ii) The computation burden of the designed controller is significantly decreased in two ways: The computation complexity of MTN-based controller has the advantage of simple structure, which contains only addition and multiplication; The structure of MTN can be determined according to the number of inputs and the highest power of product terms of the middle layer. Thus, we may conclude that the proposed method can get precise tracking results with low computational cost, and have a good real-time performance and convergence.

(iii) In each step of backstepping, only a MTN is employed to approximate for all unknown functions, which different from the control idea in (Chen & Li, 2009; Huang, Tan, & Lee, 2009), and therefore, the design procedure presented is more simplified and the control can be easily realized.

The remainder of the paper is organized as follows. Section 2 provides some problem formulation and preliminaries. The MTN-based adaptive tracking control design procedure and stability analysis of the closed-loop system are given in Section 3. Section 4 gives one numerical example and one practical example to illustrate the effectiveness of our results. Section 5 concludes the work.

2 Problem formulation and preliminaries

The following notations are used throughout this paper. R denotes the set of all real numbers, R^n indicates the real n-dimensional space, $R^{n\times r}$ denotes the set of all $n\times r$ real matrices. For a given vector or matrix X, X^{T} stands for its transpose and $\|X\|$ denotes the Euclidean norm of a vector X; $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ denote the minimal eigenvalue and maximal eigenvalue of real matrix X, respectively. I_i is $i\times i$ identity matrix.

2.1 Problem descriptions

Consider the following nonlinear system

$$\begin{cases} \dot{x}_i = g_i(\bar{\boldsymbol{x}}_i)x_{i+1} + f_i(\bar{\boldsymbol{x}}_i) \\ \quad 1 \le i \le n-1 \\ \dot{x}_n = g_n(\bar{\boldsymbol{x}}_n)u + f_n(\bar{\boldsymbol{x}}_n) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{\boldsymbol{x}}_n = [x_1, \cdots, x_n]^{\mathrm{T}} \in R^n$ is the system state, $y \in R$ is the system output, $u \in R$ is the control input, $\bar{\boldsymbol{x}}_i = [x_1, \cdots, x_i]^{\mathrm{T}} \in R^i$. $f_i(\cdot): R^i \to R$ and $g_i(\cdot): R^i \to R$ are unknown smooth functions with $f_i(\mathbf{0}) = 0$.

The objective of this paper is to design an adaptive controller for system (1), such that the system output y tracks the given reference signal y_d, and all the signals in the closed-loop systems remain bounded.

The results of this paper are based on the following Assumptions.

Assumption 2.1: All the state variables of system (1) are assumed to be available and measurable.

Assumption 2.2: For any $i = 1, \cdots, n$, the sign of $g_i(\bar{\boldsymbol{x}}_i)$ does not change, without loss of generality, we further assume that there exist two known constants $\underline{g}$ and $\bar{g}$, such that

$$0 < \underline{g} \le g_i(\bar{\boldsymbol{x}}_i) \le \bar{g}, \forall \bar{\boldsymbol{x}}_i \in R^i \tag{2}$$

Assumption 2.3: The given reference signal y_d and its time derivatives up to the n-th order are continuous and bounded.

Lemma 2.1 (Deng & Krstic, 1997)**:** For $\forall (x, y) \in R^2$ and $\varepsilon > 0$, the following inequality holds

$$xy \le \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q \tag{3}$$

where p and q are constants and satisfy $p > 1$, $q > 1$ and $(p-1)(q-1) = 1$.

2.2 Multi-dimensional Taylor Network

MTN is a three-layer network structure: input layer, middle layer and output layer, its topology structure is shown in Figure 1:

[*Figure 1 near here*]

where $s_1,\cdots,s_n$ are the input of MTN. The middle layer is the polynomial combination of inputs, and maps the input space to a new space, and its mathematical expression is given by following equation

$$f_{\mathrm{MTN}}(\boldsymbol{s})=\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{P}_{m_n}(\boldsymbol{s}) \tag{4}$$

where $\boldsymbol{s}=[s_1,\cdots,s_n]^{\mathrm{T}}\in\Omega_n\subset R^n$, $\boldsymbol{\theta}=[\theta_1,\cdots,\theta_l]^{\mathrm{T}}\subset R^l$ are input vector and weight vector of MTN, respectively. The vector $\boldsymbol{P}_{m_n}(\boldsymbol{s})$ is constructed in the following form

$$P_{m_n}(\boldsymbol{s})=[\underbrace{s_1,\cdots,s_n}_{1\,term},\underbrace{s_1^2,\cdots,s_n^2}_{2\,term},\cdots,\underbrace{s_1^m,\cdots,s_n^m}_{m\,term}]^{\mathrm{T}}\subset R^l \tag{5}$$

where n and l are the number of input dimensions and middle layer of MTN, respectively.

Remark 2.1: According to (3), the elements of vector $\boldsymbol{P}_{m_n}(\boldsymbol{s})$ are $\prod_{i,j=1}^{n}s_i^{\sigma_i}s_j^{\sigma_j}$, where σ_i and σ_j are nonnegative integers and satisfy $1\le\sigma_i+\sigma_j\le m$.

Remark 2.2: The difference between MTN and NN has been discussed in detail in our recent work (Han & Yan, 2018).

Lemma 2.2 (Han & Yan, 2018)**:** Assume that $\varphi(\boldsymbol{s})$ is a continuous function defined on a compact set Ω_s. Then, for any given desired level of accuracy $\varepsilon>0$, there exists a MTN, such that

$$\varphi(\boldsymbol{s})=\boldsymbol{\theta}^{*\mathrm{T}}\boldsymbol{P}_{n_m}(\boldsymbol{s})+\delta(\boldsymbol{s}) \tag{6}$$

where $\boldsymbol{\theta}^*$ is the ideal constant weight vector and defined as

$$\boldsymbol{\theta}^* := \arg\min_{\theta \in R^l} \left\{ \sup_{s \in \Omega_s} \left| \varphi(\boldsymbol{s}) - \boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{P}_{m_n}(\boldsymbol{s}) \right| \right\}$$

where $\delta(\boldsymbol{s})$ denotes the approximation error and satisfies $|\delta(\boldsymbol{s})| \le \varepsilon$.

3 Adaptive MTN design

In this section, an MTN-backstepping-based control design procedure will be developed based on the following coordinate transformation:

$$\begin{aligned} z_1 &= x_1 - y_d \\ z_i &= x_i - \alpha_{i-1}, i = 2, \cdots, n \end{aligned} \tag{7}$$

where $\alpha_{i-1}\ (i = 2, \cdots, n)$ are the intermediate control signals to be designed.

Combining (1) with (7) gives

$$\begin{cases} \dot{z}_1 = g_1(\overline{\boldsymbol{x}}_1) x_2 + f_1(\overline{\boldsymbol{x}}_2) - \dot{y}_d \\ \dot{z}_i = g_i(\overline{\boldsymbol{x}}_i) x_{i+1} + f_i(\overline{\boldsymbol{x}}_i) - \dot{\alpha}_{i-1} \\ 2 \le i \le n-1 \\ \dot{z}_n = g_n(\overline{\boldsymbol{x}}_n) u + f_n(\overline{\boldsymbol{x}}_n) - \dot{\alpha}_{n-1} \end{cases} \tag{8}$$

where $\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \dot{x}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\boldsymbol{\theta}}_j} \dot{\hat{\boldsymbol{\theta}}}_j + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)}$.

In the following, for simplicity, $f_i(\overline{\boldsymbol{x}}_i)$, $g_i(\overline{\boldsymbol{x}}_i)$, $\boldsymbol{P}_{m_i}(\boldsymbol{z}_i)$ and $\delta_i(\boldsymbol{z}_i)$ are abbreviated as f_i, g_i, $\boldsymbol{P}_{n_m}$ and δ_i, respectively.

3.1 Design of MTN-based controller

Step 1: Consider the following Lyapunov function

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}} \Gamma_1^{-1} \tilde{\boldsymbol{\theta}}_1 \tag{9}$$

where $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$ is the parameter error and $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$ is symmetric positive definite matrix.

The time-derivative of V_1 is given by

$$\begin{aligned}\dot{V}_1 &= z_1\left(g_1 x_2 + f_1 - \dot{y}_d\right) - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \\ &= z_1 g_1 x_2 + z_1\tilde{f}_1 - \frac{1}{2}z_1^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1\end{aligned} \tag{10}$$

where $\tilde{f}_1 = f_1 - \dot{y}_d + \frac{1}{2}z_1$.

By virtue of Lemma 2.2, an MTN is used to approximate the unknown function $\tilde{f}_1$, that is to say, for any given constant $\varepsilon_1 > 0$, there exists an MTN $\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{P}_{m_1}\left(\boldsymbol{z}_1\right)$, such that

$$\tilde{f}_1 = \boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{P}_{m_1}\left(\boldsymbol{z}_1\right) + \delta_1\left(\boldsymbol{z}_1\right), \left|\delta_1\left(\boldsymbol{z}_1\right)\right| \le \varepsilon_1 \tag{11}$$

where $\delta_1\left(\boldsymbol{z}_1\right)$ is the approximation error and $\boldsymbol{z}_1 = \left[z_1\right]^{\mathrm{T}}$.

Taking the intermediate control signal α_1 as

$$\alpha_1 = -\frac{1}{\underline{g}}\left(r_1\left|z_1\right| + \left|\hat{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{P}_{m_1}\right|\right)\operatorname{sgn}\left(z_1\right) \tag{12}$$

where $r_1 > 0$ is a design constant.

Combining Lemma 2.1 with (11), (12) and $x_2 = z_2 + \alpha_1$ gives

$$z_1\tilde{f}_1 \le z_1\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{P}_{m_1} + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2 \tag{13}$$

$$z_1 g_1 x_2 \le z_1 g_1 z_2 - r_1 z_1^2 - \left|z_1\hat{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{P}_{m_1}\right| \tag{14}$$

Substituting (13) and (14) into (10) gives

$$\dot{V}_1 \le z_1 g_1 z_2 - r_1 z_1^2 + \frac{1}{2}\varepsilon_1^2 + \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\left(z_1\boldsymbol{P}_{m_1} - \Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1\right) \tag{15}$$

Step $2 \le k \le n-1$***:*** At this step, we can obtain the similar property to (15) for the system (1), such a result is presented by the following lemma.

Lemma 3.1: For every $i = 2, \cdots, n-1$, consider following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i \tag{16}$$

we have

$$\dot{V}_i \le \sum_{j=1}^{i} z_j g_j z_{j+1} - \sum_{j=1}^{i} r_j z_j^2 + \frac{1}{2}\sum_{j=1}^{i}\varepsilon_j^2 + \sum_{j=1}^{i}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j\boldsymbol{P}_{m_j} - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right) \tag{17}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ are parameter errors, $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ are symmetric positive definite matrices and $r_i > 0$ are positive design parameters, respectively.

Proof: We demonstrate Lemma 3.1 by induction. Assume that Lemma 3.1 is true for $i-1$, then, we have

$$\dot{V}_{i-1} \le \sum_{j=1}^{i-1} z_j g_j z_{j+1} - \sum_{j=1}^{i-1} r_j z_j^2 + \frac{1}{2}\sum_{j=1}^{i-1}\varepsilon_j^2 + \sum_{j=1}^{i-1}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j \boldsymbol{P}_{m_j} - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right) \tag{18}$$

we will show that Lemma 3.1 is still true for i.

According (17), the time-derivative of V_i is given by

$$\dot{V}_i = \dot{V}_{i-1} + z_i g_i x_{i+1} + z_i \tilde{f}_i - \frac{1}{2}z_i^2 - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i \tag{19}$$

where $\tilde{f}_i = f_i - \dot{\alpha}_{i-1} + \frac{1}{2}z_i$.

Similarly, by virtue of Lemma 2.2, for any given constant $\varepsilon_i > 0$, there exists an MTN $\boldsymbol{\theta}_i^{\mathrm{T}}\boldsymbol{P}_{m_i}(\boldsymbol{z}_i)$, such that

$$\tilde{f}_i = \boldsymbol{\theta}_i^{\mathrm{T}}\boldsymbol{P}_{m_i}(\boldsymbol{z}_i) + \delta_i(\boldsymbol{z}_i), \quad \left|\delta_i(\boldsymbol{z}_i)\right| \le \varepsilon_i \tag{20}$$

where $\delta_i(\boldsymbol{z}_i)$ is the approximation error and $\boldsymbol{z}_i = [z_1, \cdots, z_i]^{\mathrm{T}}$.

Taking the intermediate control signal α_i as

$$\alpha_i = -\frac{1}{\underline{g}}\left(r_i|z_i| + \left|\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{P}_{m_i}\right|\right)\mathrm{sgn}(z_i) \tag{21}$$

Combining Lemma 2.1 with (20), (21) and $x_{i+1} = z_{i+1} + \alpha_i$ gives

$$z_i \tilde{f}_i \le z_i \boldsymbol{\theta}_i^{\mathrm{T}}\boldsymbol{P}_{m_i} + \frac{1}{2}z_i^2 + \frac{1}{2}\varepsilon_i^2 \tag{22}$$

$$z_i g_i x_{i+1} \le z_i g_i z_{i+1} - r_i z_i^2 - \left|z_i \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{P}_{m_i}\right| \tag{23}$$

Substituting (22) and (23) into (19) gives

$$\dot{V}_i \le \sum_{j=1}^{i} z_j g_j z_{j+1} - \sum_{j=1}^{i} r_j z_j^2 + \frac{1}{2}\sum_{j=1}^{i}\varepsilon_j^2 + \sum_{j=1}^{i}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j \boldsymbol{P}_{m_j} - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right) \tag{22}$$

Therefore, the proof of Lemma 3.1 is completed.

Step n***:*** Consider the following Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}} \Gamma_n^{-1} \tilde{\boldsymbol{\theta}}_n \tag{23}$$

where $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n - \hat{\boldsymbol{\theta}}_n$ is the parameter error and $\Gamma_n = \Gamma_n^{\mathrm{T}} > 0$ is symmetric positive definite matrix.

According (23), the time-derivative of V_n is given by

$$\dot{V}_n = \dot{V}_{n-1} + z_n g_n u + z_n \tilde{f}_n - \frac{1}{2} z_n^2 - \tilde{\boldsymbol{\theta}}_n^T \Gamma_n^{-1} \dot{\hat{\boldsymbol{\theta}}}_n \tag{24}$$

where $\tilde{f}_n = f_n - \dot{\alpha}_{n-1} + \frac{1}{2} z_n$.

Similarly, by virtue of Lemma 2.2, for any given constant $\varepsilon_n > 0$, there exists a MTN $\boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{P}_{m_n}(\boldsymbol{z}_n)$, such that

$$\tilde{f}_n = \boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{P}_{m_n}(\boldsymbol{z}_n) + \delta_n(\boldsymbol{z}_n), \left|\delta_n(\boldsymbol{z}_n)\right| \le \varepsilon_n \tag{25}$$

where $\delta_n(\boldsymbol{z}_n)$ is the approximation error and $\boldsymbol{z}_n = [z_1, \cdots, z_n]^{\mathrm{T}}$.

Taking the control input u as

$$u = -\frac{1}{\underline{g}} \left(r_n |z_n| + \left| \hat{\boldsymbol{\theta}}_n^{\mathrm{T}} \boldsymbol{P}_{m_n} \right| \right) \mathrm{sgn}(z_n) \tag{26}$$

Combining Lemma 2.1 with (25) and (26) gives

$$z_n \tilde{f}_n \le z_n \boldsymbol{\theta}_n^{\mathrm{T}} \boldsymbol{P}_{m_n} + \frac{1}{2} z_n^2 + \frac{1}{2} \varepsilon_n^2 \tag{27}$$

$$z_n g_n u \le -r_n z_n^2 - \left| z_n \hat{\boldsymbol{\theta}}_n^{\mathrm{T}} \boldsymbol{P}_{m_n} \right| \tag{28}$$

Now, substituting (22) with $i = n-1$, (27) and (28) into (24) gives

$$\dot{V}_n \le \sum_{j=1}^{n-1} z_j g_j z_{j+1} - \sum_{j=1}^{n} r_j z_j^2 + \frac{1}{2} \sum_{j=1}^{n} \varepsilon_j^2 + \sum_{j=1}^{n} \tilde{\boldsymbol{\theta}}_j^{\mathrm{T}} \left(z_j \boldsymbol{P}_{m_j} - \Gamma_j^{-1} \dot{\hat{\boldsymbol{\theta}}}_j \right) \tag{29}$$

3.2 Stability Analysis

The main results of this paper can be summarized by the following theorem.

Theorem 3.1: Under Assumptions 2.1-2.3, consider the nonlinear system (1), if the control law is chosen as

$$u = -\frac{1}{\underline{g}} \left(r_n |z_n| + \left| \hat{\boldsymbol{\theta}}_n^{\mathrm{T}} \boldsymbol{P}_{m_n} \right| \right) \mathrm{sgn}(z_n) \tag{30}$$

and the intermediate control signals described as

$$\alpha_j = -\frac{1}{\underline{g}}\left(r_j\left|z_j\right| + \left|\hat{\boldsymbol{\theta}}_j^{\mathrm{T}}\boldsymbol{P}_{m_j}\right|\right)\operatorname{sgn}\left(z_j\right) \tag{31}$$

and the adaptive laws defined as

$$\dot{\hat{\boldsymbol{\theta}}}_j = -\eta_j\Gamma_j\hat{\boldsymbol{\theta}}_j + z_j\Gamma_j\boldsymbol{P}_{m_j} \tag{32}$$

where $r_j > 0$ and $\eta_j > 0$ are designed parameters, $\Gamma_j = \Gamma_j^{\mathrm{T}} > 0$ are designed matrices. Then, for any bounded initial conditions, the tracking error finally converges to a small neighborhood of the origin and all signals of the closed-loop system remain bounded.

Proof: Choose the following Lyapunov function for the system as

$$V = \frac{1}{2}\sum_{j=1}^{n} z_j^2 + \frac{1}{2}\sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\Gamma_j^{-1}\tilde{\boldsymbol{\theta}}_j \tag{33}$$

It follows from (29), one has

$$\dot{V} \le \sum_{j=1}^{n-1} z_j g_j z_{j+1} - \sum_{j=1}^{n} r_j z_j^2 + \frac{1}{2}\sum_{j=1}^{n}\varepsilon_j^2 + \sum_{j=1}^{n}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\left(z_j\boldsymbol{P}_{m_j} - \Gamma_j^{-1}\dot{\hat{\boldsymbol{\theta}}}_j\right) \tag{34}$$

By Lemma 2.1, the following inequality holds

$$\sum_{j=1}^{n-1} z_j g_j z_{j+1} \le \bar{g}\sum_{j=1}^{n} z_j^2 \tag{35}$$

Substituting (32) and (35) into (34) gives

$$\dot{V} \le -\sum_{j=1}^{n}\left(r_j - \bar{g}\right)z_j^2 + \frac{1}{2}\sum_{j=1}^{n}\varepsilon_j^2 + \sum_{j=1}^{n}\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j \tag{36}$$

with

$$\begin{aligned}\sum_{j=1}^{n}\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\hat{\boldsymbol{\theta}}_j &\le -\frac{1}{2}\sum_{j=1}^{n}\eta_j\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_j + \frac{1}{2}\sum_{j=1}^{n}\eta_j\left\|\boldsymbol{\theta}_j\right\| \\ &\le -\frac{1}{2}\sum_{j=1}^{n}\frac{\eta_j}{\lambda_{\max}\left(\Gamma_j^{-1}\right)}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\Gamma_j^{-1}\tilde{\boldsymbol{\theta}}_j + \frac{1}{2}\sum_{j=1}^{n}\eta_j\left\|\boldsymbol{\theta}_j\right\|\end{aligned} \tag{37}$$

It follows from (36) and (37), one has

$$\dot{V} \le -\sum_{j=1}^{n}c_j z_j^2 - \frac{1}{2}\sum_{j=1}^{n}\frac{\eta_j}{\lambda_{\max}\left(\Gamma_j^{-1}\right)}\tilde{\boldsymbol{\theta}}_j^{\mathrm{T}}\Gamma_j^{-1}\tilde{\boldsymbol{\theta}}_j + \frac{1}{2}\sum_{j=1}^{n}\varepsilon_j^2 + \frac{1}{2}\sum_{j=1}^{n}\eta_j\left\|\boldsymbol{\theta}_j\right\| \tag{38}$$

where $c_j = r_j - \bar{g} > 0$, $\underline{\eta}_j = \min\left\{ \frac{\eta_j}{\lambda_{\max}\left(\Gamma_j^{-1}\right)} \right\}$, $j = 1, \cdots, n$.

Let $a_0 = \min\left\{2c_j, \underline{\eta}_j \mid j = 1, \cdots, n\right\}$ and $b_0 = \frac{1}{2}\sum_{j=1}^{n} \varepsilon_j^2 + \frac{1}{2}\sum_{j=1}^{n} \eta_j \left\|\boldsymbol{\theta}_j\right\|$, yields

$$\dot{V} \le -a_0 V + b_0 \tag{39}$$

Solving inequality (39) gives

$$0 \le V(t) \le \left(V(0) - \frac{a_0}{b_0} \right) e^{-b_0 t} + \frac{a_0}{b_0} \tag{40}$$

From inequality (40), we can get the conclusions that all signals of the resulting closed-loop system remain bounded and the tracking error finally converges to a small neighborhood of the origin.

The proof of Theorem 3.1 is completed.

4 Simulation examples

In this section, three examples used to illustrate the effectiveness of the proposed control method.

Example 1 (Numerical example): Consider the following third-order nonlinear system

$$\begin{cases} \dot{x}_1 = \left(1.2 + \sin x_1^2\right) x_2 + 0.5 x_1^2 e^{-x_1} \\ \dot{x}_2 = \left(1.5 - \sin x_1 x_2\right) x_3 + x_1 \cos x_1 x_2^2 \\ \dot{x}_3 = \left(1 + \dfrac{x_2^2}{0.1 + x_2^2 + x_3^2}\right) u + x_1 x_2 x_3^3 \\ y = x_1 \end{cases} \tag{41}$$

with the initial states $x_1 = 0.01$, $x_2 = 0.01$ and $x_3 = 0.01$. The given reference signal $y_d = 0.5 \sin t$.

According to Theorem 3.1, the intermediate control signals, the actual control law, and the adaptive laws are defined as

$$\alpha_i = -\frac{1}{\underline{g}}\left(r_i \left|z_i\right| + \left|\hat{\boldsymbol{\theta}}_i^{\mathrm{T}} \boldsymbol{P}_{m_i}\left(\boldsymbol{z}_i\right)\right| \right) \operatorname{sgn}\left(z_i\right), i = 1, 2$$

$$u=-\frac{1}{\underline{g}}\left(r_3\left|z_3\right|+\left|\hat{\boldsymbol{\theta}}_3^{\mathrm{T}}\boldsymbol{P}_{m_3}\left(\boldsymbol{z}_3\right)\right|\right)\operatorname{sgn}\left(z_3\right)$$

$$\dot{\hat{\boldsymbol{\theta}}}_i=-\eta_i\Gamma_i\hat{\boldsymbol{\theta}}_i+z_i\Gamma_i\boldsymbol{P}_{m_i}\left(\boldsymbol{z}_i\right),\, i=1,2,3$$

where $z_1=x_1-y_d$, $z_2=x_2-\alpha_1$, $z_3=x_3-\alpha_2$ and $\boldsymbol{z}_1=\left[z_1\right]^{\mathrm{T}}$, $\boldsymbol{z}_2=\left[z_1,z_2\right]^{\mathrm{T}}$, $\boldsymbol{z}_3=\left[z_1,z_2,z_3\right]^{\mathrm{T}}$.

The design parameters are taken as follows: $\underline{g}=0.2$, $r_1=3$, $\eta_1=4$, $\Gamma_1=0.5I_5$, $r_2=3$, $\eta_2=2$, $\Gamma_2=10I_{19}$, $r_3=3$, $\eta_3=0.1$, and $\Gamma_3=5I_{19}$. The simulation results are shown in Figures 2-5, respectively. Figure 2 shows the system output y and the reference signal y_d, it can be seen that the good tracking performance has been achieved. Figure 3 displays the control signal u. Figure 4 show that the state variables x_2 and x_3 are bounded. Figure 5 depicts the response of the tracking error $y-y_d$. From Figures 2-5, it can be seen that the system output y tracks the reference signal y_d well, and all the closed-loop signals are all remain bounded.

[*Figures 2-5 near here*]

Example 2 (Practical example): Consider a rigid robot manipulator system, which can be described as follows (Feliu, Rattan, & Brown, 1993; Liu, Wang, Gao, & Chen, 2017):

$$\begin{cases}\dot{x}_1=x_2\\ x_2=\dfrac{1}{J}u-\dfrac{m_r g_v l_r}{J}\cos x_1\\ y=x_1\end{cases}\tag{42}$$

where x_1 is the angular position of manipulator, x_2 is the relative angular velocity, m_r is the load mass, g_v is the gravity, l_r is the length of manipulator, and $J=4m_r l_r^2/3$ is the inertia coefficient. The given reference signal

$y_d = 0.5(\sin t + \sin(0.5t))$.

Similarly, according to Theorem 3.1, the intermediate control signals, the actual control law, and the adaptive laws are defined as

$$\alpha_1 = -\frac{1}{\underline{g}}\left(r_1|z_1| + \left|\hat{\boldsymbol{\theta}}_1^{\mathrm{T}}\boldsymbol{P}_{m_1}(\boldsymbol{z}_1)\right|\right)\operatorname{sgn}(z_1)$$

$$u = -\frac{1}{\underline{g}}\left(r_2|z_2| + \left|\hat{\boldsymbol{\theta}}_2^{\mathrm{T}}\boldsymbol{P}_{m_2}(\boldsymbol{z}_2)\right|\right)\operatorname{sgn}(z_2)$$

$$\dot{\hat{\boldsymbol{\theta}}}_i = -\eta_i\Gamma_i\hat{\boldsymbol{\theta}}_i + z_i\Gamma_i\boldsymbol{P}_{m_i}(\boldsymbol{z}_i), i = 1,2$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$ and $\boldsymbol{z}_1 = [z_1]^{\mathrm{T}}$, $\boldsymbol{z}_2 = [z_1, z_2]^{\mathrm{T}}$.

In simulation, the model parameters are set as $m_r = 5$, $g_v = 9.8$ and $l_r = 0.25$; The design parameters are taken as follows: $\underline{g} = 0.2$, $r_1 = 7$, $\eta_1 = 4$, $\Gamma_1 = 0.5I_5$, $r_2 = 3$, $\eta_2 = 0.1$ and $\Gamma_2 = 5I_9$. The simulation is run with the initial condition $[x_1(0), x_2(0)]^T = [0,0]^T$. The simulation results are shown in Figures 6-9, respectively. It can be seen that the tracking control performance is still fairly satisfactory, which further verify the effectiveness of the adaptive MTN control approach proposed in this paper.

[*Figures 6-9 near here*]

Remark 4.1: From examples 4.1-4.2, it can be seen that the good tracking performances can be achieved by suitably choosing the design parameters. Generally speaking, the tracking error converges to a small residual set around the origin by properly adjusting the parameters r_i, η_i and matrices Γ_i.

Example 3 (Contrast simulation): To further demonstrate the advantages of our proposed method, a contrasting experiment has been done in this section. We select radial basis function neural network (RBFNN) controller as a contrasting reference.

Next, three MTNs are replaced by three RBFNNs, respectively, and all the control parameters are kept as that in Example 1. The simulation result is shown Figure 10.

[*The position of Figure 10*]

Figure 10 shows the tracking performance of two control strategies. As we see from Figure 10, though both MTN-based controller and RBFNN-based controller can well realize the tracking control, the former promises more satisfactory performance than the latter. Contrasting experiment results further verify the effectiveness of our proposed method.

5 Conclusions

In this paper, an adaptive MTN-based control approach based on backstepping design is proposed for a class of nonlinear systems with unknown control direction. By employing MTNs to estimate the nonlinearities of system, a new adaptive MTN controller is proposed to guarantee that all the signals in the closed-loop system remain bounded and the tracking error finally converges to a small neighborhood of the origin.

There exist several topics left to be studied, e.g., how to extend the proposed approach in this paper to other types of nonlinear systems (such as nonlinear systems with input saturation, switched nonlinear systems, stochastic nonlinear systems with time-delay systems and so on). These topics will be considered in the following study.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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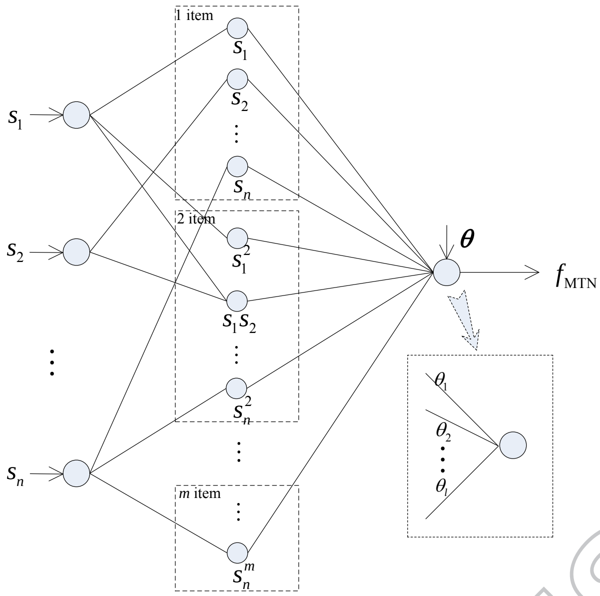

Figure 1. The structure of MTN.

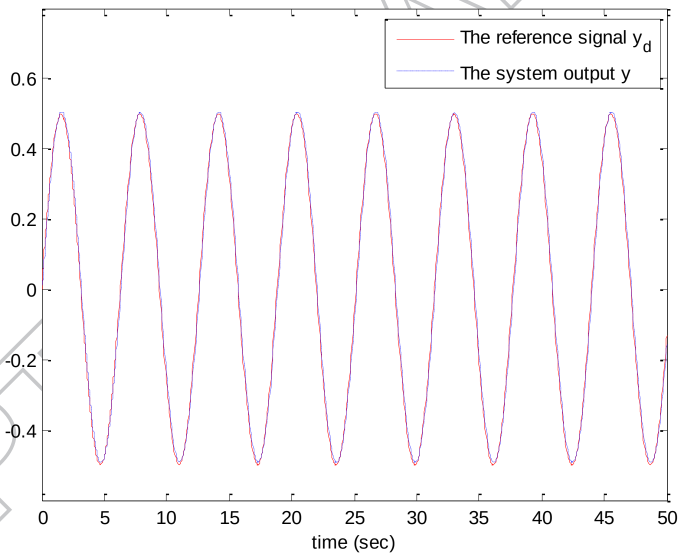

Figure 2. System output y and reference signal y_d of Example 1.

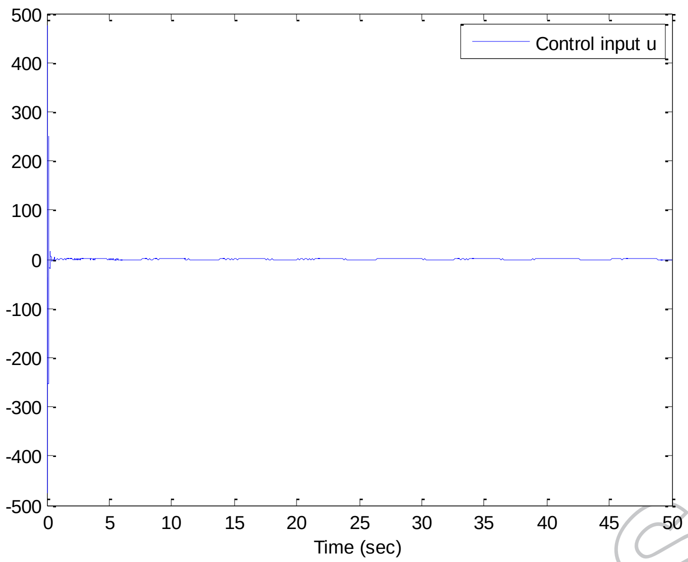

Figure 3. The true control input u of Example 1.

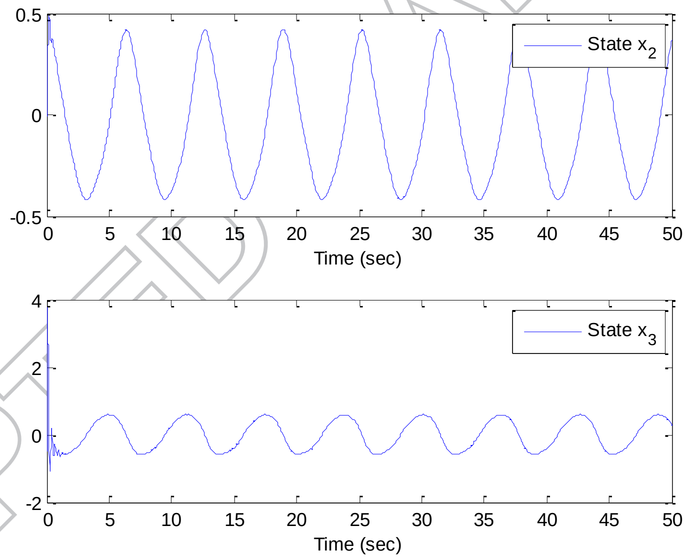

Figure 4. State variables x_2 and x_3 of Example 1.

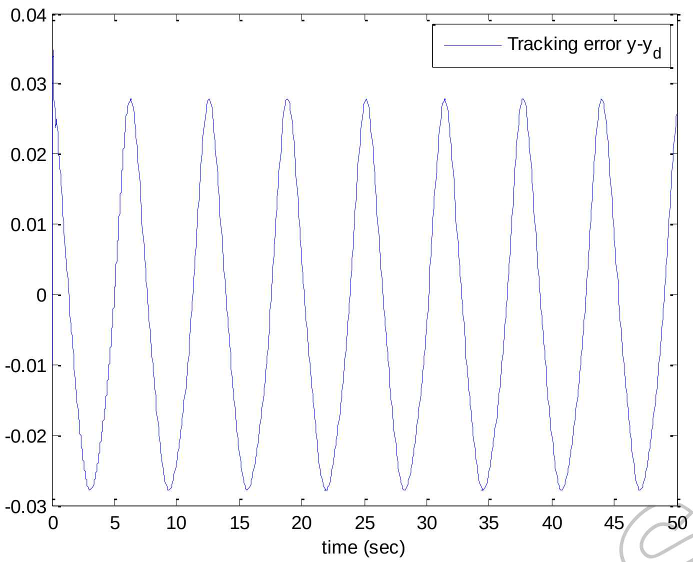

Figure 5. Tracking error of Example 1.

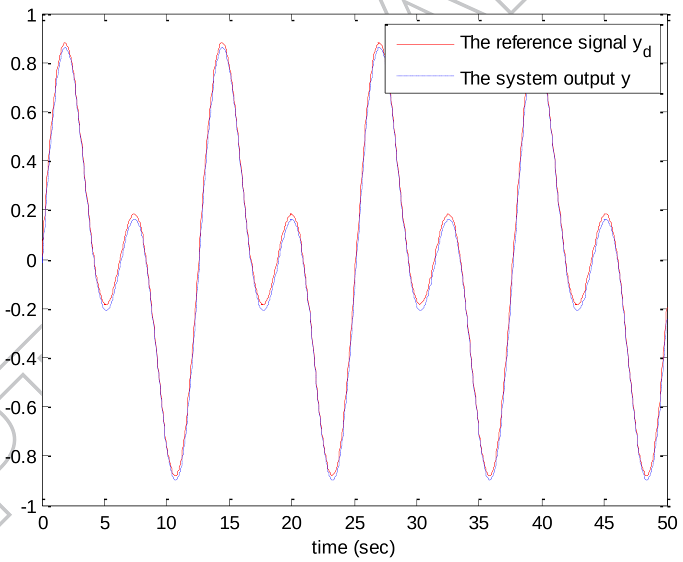

Figure 6. System output y and reference signal y_d of Example 2.

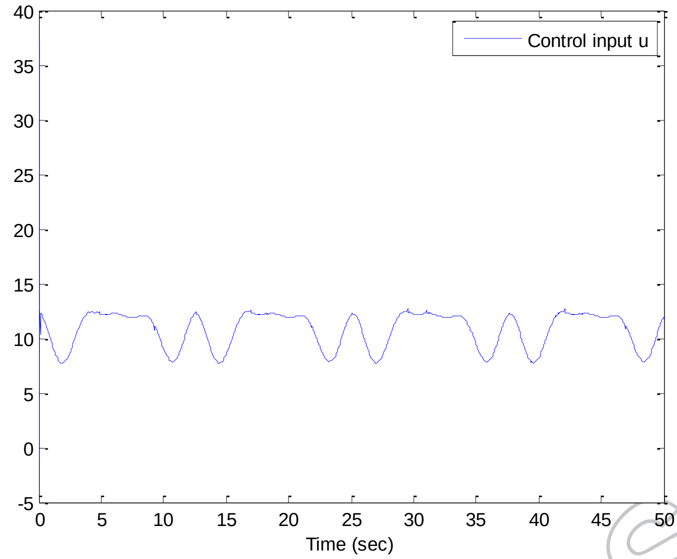

Figure 7. The true control input u of Example 2.

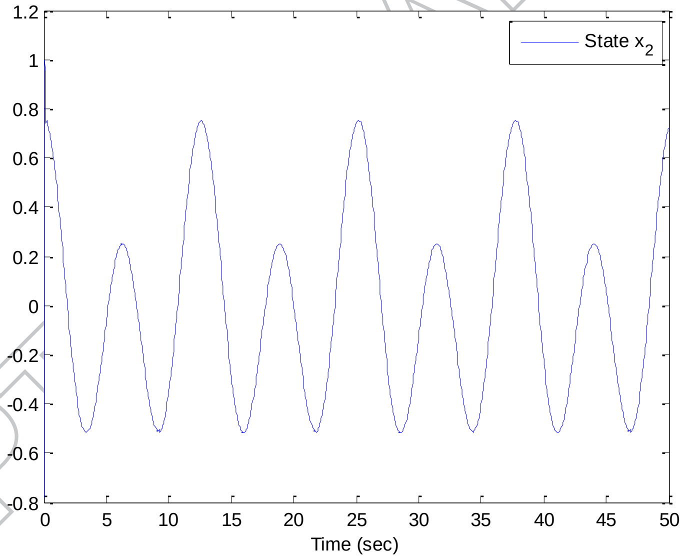

Figure 8. State variable x_2 of Example 2.

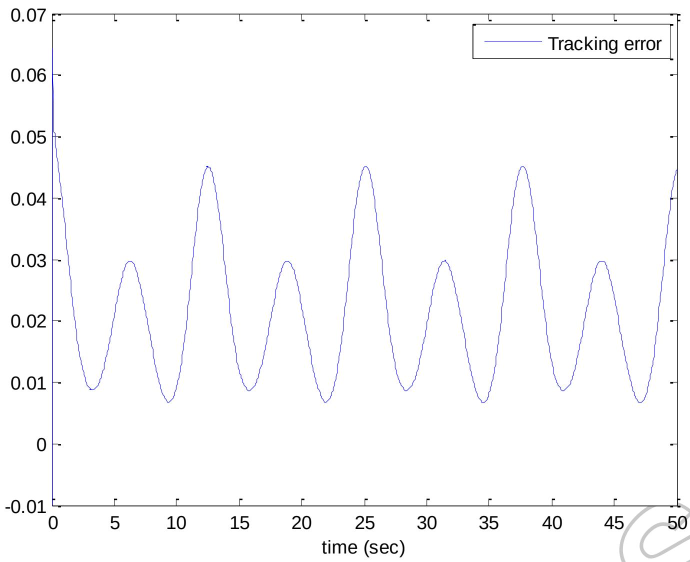

Figure 9. Tracking error of Example 2.

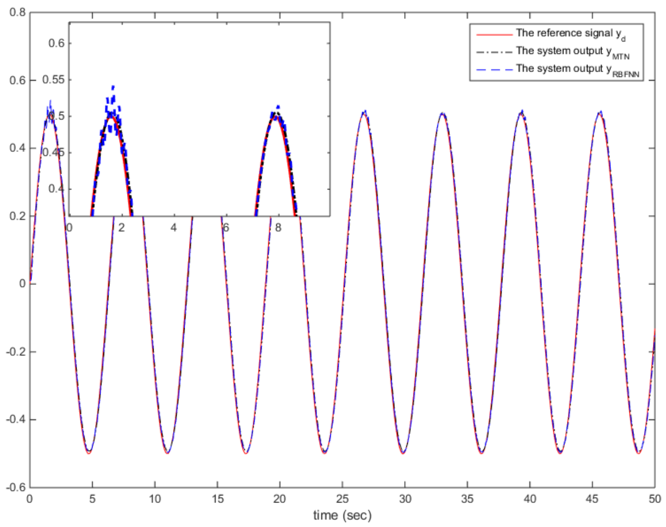

Figure 10. Comparison curves of tracking